

LOAD SHARING ANALYSIS OF BENDING STRENGTH IN ALTERED TOOTH-SUM GEARS OPERATING BETWEEN A STANDARD CENTER DISTANCE AND MODULE

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ABSTRACT

Power transmission between shafts that need positive drive is well assured by gears. The size and shape of the gear tooth is a critical consideration if specific drive requirements are imposed. Thus, design of gears calls for a detailed study of the tooth geometry that defines the performances like load carrying capacity, wear characteristics, efficiency and noise. Among them load carrying capacity is one of the fundamental requirements which depends on bending strength that can be computed using well known Lewis equation. Most often, tooth geometry is modified using S-gearing to influence the performance that may be either S_o or S_{\pm} type. Tooth geometry can also be modified by way of altering the tooth-sum of meshing gears for a standard center distance and module. Such modifications have some inherent benefits that are unique. Hence, this study is focused on analysis of bending stress as well as contact ratio in altered tooth-sum gears using load sharing diagram. The diagram clearly shows the loading pattern and contact length for both standard and altered tooth-sum gears at different points of contact. The load sharing analysis is carried out for altered tooth-sum spur gears having tooth-sum 100, 2 mm module, face width 10 mm and 20° pressure angle with a tangential load of 9.81N/mm of face width.

Keywords: Altered tooth-sum, Bending strength, Load sharing, Profile shift, Spur gears.

Abbreviations:

STS-Standard tooth-sum

ATS-Altered tooth-sum

BS-Bending stress

HPSTC-Highest point of single tooth contact

BS-HPSTC- Bending stress for load at
HPSTC

BS-Tip- Bending stress for load at tip

SPA/WPA Standard/Working Pressure angle

CR-Contact ratio

NCR/HCR-Normal/High contact ratio

GR-Gear ratio

Nomenclature:

Z_1/Z_2 -Number of teeth on standard
pinion/gear

Z_e -Number of teeth altered.

Z_1'/Z_2' -Altered number of teeth on
pinion/gear.

Z_s/Z_s' - Standard tooth-sum /Altered tooth-
sum.

A,B1,B2,C,D2,D1 and E – Salient points of
contact

α/α_e –Standard pressure angle/Operating
pressure angle.

X_e -Total profile shift.

X_1/X_2 - profile shift coefficient on
pinion/gear.

y_m -Tooth topping (mm).

INTRODUCTION

Gears are immense elements used worldwide in vast mechanical devices ranging from domestic to heavy engineering and aerospace applications. The design features built in the gears rule its operating characteristics. The power that a gear can transmit depends on the maximum permissible tooth load during mesh. Modern day industry requires gears that can transmit huge power while still maintaining high performance. To achieve this requirement, it is necessary to design gears in which low tooth stresses are induced. For increasing the load carrying capacity of gears, apart from surface treatments, several alternatives are used most of which involves modification of tooth geometry by altering the pressure angle, introducing profile shift, fillet optimization, tip modification, asymmetric tooth design etc. A stronger gear is one in which less tooth bending stress is induced. The common practice of modifying the tooth geometry is to use different portions of involute to make a stronger gear tooth which is known as profile shift, profile correction or addendum modification. Profile shift is used when small numbers of teeth have to be generated in order to overcome the problems of interference and undercutting. Such modifications are done by S-gearing which may be either S_0 or S_{\pm} gearing. Among them S_0 gearing is widely practiced, S_{\pm} gearing being seldom.

Understanding the effort of earlier researchers regarding to the bending stress in gear tooth helps this study in different ways. The practical advantage of profile shifting is in avoiding the tooth contact below the base circle or at the beginning of the involute profile. The amount of profile shift required, its calculation and use have been discussed in [1-2]. Maag [3] was the first to use the principle of generating involute tooth using rack type cutter, he was also first to generate the involute gear tooth having profile shift. Buckingham [4] suggests that load on the gear tooth should be considered as acting at the tip for computing the bending stress as this consideration takes care of any misalignments. Altering the tooth-sum and its effect on few gear parameters have been discussed in detail by Gonsalvis [5]. Dr. Joseph Gonsalvis and Sachidananda. H.K [6] has shown that the contact stresses for altered tooth-sum gears are much lower than standard tooth-sums gears. Dr. Joseph Gonsalvis and H.R.Prakash [7] report that minimum power loss is observed in altered tooth-sum gears so that power transmission can be maximized when negative teeth alterations are considered. A.R.Rajesh et.al [8] has studied the influence of altering the tooth-sum on bending stress by plotting the induced bending stress against profile shift co-efficient for different loading locations along the tooth profile. It is reported that the bending stress is reduced by 35.28% with negative teeth alteration having positive profile shift while the contact ratio is increased by 24% with positive teeth alteration having negative profile shift. M.Rameshkumar et.al [9] has performed load sharing analysis and comparison of NCR and HCR gears in terms of percentage load shared from root to tip of a gear tooth. Here, load distribution is plotted with respect to rotation angle for a single tooth from load at root to load at tip. The load sharing diagrams clearly show the point of maximum bending stress on the tooth profile, i.e., HPSTC. It is reported that the load carrying capacity of HCR gears is 18% more than NCR gears having same weight and volume for fixed module and center distance. P.Marimuthu et.al [10] has studied the influence of different gear parameters such as addendum height, tooth number and module on load sharing aspect followed by stress analysis. As per the load sharing analysis, increase in addendum height increases bending stress due to increased moment arm.

OBJECTIVES

After a thorough review of available literature, the objectives of this study are identified and enlisted as below:

- i. To study the tooth profile of standard gears and profile shifted gears.
- ii. To understand profile shifting by way of altering the tooth-sum in the gear pair.
- iii. To compute the total amount of profile shift resulting from altering the tooth-sum.
- iv. To study the effects of altering the tooth-sum on bending stress and contact ratio.
- v. To identify the optimum profile shift for each teeth alteration and its judicious distribution.
- vi. Analysis of bending stress and contact ratio in STS and ATS gears using comparative load sharing diagrams.
- vii. Evaluation of the outcome of altered tooth-sum gearing and draw conclusions for better gear design.

ALTERED TOOTH-SUM GEARING

From the available literature, it can be stated that until recently there has been little information regarding introducing profile shift by way of altering the tooth-sum among the mating gears. Very few researchers have reported about this type of investigation. For a given module and tooth numbers if profile shift is introduced on the gears the center distance changes, conversely for a given center distance if profile shift is introduced different tooth numbers can be accommodated for the same module, but such altered tooth-sum gears operate on a different pressure angle. Hence this is a unique and promising design approach in which the tooth numbers can be varied for a given standard center distance and module.

As the tooth-sum of a standard gear pair is altered the following geometrical changes takes place:

- i. Modifications in tooth profile with respect to tooth root thickness, tip thickness and tooth topping.
- ii. Modifications in size of the base circle that alters the pressure line.
- iii. Modification in pressure angle that introduces operating/working pressure angle.
- iv. Introduction of computed total profile shift that has to be conveniently distributed between the gear pair.

The above geometrical changes affect other aspects like length of active profile, radius of involute along the tooth profile etc., thus affecting one or more parameters like bending stress, contact stress, contact ratio etc. The size of the base circles of standard tooth-sum gears in mesh changes if alteration in tooth numbers is effected, while the center distance and module being standard, the common tangent to altered base circles define an operating pressure angle α_e . This will be larger or smaller than the standard pressure angle depending on whether it is negative teeth alteration or positive teeth alteration. The corresponding operating pressure angle and amount of total profile shift for accommodating altered tooth-sum gears between standard center distances can be calculated by using equations (1) and (2).

$$\alpha_e = \cos^{-1}(Z_s' \cos(\alpha)/Z_s) \quad - (1)$$

$$X_e = (Z_1' + Z_2') (\text{inv } \alpha_e - \text{inv } \alpha) / (2 \tan \alpha) \quad - (2)$$

A judicious distribution of this resulting profile shift is possible to benefit gearing in many ways.

Table. I shows the details of various altered tooth-sums and its design parameters for tooth-sum 100 having module 2 mm. It can be seen that for the standard center distance 100 mm it is possible to accommodate a maximum negative teeth alteration of -4 resulting in tooth-sum 96 and a maximum positive teeth alteration of +5 resulting in tooth-sum 105. Teeth alteration

beyond these limits lead to either a pointed tooth lacking permissible shear stress or extremely low operating pressure angle which is less than prescribed along with other geometrical impossibilities. Hence, permitted design conditions caring undercutting, minimum top land thickness, minimum contact ratio and location of pitch point are considered to identify an operable domain. For each tooth altered over the standard tooth-sum the gears will receive profile shifts accordingly.

Table. I - DETAILS OF ALTERED TOOTH-SUM 100 GEARS FOR GR 1:1
(For equal distribution of profile shift, i.e., $X_1=X_2$)

Z_e	Z_s'	$Z_1' \times Z_2'$	α_e , (deg)	X_e	X_1	X_2	y_m (mm)
- 4	96	48x48	25.564	2.277	1.139	1.139	0.554
- 3	97	48x49	24.286	1.659	0.829	0.829	0.318
- 2	98	49x49	22.942	1.072	0.536	0.536	0.144
-1	99	49x50	21.519	0.518	0.259	0.259	0.037
0	100	50x50	20.000	0.000	0.000	0.000	0.000
+1	101	50x51	18.361	-0.481	-0.24	-0.24	0.039
+2	102	51x51	16.567	-0.920	-0.46	-0.46	0.160
+3	103	51x52	14.560	-1.314	-0.657	-0.657	0.371
+4	104	52x52	12.237	-1.657	-0.828	-0.828	0.686
+5	105	52x53	9.363	-1.938	-0.969	-0.969	1.124

LOAD SHARING PATTERN ON TOOTH PROFILE

As gear tooth begins and progresses, the salient points of contact during mesh along the tooth profile till disengagement is shown in Fig. 1. The bending stress induced in the gear tooth can be computed when load acts at these salient points. But, among them D2 representing HPSTC loading and E representing tip loading are important, hence only those points are considered in this analysis.

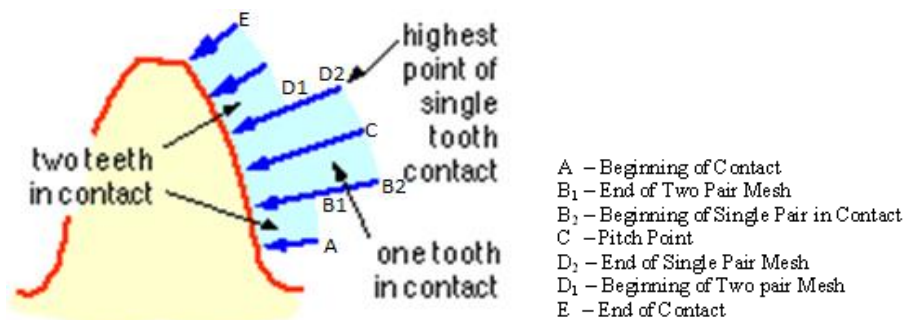


Fig.1 Salient points of mesh along tooth profile.

BENDING STRESS AND CONTACT RATIO

The bending stress is calculated using Lewis equation. Bending stress in gears depends on load acting normal to the tooth profile at the point of contact. Further the load shared by the gear tooth along the path of contact varies from point to point as illustrated in Fig .1. As both profile shift and tooth topping occurs in altered tooth-sum design, these modifications contribute for the significant changes in bending stress as well as contact ratio.

Due to the geometry of altered tooth-sum gears, a negative teeth alteration needs positive profile shift while positive teeth alteration needs negative profile shift. From this analysis it can be understood that negative teeth having positive profile shift results in a stronger gear tooth, obviously having lower bending stress and positive teeth alteration having negative profile shift results in higher contact ratio leading to HCR gearing [8]. Though it is possible to obtain the bending stress for contacts at all the salient points A to E, only D2 and E are considered as important for the analysis. Each tooth alteration in altered tooth-sum gearing has different optimum profile shift co-efficient that results in lowest bending stress for negative teeth alteration or highest contact ratio for positive teeth alteration. But, at the same time the bending stress induced in the mating gear tooth and the contact ratio of the gearing should also be viewed. Hence Fig. 2 and Fig. 3 are plotted which shows equal bending stress situation for negatively altered tooth-sum and highest contact ratio situation for positively altered tooth-sum.

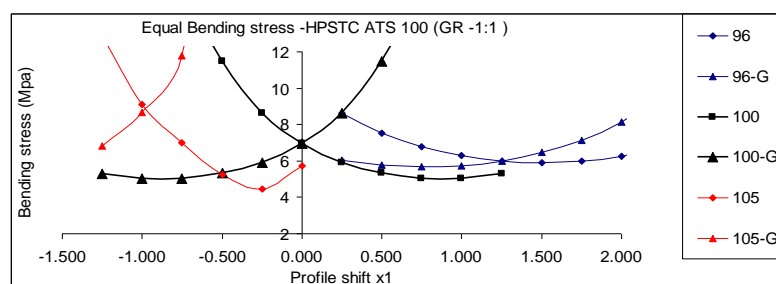


Fig. 2 Equal Bending stress BS-HPSTC vs. X1

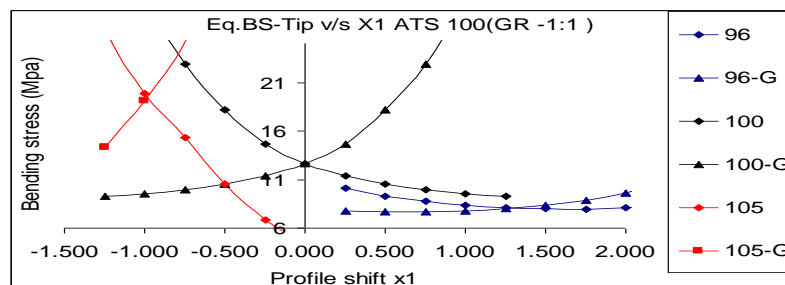
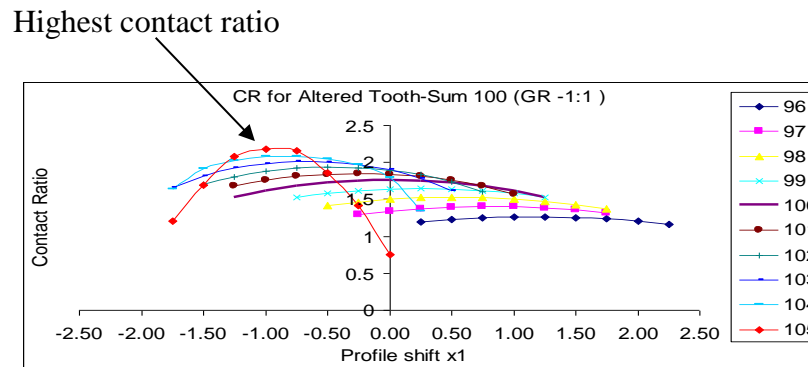


Fig. 3 Equal Bending stress BS-Tip vs. X1.

Contact ratio of a gear pair in mesh is the ratio of the length of the path of contact to the base pitch. The contact ratio of a standard gear pair is affected only when the tooth profile is modified. Fig. 4 shows the variation of contact ratio against profile shift co-efficient for different teeth altered over a tooth-sum 100 allowing the profile shift X_1 on pinion and $X_2 = X_e - X_1$ on the mating gear. From the illustration it is seen that the contact ratio increases with increase in teeth alteration Z_e upto certain limit beyond which it decreases, it means that there is an optimum profile shift coefficient for each tooth-sum leading to maximum contact ratio. For any altered tooth-sum gears having GR 1:1 the value of contact ratio peaks when the resulting total profile shift due to teeth alteration is equally distributed between the mating gears. Under such distribution the length of approach will be equal to length of recess. From altered tooth-sum design HCR gearing can be achieved for higher positive values of Z_e which is not possible with standard gears for 20° pressure angle.

Fig. 4 Contact ratio vs. X_1

LOAD SHARING ANALYSIS

As load on the gear tooth varies during the meshing cycle, the bending stress also varies. A load sharing diagram shows the bending stress on the ordinate and length of contact along the abscissa. The load sharing pattern during engagement influences the bending stress induced in a gear tooth, it depends on whether it is NCR ($CR < 2$) or HCR ($CR > 2$) gearing which is decided by the tooth geometry. Fig. 5 shows the basic load sharing pattern along the length of contact. When the contact ratio is less than 2 the tooth load on the gear tooth is half the full load (shared by two pair of teeth) for some time of engagement and full load (shared by one pair of teeth) for the remaining time of engagement as seen in Fig. 5(a). When contact ratio is 2, the load on the gear tooth never exceeds half the full load throughout the meshing cycle because minimum two pairs of teeth will always be in mesh as shown in Fig. 5(b).

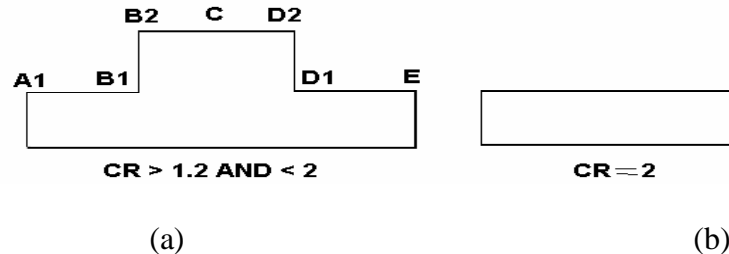


Fig. 5- Basic load sharing along the length of contact. (a) NCR gearing, (b) HCR gearing

As tooth profile is modified due to change in tooth numbers, pressure angle or profile shift, the load shared along the tooth profile will change, hence study of the following cases are essential :

- Standard gears with GR 1:1.
- Standard gears with different GR.
- Standard gears with different pressure angle.
- Profile shifted gears.
- Altered tooth sum gears.

When tooth-sum of a gear pair is varied, the tooth number changes along with introduction of computed and distributed profile shift as well as necessary tooth topping, while operating on a different pressure angle other than generating pressure angle. Hence study of case (A) for standard gears along with cases (B), (C) and (D) are essential. The load sharing diagrams are drawn on comparative basis so that load sharing pattern for any three considerations of tooth

number and/or profile shift can be analyzed at a time. Such comparison between the standard and altered tooth-sum gears lead to better understanding and analysis of bending stress and contact ratio in altered tooth-sum gears. In the load sharing diagrams presented ahead, the notations and values in the legend clearly show the details of various parameters considered in the analysis.

A. Standard Gears (GR(1:1))

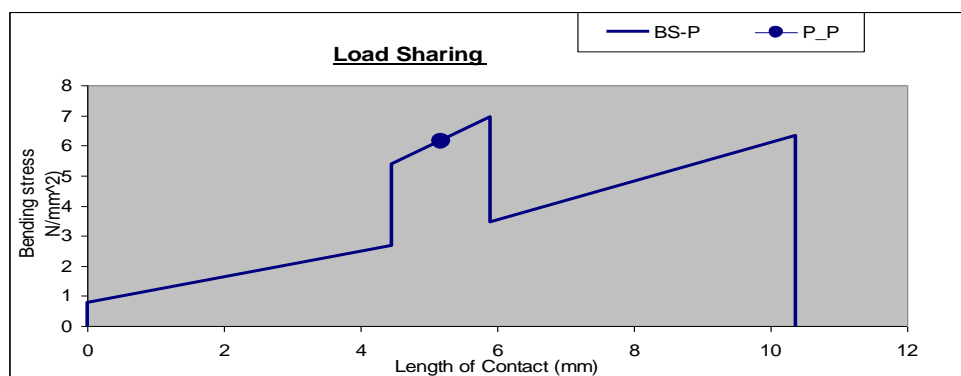


Fig. 6-Load sharing along length of contact for STS 100

Fig. 6 shows the load sharing diagram along the salient points of contact A,B1,B2,C,D2,D1 and E which are in congruence with Fig. 5(a). As the contact establishes at A and progresses the contact radius increases, accordingly the bending stress. The contact begins at A and ends at E both having double pair contact, while in the middle of contact journey, the gear tooth encounters single pair contact because the contact shifts from double pair to single pair situation (B1 to B2) and again from single pair to double pair situation (D2 to D1). This results in variation of bending stress along the length of contact. At these transition points from double pair to single pair contact or otherwise the bending stress increases or decreases suddenly and repeatedly for every rotation on every tooth, the bending stress being practically maximum at D2. This point where maximum stress is encountered is known as HPSTC (Highest point of single tooth contact). From Fig. 6 it can be seen that for standard gears with gear ratio 1:1 the pitch point is at middle of contact journey, having length of approach equal to length of recess.

B. Standard Gears With Different Gear Ratio

For a given tooth-sum, increase in gear ratio means decrease in number of teeth on pinion, hence change in GR alters the tooth geometry. As seen in Fig. 7, higher bending stress is induced for higher gear ratio due to reduction in tooth root thickness. The length of contact decreases as gear ratio increases, therefore the contact ratio decreases. As the gear ratio increases the pitch point moves towards the last point of contact making the drive recess action gearing.

By observing the length of contact in load sharing diagram, it can be said that the contact ratio slightly decreases for higher gear ratio and vice versa.

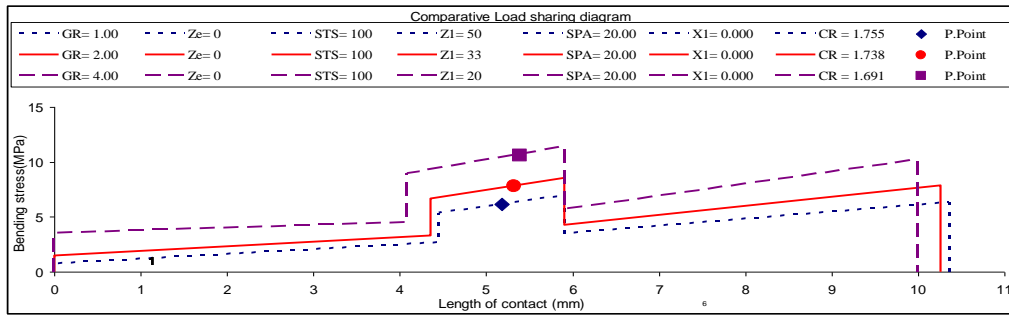


Fig. 7-Load sharing diagram for STS 100, different GR.

C. Standard Gears With Different Pressure Angle

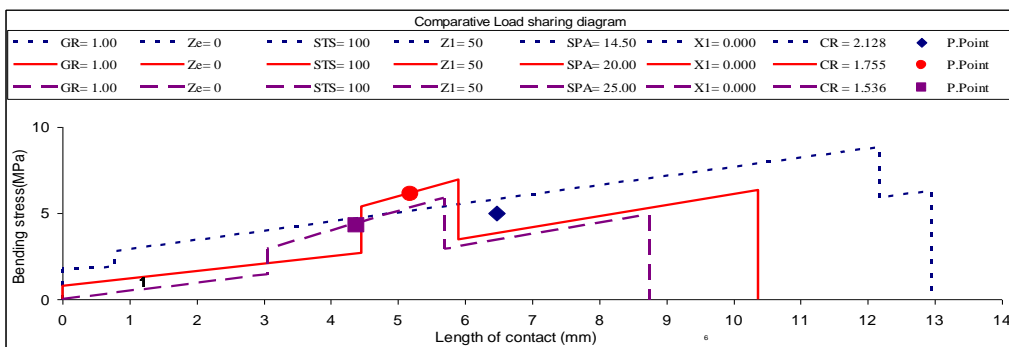


Fig. 8- Load sharing diagram for STS 100, different pressure angle.

The pressure angle of a gear being one of the significant parameter, when changed, alters the shape of the gear tooth to a greater extent compared to case (B) above. The load sharing pattern shown in Fig. 8 considers three pressure angles 14.5°(short dashed blue line), 20°(continuous red line) and 25°(long dashed purple line). It is seen that the bending stress is higher for lower pressure angle and vice versa.

By observing the length of contact in the load sharing diagram, it can be said that the contact ratio increases greatly for lower pressure angle and vice versa. A lower pressure angle of 14.5° leads to HCR gearing with contact ratio 2.128.

D. Profile Shifted Gears

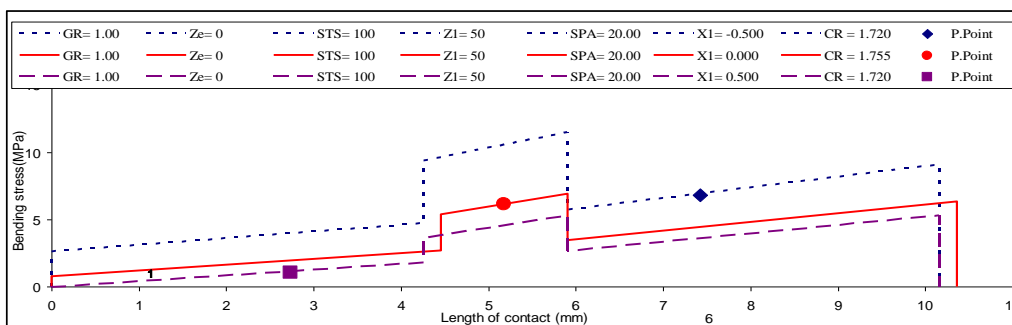


Fig. 9-Load sharing diagram for STS 100 with profile shift

As the tooth geometry changes by S_o gearing both bending stress and contact ratio are affected. The bending stress decreases for positive shift due to increased tooth thickness and vice versa, while the length of contact decreases reducing the contact ratio due to decreased length of contact. Fig. 9 illustrates that the location of pitch point moves either towards A or E depending on whether the profile shift is positive or negative. In S_o gearing the length of approach will not be equal to length of recess, for positive correction the length of approach will be shorter than length of recess and vice versa. For both positive and negative shifts the length of contact reduces, hence the contact ratio decreases due to profile shifting. The maximum contact ratio is achieved with standard gears. For profile shifted gears, whether positive or negative, the contact ratio decreases because of decrease in length of contact. For equal amount of negative and positive corrections i.e., $X1=-0.5$ and $X1=+0.5$ the deviation in bending stress will be opposite, this is being more for negative correction. In most cases, the pinion is provided with positive correction as it is the decisive element.

By observing the length of contact in the load sharing diagram, it can be said that the contact ratio slightly decreases for both positive and negative profile corrected gears.

E. Altered Tooth-sum Gears

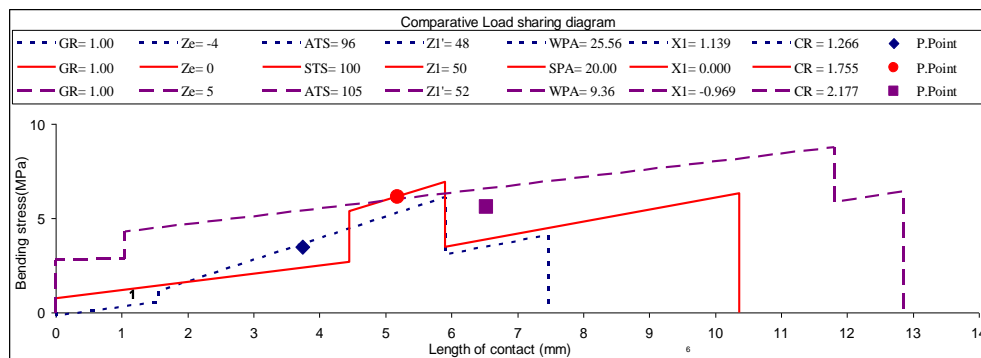


Fig. 10- Load sharing diagram for ATS 100

Fig. 10 shows the load sharing pattern from first point of contact A to last point of contact E. For tooth-sum 100, reduced equal bending stress occurs at $X1=1.139$ for a negatively altered tooth-sum 96 and maximum contact ratio occurs at $X1=-0.969$ for positively altered tooth-sum 105 [8]. The length of contact decreases greatly for negative teeth alteration due to less tooth topping resulting in NCR gearing, while the length of contact increases greatly with positive teeth alteration due to more tooth topping resulting in HCR gearing. It is observed that for a given tooth-sum 100 having GR 1:1, when the total profile shift resulting from altering the tooth-sum is equally distributed between the mating gears maximum contact length is achieved leading to high contact ratio. For such distribution the length of single pair journey for tooth-sum 96 is more than the length of single pair journey for tooth-sum 100, total length of contact being very less. Hence the contact ratio for tooth-sum 96 is 1.266 which is less than the contact ratio of standard tooth-sum 100 i.e., 1.755. On the other side, for tooth-sum 105 the teeth engagement shifts from double pair to triple pair contact, under such circumstances the length of contact becomes more than 2 times the base pitch. Hence it results in HCR gears having contact ratio of 2.177 which would otherwise result in NCR gears encountering 100% greater levels of bending stress at HPSTC and 50% greater levels at tip.

This shows that altered tooth-sum gearing can be used to obtain stronger gears having lower bending stress or higher contact ratio gears for quiet running.

RESULTS AND DISCUSSION

Analytical results are presented in Table. II(a) and Table. II(b) for involute spur gears of tooth-sum 100, 2 mm module, 20° pressure angle and 10 mm face width considering a static load of 9.81 N/mm of face width. Though many combinations of altered tooth-sum and profile shift are possible, design conditions like undercutting, minimum top land thickness, minimum contact ratio and location of pitch point are taken care to identify an operable domain.

Table II. RESULTS OF ATS 100 GEARS, GR 1:1

For Minimum Bending stress (BS) condition	$Z_s=$	100(50x50)	$Z_e=$	-4
	$Z_s'=$	96(48x48)	$X_e=$	2.277
	$X_1=$	1.139	$X_2=$	1.138
Parameter	BS-HPSTC (MPa)	BS-Tip (MPa)	CR	
ATS gears	6.108	8.193	1.266	
Standard gears	6.94	12.66	1.755	
% change	-12% (decrease)	-35.28% (decrease)	-27.86% (decrease)	

(a)

For Highest Contact ratio (CR) condition	$Z_s=$	100(50x50)	$Z_e=$	+5
	$Z_s'=$	105(52x53)	$X_e=$	-1.938
	$X_1=$	-0.969	$X_2=$	-0.969
Parameter	BS-HPSTC (MPa)	BS-Tip (MPa)	CR	
ATS gears	8.768	19.218	2.177	
Standard gears	6.94	12.66	1.755	
% change	+26.34% (increase)	+51.80% (increase)	+24.05% (increase)	

(b)

For cases (B), (C) and (D) in the previous section, the profile modification leads to changes in tooth root thickness, top land thickness and change in pressure angle by varying amounts, while in all the above cases the tooth height remains unchanged. But, in altered tooth-sum gearing along with the above changes, the tooth height also changes due to tooth topping that makes this a unique approach. In this study altered tooth-sum design is analytically implemented and understood. Based on the results, it is identified that negative teeth alteration having positive profile shift remarkably reduces the bending stress, while positive teeth alteration having negative profile shift increases contact ratio, some cases even leading to HCR gearing.

From this novel gear design approach a reduction in bending stress up to 35.28% for tip load condition leading to higher load carrying capacity and increase in contact ratio upto 24% leading to quiet running is obtained compared to standard gears.

From the above analysis the following results are observed:

- a. The tooth-sum can be altered within certain limits.
- b. Profile shifts greater than 1 module is possible by altering the tooth-sum which is uncommon.
- c. Negative teeth alteration results in positive profile shift and vice versa.
- d. Negative teeth alteration results in the working pressure angles greater than the standard pressure angle and vice versa.
- e. Considering reduced equal bending stress condition, the negatively altered tooth sum gears with equally distributed positive shift is found to yield stronger teeth with low bending stress for GR 1:1.
- f. The positively altered tooth-sum gears with equally distributed negative profile shift results in highest contact ratio (greater than 2) which ensures a minimum of two pairs of teeth in mesh across the path of contact. More pairs of teeth in mesh lowers tooth load, which results in lower stresses and quiet operation.
- g. No special tooling or manufacturing facility is required to generate gears for altered tooth-sum gearing.
- h. often pinion is the decisive element, if bending stress on the pinion alone is considered, further reduction in bending stress is possible compared with other tooth-sum and profile shift combinations, but at the cost of a corresponding increase in bending stress on the mating gear, moreover it is associated with certain geometrical irregularities.

CONCLUSION

Stress reduction using altered tooth-sum design is possible. The tooth numbers of a standard tooth-sum gear pair operating between a standard center distance and module can be varied with negative or positive teeth alteration either to have reduced tooth stress or increased contact ratio respectively. In order to enhance the bending load capacity of the gear tooth, altered tooth-sum gearing that operates on a different pressure angle while in mesh is considered. A gear pair with a tooth sum of 100 is altered in both negative and positive directions ranging from 96 teeth to 105 teeth. The module of the gears considered is 2 mm with operating centre distance of 100 mm and pressure angle of 20° to study the effect on bending stress and contact ratio. With judicious selection of teeth alteration and distribution of resulting profile shift, it is possible to design gears either to have lower tooth bending stress or higher contact ratio.

Considering the material needs, heat treatment and vibration aspects, altered tooth-sum gear design can be traded off for

- i. Increased load carrying capacity.
- ii. High stiffness.
- iii. Quiet operation.
- iv. Longer life.
- v. Reduced cost.
- vi. Size and weight reductions.
- vii. No structural changes in gear box.

Since the above benefits can be achieved from the proposed analysis without resorting to any structural changes, it may be concluded that altered tooth-sum design can be considered as a novel, promising and unique approach to gear design.

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