ALGORITHMS FOR EVALUATING ADAPTIVE EQUALIZATION IN WIRELESS COMMUNICATION SYSTEMS

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ABSTRACT

One of the new challenges in modern communication is to conceive highly reliable and fast communication system that is devoid of problems caused by multipath fading in wireless channels. The search is geared toward removing one of the obstacles in the way of achieving ultimately fast and reliable wireless digital communication such as inter-symbol interference (ISI). The job here is to apply adaptive equalization technology to minimize the communication errors resulting from the multipath signal effects. Adaptive equalization method is applied in this research based on the Least Mean Square (LMS) algorithms. The approach to the work is based on one methodology but several algorithms and configurations such as trained LMS algorithm, decision-directed algorithm and dispersion minimization algorithm. Different step size values are considered and compared for each of the three algorithms. Result of the simulation reveals that decision directed linear equalizer performs significantly better than others.

Keywords: Adaptive Algorithm, Dispersion Minimization, Decision Directed, LMS, ISI.

INTRODUCTION

In an ideal communication system, it is assumed that if there is no interaction between successive symbols; each symbol arrives at its own time and is decoded independently of all others. But when symbols interact, if the waveform of one symbol corrupts the value of a nearby symbol, then the received signal gets distorted [1]. It is then difficult to extract the message from such received signal. This type of problem is called Inter-symbol Interference (ISI). Adaptive filtering is a specialized branch of digital signal processing, dealing with adaptive filters and system design. They are used in a wide range of applications including system identification, noise cancellation, interference removal, signal prediction, echo cancellation, beam forming and adaptive channel equalization.

Filtering is the extraction of information about a quantity of interest at time ‘t’ by using data measured up to and including time t. If the input to the filter is stationary, the resulting solution to the filtering problem is known as the Wiener filter, which is said to be optimum in the mean square sense [3]. But it requires prior information about the statistics of the data to be processed. If the environment is unknown, another efficient method is to use an adaptive filter using recursive algorithm. The algorithm starts with some predetermined set of initial conditions, representing whatever is known about the environment.

To solve the ISI problem, many algorithms and structures have been proposed. The general solution to reduce the effect of ISI is the application of adaptive equalization. Adaptive algorithms are used to update the coefficients of equalizer when a channel is unknown and time varying. The initialization of the coefficients is done by transmitting a training sequence.
from the transmitter to the receiver [5]. It is then followed by a decision directed mode for normal reception of data. Among many algorithms and structures suggested, this work is going to treat trained equalization, Decision directed equalization and Dispersion minimization.

**LEAST MEAN SQUARE ALGORITHM (LMS)**

This is a class of algorithm that is used to imitate a desired filter by finding the filter coefficients that relate to producing the least mean square of the error signal (i.e., the difference between the desired and actual signal). It is a stochastic gradient descent method in that the filter is only adapted based on the error at the current time [4]. The least mean square filter is built around a transversal (i.e. tap delay line) structure.

![Diagram of Basic Linear Transversal Equalizer](image)

The least mean square algorithm is simple to design, yet highly effective in performance and this has made it popular in various applications. LMS filter employs, small step size statistical theory, which provides a fairly accurate description of the transient behavior [2]. It is robust. A weighting control mechanism responsible for performing the adaptive control process on the tap weight of the transversal filter is illustrated in fig 1. The iterative procedure of the LMS involves computing the output of a finite impulse response (FIR) filter produced by a set of filter coefficients, followed by the generation of an estimated error by comparing the output of a filter to a desired response and finally, adjusting the filter coefficients based on the estimation error [3].

The following equations explain the mentioned process.

\[ y(k) = f^T(k)x(k) \]  
Filter output   

\[ e(k) = d(k) - y(k) \]  
Error

\[ f(k) = [f_0(k) \ f_1(k) \ f_2(k) \ ... \ f_{M-1}(k)]^T \]  
Filter coefficient at time n

\[ x(k) = [x(k) \ x(k-1) \ x(k-2) \ ... \ x(k-M+1)]^T \]  
Input Data

The adaptive process which involves the automatic adjustment of the parameters of the equalizer in accordance with the estimation error results to the weight or coefficient update given by:

\[ f(k+1) = f(k) + 2\mu e(k)x(k) \]  

![Diagram of Basic Linear Transversal Equalizer](image)
Where $\mu$ is the step size. The algorithm requires that at each iteration, $x(k), d(k)$ and $f(k)$ are known. As the step size decreases, the convergence speed to the optimal value is slower. This implies that the LMS algorithm is a stochastic gradient algorithm if the input is a stochastic process.

The LMS is built around the transversal structure. Its two practical features include design simplicity, yet highly effective in performance which have made it highly popular in various applications. The LMS filter employs small step-size statistical theory, which provides a fairly accurate description of the transient behavior. It also includes infinite impulse ($H_\infty$) theory which provides the mathematical basis for the deterministic robustness of the LMS filter [6]. A weighted control mechanism responsible for performing the adaptive control process on the tap-weight of the transversal filters is illustrated in fig 2. The block diagram illustrates the combination of filtering process and adaptation process working together in a feedback formation to perform the adaptive control process on the tap-weight of the transversal filter.

It is deduced from experiment that:

- The LMS algorithm is a well-known adaptive algorithm which adapts by a value that is proportional to the product of input to the equalizer and the output error.
- The LMS algorithm executes quickly but converges slowly. Its complexity grows linearly with the number of Tap-weights.
- The LMS algorithm is computationally simple.
- The channel parameters of an LMS algorithm vary slowly

### TRAINED ADAPTIVE EQUALIZATION

The equalizer shown in fig.2 also explains an adaptive equalization in a training mode of operation. Here, consideration is put in place to use an adaptive element to minimize the mean square error.

$$J_{LMS} = \frac{1}{2} \{e[k]\}^2 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (6)$$

Where the error is expressed as

$$e[k] = s[k - \delta] - y[k] = s[k - \delta] - \sum_{j=0}^{n} f_j r[k - j] \ldots \ldots (7)$$
Therefore, an algorithm for minimizing the performance function $J_{LMS}$ with respect to the equalizer coefficient $f_i$ is

$$f_i[k + 1] = f_i[k] - \mu \frac{dJ_{LMS}}{df_i}|_{f_i=f_i[k]} ....... (8)$$

By computing the derivatives and making necessary substitutions, a final result of the LMS adaptation coefficient ($f_i$) update is

$$f_i[k + 1] = f_i[k] + \mu e[k]r[k - i] ....... (9)$$

Provided $\mu$ is nonzero, the equalizer is made adaptive if underlying composition of the received signal changes so that the error increases and the desired equalizer changes, then, $f_i$ reacts accordingly. This is adaptive tracking of the system.

**BLIND CHANNEL EQUALIZATION**

For over twenty years, research has centered on developing new algorithms and formulating a theoretical justification for these algorithms. Blind channel equalization is also known as self-recovery equalization. The essence of blind equalization is to recover the unknown input sequence to the unknown channel based solely on the probabilistic and the statistical properties of the input sequence. The receiver can synchronize to the received signal and adjust the equalizer without the training sequence. The term blind is used in this argument because it performs the equalization on the data without reference signal. Instead the blind equalizer relies on knowledge of the signal structure and its statistics to perform the equalization.

i. Blind signal is the unknown signal which would be identified in output signal with accommodated noise at the receiver.

ii. Channel equalization uses the idea and knowledge of the training sequences for channel estimation whereas blind channel equalization does not utilize the characteristics of the training sequences for frequency and impulse response analysis of channel.

iii. Blind channel equalization differs from channel equalization and without knowing the channel characteristics like transfer function and SNR it efficiently estimates the channel and reduces ISI by blind signal separation at receiver side by suppressing noise in the received signal.

The algorithms whose operations are based on the principle of blind equalization are the Decision Directed Equalization algorithm and the Dispersion Minimization Algorithm treated below.

**DECISION-DIRECTED LINEAR EQUALIZATION.**

The equalizer parameters can be adapted without application of the training data. This method helps to improve the channel capacity as well as reduce cost. Considering the situation in which some procedure has produced an equalizer setting that opens the eye of the channel. Thus all decisions are perfect, but the equalizer parameters may not yet be at their optimal values. In such a case, the output of the decision device is an exact replica of the delayed source, i.e. it is as good as a training signal. For a binary±1 source and decision device that is a sign operator, the delayed source recovery error can be computed as sign $\{y[k]\} - y[k]$ where $y[k]$ is the equalizer output and sign$\{y[k]\}$ equals the delayed source.
s[k – δ] (i.e. the transmitted signal delayed by δ). Thus, the trained adaptive equalizer of fig (2), above can be replaced by the decision directed device as shown in fig (3) below. This converts eqn(9) to decision directed least mean square (LMS), which has an update as

\[ f_i[k + 1] = f_i[k] + \mu (\text{sign}(y[k]) - y[k])r[k - i] \]

(10)

It is observed that the source s[k] does not appear in eqn(10). Thus, no training signal is required for its implementation and the decision directed LMS equalizer adaptation law of eqn (10) is called a ‘blind’ equalizer. Given its genesis, one should expect decision directed LMS equalizer to exhibit poor behavior when the assumption regarding perfect decision is violated. The basic rule of thumb is that 5% (or so) decision errors can be tolerated before decision directed LMS fails to converge properly.

**DISPERSION MINIMIZING LINEAR EQUALIZATION**

This section considers an alternative performance function that leads to another kind of blind equalization. Observe that for a binary ±1 source, the square of the source is known even when the particular values of the source are not. Thus \( s^2[k] = \gamma = 1 \) for all k. This suggests creating a performance function that penalizes the deviation from the known square value \( \gamma = 1 \). In particular, consider

\[ J_{DMA} = \frac{1}{N} \text{avg}(\gamma - y^2[k]) \]

(11)

Which measures the dispersion of the equalizer output about its desired square value. The associated adaptive element for updating the equalizer coefficients is

\[ f_i[k + 1] = f_i[k] + \mu \frac{dJ_{DMA}}{df_i} |_{f_i=f_i[k]} \]

(12)

Mimicking the previous equations the Dispersion Minimization Algorithm (DMA) for blindly adapting the coefficients of a linear equalizer which is

\[ f_i[k + 1] = f_i[k] + \mu \text{avg}(1 - y^2[k])y[k]r[k - 1] \]

(13)

Suppressing the averaging operation, this becomes

\[ f_i[k + 1] = f_i[k] + \mu (1 - y^2[k])y[k]r[k - 1] \]

(14)
SIMULATION RESULTS.

At step-size 0.009, the algorithm tends to converge to the optimal filter coefficients $f$ in about 150 iterations and the MMSE is at about 11.01. The next step-size that follows in rank is 0.0067, 0.0045, 0.0031 and 0.0011 respectively. The least in the ranked step-sizes is 0.0011 which could not converge even after 500 iterations and it require more delay to reach optimal filter coefficient. This value of step-size is considered too small since it cannot converge at an acceptable time cycle.
Figure (6) is shows the Convergence graph of Decision Directed equalizer at five levels of step-size values. The optimal result is achieved at step-size value of 0.009 followed by 0.0067, 0.0045, 0.0031 and 0.0011. The graph shows that decision directed equalizer has a very fast convergence speed as compared to trained LMS and Dispersion Minimization algorithms. Yet the mean square error level is higher than Dispersion minimization as shown by the graph of fig 6. It converges to the optimal filter coefficient $f_i$ in 50 iterations. The MMSE is approximately 0.31.

Fig 7 shows the Convergence graph of Dispersion Minimization algorithm at five levels of step-size values. The optimal result is achieved at step-size value of 0.009 followed by 0.0067, 0.0045, 0.0031 and 0.0011. Although it converges to optimal filter coefficient in about 100 iteration, the MMSE is approximately 0.20.
CONCLUSION

The relationship that is common to the three algorithms is the LMS algorithm. Their individual differences occur due to the method of error minimization. As shown in equations 9, 10 and 14 for trained equalizer, decision directed equalizer and dispersion minimization equalizer respectively, the error method used to calculate the coefficient update vary. The different error minimization method helps to determine the optimal algorithm. The step-size value of 0.009 achieves the best convergence in all the three algorithms. The decision directed equalizer has a very fast convergence speed as compared to trained LMS and Dispersion Minimization algorithms. It converges to the optimal filter coefficient in 50 iterations with the MMSE of approximately 0.31. Although dispersion minimization equalizer converges to optimal filter coefficient in about 100 iteration, the MMSE is approximately 0.20. The trained equalizer algorithm tends to converge to the optimal filter coefficients in about 150 iterations at the MMSE of about 11.01. This shows that the trained equalizer has the highest delay. Also it is shown from the graphs that decision directed equalizer has all the step-size values converged earlier than the others. Having investigated the effect of the step-size parameter on the convergence speed and MMSE performance of the LMS algorithm, it was demonstrated that step-size plays a significant role in the performance of LMS equalizer design.

REFERENCES