PORTFOLIO INSURANCE WITH A DYNAMIC RISK MULTIPLIER BASED ON PRICE FLUCTUATION

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ABSTRACT

In this paper we extend the Constant Proportion Portfolio Insurance Strategy (CPPI) and the Time-Invariant Portfolio Protection Strategy (TIPP) to dynamic CPPI (D-CPPI) and dynamic TIPP (D-TIPP) by using a novel dynamic risk multiplier based on the price fluctuation of the risky asset. The multiplier *m* is adjusted by the movement of the risky asset price, that is, when the risky asset price rises, the dynamic multiplier *m* rises along with it; when the risky asset price falls, the dynamic multiplier *m* also falls. Accordingly the dynamic strategies would better allow potential benefits in rising markets and limits downside risk in falling markets. By using the real data of Chinese stock market, we evaluate the performances of the D-CPPI and D-TIPP strategy under bull, bear and deer markets with different parameters and simultaneously compare the simulation results with traditional CPPI and TIPP strategy. The empirical results show that the performances of D-CPPI and D-TIPP strategy significantly exceed the traditional CPPI and TIPP strategy under most circumstances. This paper extends the traditional portfolio strategy with the dynamic risk multiplier, which provides solid foundations for further research of domestic portfolio insurance in emerging market. It contribute to both academic research and practical investment by laying theoretical supports for hedge using proper financial derivatives in Chinese market and technical supports for investors using portfolio insurance to avoid market risks.

Keywords: Dynamic multiplier; portfolio insurance; CPPI; TIPP.

INTRODUCTION

Portfolio insurance allows investors to recover at maturity a given percentage of their initial investment, whatever financial market evolutions. The Portfolio insurance strategy limits downside risk in falling markets, while it allows potential benefits in rising markets. The first main portfolio insurance method has been introduced by (Leland& Rubinstein, 1976).It is the Option Based Portfolio Insurance (OBPI), which consists of a portfolio invested in a risky asset, *S* (usually a financial index) covered by a listed put option written on it. Whatever the value of *S* at maturity *T*, the portfolio value will be always greater than the strike price (*K*) of the put option. The purpose of the OBPI method is to guarantee a fixed amount only at maturity.

The second important insurance portfolio strategy is the Constant Proportion Portfolio Insurance (CPPI) considered by (Pernold, 1986) and further studied by (Perold & Sharpe, 1988)for fixed-income instruments and (Black & Jones, 1987)for equity instruments. This strategy is based on a dynamic asset allocation over time. The investor starts by setting a floor which equals to the lowest acceptable value of the portfolio. Then, he determines the cushion as the excess of the portfolio value over the floor. The amount allocated to the risky asset is equal to the cushion multiplied by a predetermined multiplier. There maining funds are invested in the reserve asset, usually T-bills.

However, (Estep & Kritzman, 1988)argue that investors will not only be interested in a protection of their initial wealth, but also in the protection of any capital gains. They suggest a modification of the CPPI strategy, which they call the 'time invariant portfolio protection' (TIPP) strategy. The adjustment of TIPP is very similar to CPPI. The only difference is the assumption in respect to the initial floor: it is not constant; the new initial floor will be the maximum value through the comparison of original amount and a constant proportion of assets at that time.

While the principle idea behind the TIPP strategy seems attractive,(Kenneth & Eric, 1989)argue that this strategy suffers from a major shortcoming. Compared with the traditional CPPI strategy, the TIPP strategy transfers all holdings of the risky asset into an irreversible manner of the risk-free asset once the floor has been reached. Accordingly, the TIPP strategy cannot participate from any subsequent upward market movements. However, due to the continuous 'ratcheting up' of the floor to the highest portfolio value, the likelihood that the portfolio value reaches or falls below the prevailing floor increases, and hence the TIPP strategy will more often end up fully invested in the risk-free asset.

The work of (Do, 2002)uses simulation analysis to compare the synthetic put strategy with the CPPI strategy. Although he claims that neither strategy can be justified based on either a loss minimization or again participation point of view, the CPPI strategy seems to dominate in terms of floor protection and the costs of insurance. The simulation results in (Cesari & Cremonini, 2003)indicate that the relative performance of portfolio insurance strategies depends on the market phase. They report a dominant role of the CPPI strategy against all other portfolio insurance strategies in bear and sideway markets. If define the multiplier by a quantile of dynamic auto regressive model based on Value-at-Risk (Benjamin, Bertrand, & Prigent, 2009), it improves the benefits of CPPI depending on market conditions. The equal amount dynamic floor discipline is proposed in (Huaii, Hsinan, & Min-Hsien, 2010) and their results show that the dynamic discipline outperforms the fixed floor discipline in both better downside protection and Sharpe ratios generation in the long run.

Furthermore, variations of the CPPI strategy have been proposed by recent researches. (Joossens & Schoutens, 2010) compares between the CPPI strategy and the Constant Proportion Debt Obligations (CPDO) strategy. The CPDO strategy is a variation of the CPPI strategy. It borrows certain features such as a 'constant proportion' approach to determining leverage and the re-balancing of the portfolio between the risky asset and the riskless asset. The D-CPPI strategy and the D-TIPP strategy are introduced by (Yuan & Shanshan, 2012) with a dynamic multiplier. The dynamic multiplier is adjusted by the movement of stock price. When stock price rises, the dynamic multiplier rises along with it; when the stock price falls, the dynamic multiplier falls. Accordingly investors will gain profits from the strategies when the stock price rises and get downward protection when the stock price falls.

Similarly, the CPPI method is extended on the basis of conditional floors which allows to keep part of the past gains and to protect the portfolio value against future high drawdown of the financial market (Ameur & Prigent, CPPI Method with a Conditional Floor, 2011). Following on, explicit upper bounds on the multiplier as a function of past asset returns and volatilities are found and it can be chosen to satisfy a certain condition with a given level of probability for various financial market conditions(Ameur & Prigent, Portfolio Insurance: Gap Risk under Conditional Multiples, 2014).

In this paper, we introduce a dynamic risk multiplier *m* of the CPPI strategy and the TIPP strategy based on the price fluctuation of the risky asset, and propose the dynamic Constant Proportion Portfolio Insurance Strategy (D-CPPI) and the dynamic Time-Invariant Portfolio Protection Strategy (D-TIPP). The multiplier *m* is adjusted by the movement of the risky asset price. That is, when the risky asset price rises, the dynamic multiplier rises along with it; when the risky asset price falls, the dynamic multiplier also falls. Accordingly the dynamic strategies would better allow potential benefits in rising markets and limits downside risk in falling markets. By using the actual data of Chinese stock market, we analyze the performances of the D-CPPI strategy and the D-TIPP strategy with under different markets and different parameters, and compare them with the traditional CPPI and TIPP strategy.

This paper extends the CPPI and TIPP strategy with the dynamic risk multiplier, which provides foundations for further research of domestic portfolio insurance, derives valuable conclusions for the theoretical studies and practical investments in emerging market, lays theoretical supports to hedge using financial derivatives in Chinese stock market and technical supports to the investors using portfolio insurance to avoid market risks.

The remainder of the paper is organised as follows: Section II describes the traditional CPPI and TIPP strategy. The the mathematical model of dynamic risk multiplier adjustment factor and the research methodology forCPPI and TIPP with the dynamic risk multiplier is further discussed in Section III and IV. Section V describes the the data sample and simulation design. In addition, the simulation performance and the results are presented at the same time. Finally, in Section VI concludes the paper.

CPPI AND TIPP

CPPI

To avoid the complexity and inconvenience of the OBPI strategy,(Black & Jones, 1987)proposed the constant proportion portfolio insurance strategy (CPPI). Investors refer to the difference between the present value of the insured portfolio and the current value of maturity floor as the expected loss. They choose the risk multiplier *m* according to the tolerance of the risk and use the simple dynamic formula to adjust the position of risky asset and riskless asset.

During the insurance time period, the value of the risky assets $E_t = mC_t = m(V_t - F_t)$,where *m* is the risk multiplier, V_t is the total portfolio at time *t*, F_t is the present value of the floor, $C_t = V_t - F_t$ is the cushion at time *t*.

At the beginning, the floor F_0 and the multiplier *m* are decided according to the investor's risk tolerance, which are generally fixed through the whole time period. The higher the multiplier, the more the investor will participate in a sustained increase in the risky asset price and more affected by the price of the risky asset. Simultaneously, the higher the multiplier, the faster the portfolio will approach the floor when there is a sustained decrease in the risky asset price. The floor F_0 grows with the risk-free rate *r*, that is $F_t = e^{rt}F_0$. *T* is the maturity of the insurance. The position of risky asset and riskless asset are adjusted according to the changed portfolio. Overall, the risk multiplier *m* of CPPI is bigger than 1 under most circumstances. When *m* equals to 1, it is equivalent to the buy-and-hold strategy. When *m* is between 0 and 1, and the initial floor F_0 is 0,it is equivalent to the constant-mix strategy.

TIPP

(Estep & Kritzman, 1988) proposed time-invariant portfolio protection(TIPP)strategy whose floor is variable which is different from the CPPI strategy. When the net value of the portfolio changes, we can choose the bigger value between the previous floor and the present floor which is the product of the proportion of guarantee and the portfolio. Apparently, when the value of portfolio rises investor can protect his current value of portfolio. The TIPP strategy is more conservative than the CPPI strategy.

TIPP is defined as $E_t = mC_t = m(V_t - F_t)$, $F_t = \max(F_{t-1}, \lambda V_t)$. Where E_t is the value of the risky asset at time *t*, *m* is the risk multiplier; V_t is the value of the portfolio at time *t*, F_t is the present value of the floor, C_t is the cushion at time *t*, λ is the proportion of guarantee. When the value of portfolio rises, the floor of the TIPP strategy increases, so the ability of gaining profit in the rising market is worse than the CPPI strategy.

The mathematical model of dynamic risk multiplier adjustment factor

The stock index is usually chosen by investors as the risky asset when they carry out the CPPI strategy or the TIPP strategy. Take CPPI as the example, at the initial:

$$C_0 = V_0 - F_0 \qquad (1)$$

Where V_0 is the initial value of portfolio, C_0 is the initial cushion, F_0 is the initial floor.

$$E_0 = m_0 C_0 = S_0 \times n_0 \qquad (2)$$

Where E_0 is the initial value of risky asset which equals to the production of the price of risky asset and the shares, S_0 is the initial price of the risky asset, n_0 is the initial shares of it.
The portfolio is composed of the risky and riskless assets:

$$V_0 = E_0 + R_0 \qquad (3)$$

Where R_0 is the initial value of riskless asset. From (2) and (3),we get

$$V_0 = S_0 \times n_0 + R_0 \qquad (4)$$

Therefore,

$$\frac{\partial V_0}{\partial S_0} = n_0 \qquad (5)$$

Similarly,

$$\frac{\partial V_i}{\partial S_i} = n_i, i = 0,1,2,...,n \qquad (6)$$

From (2), when the value of portfolio changes with the price of risky asset, we rebalance the position between the risky asset and riskless assets, then $S_1 \times n_1 = m(V_1 - F_1)$, that is

$$(S_0 + \Delta S_0)(n_0 + \Delta n_0) = (m_0 + \Delta m_0)[(V_0 + \Delta V_0) - (F_0 + \Delta F_0)] \qquad (7)$$

The *i*-adjustment is

$$(S_0 + \sum_{i=1}^{k} \Delta S_{i-1})(n_0 + \sum_{i=1}^{k} \Delta n_{i-1}) = (m_0 + \sum_{i=1}^{k} \Delta m_{i-1})[(V_0 + \sum_{i=1}^{k} \Delta V_{i-1}) - (F_0 + \sum_{i=1}^{k} \Delta F_{i-1})] \qquad (8)$$

$$n_0 + \sum_{i=1}^{k} \Delta n_{i-1} = (m_0 + \sum_{i=1}^{k} \Delta m_{i-1})[(C_0 + \sum_{i=1}^{k} (\Delta V_{i-1} - \Delta F_{i-1})]/(S_0 + \sum_{i=1}^{k} \Delta S_{i-1}) \qquad (9)$$

From (9), the accumulative amount of the changes of shares in the risky asset is composed of the accumulative amount of the changed multiplier $\sum_{i=1}^{k}\Delta m_{i-1}$, the accumulative amount of the portfolio $\sum_{i=1}^{k}\Delta V_{i-1}$, the accumulative amount of the floor $\sum_{i=1}^{k}\Delta F_{i-1}$ and the accumulative amount of the changed price of the risky asset $\sum_{i=1}^{k}\Delta S_{i-1}$.

The CPPI and TIPP strategy state that the floor grows with the risk-free rate and the risk multiplier *m* is an invariant constant which is the most flexible parameter. In order to benefit from the rising market and avoid the downside risk, this paper proposes a dynamic risk multiplier. When the price of the risky asset rises, we enlarge the multiplier ($\Delta m_{i-1} > 0$) to increase the position of the risky asset for the upward potential profits. When the price of the risky asset falls ($\Delta m_{i-1} < 0$), we reduce the multiplier to decrease the position of the risky asset for protecting the profits. So, we adjust the risk multiplier *m* simply according to the changing price of the risky asset for increasing the profits or decreasing the risks.

So, let the dynamic risk multiplier equal to:

$$m_i = m_{i-1} + a\ln(S_i / S_{i-1}), i = 1,2,3... \qquad (10)$$

Where m_i is the dynamic multiplier in the *i*- adjustment, m_0 is the initial multiplier(similar to the fixed multiplier *m* in CPPI and TIPP), S_i is the current price of the risky asset. Depending on the initial risk multiplier, we dynamically adjust *m* by the ratio of logarithm returns of the current and the previous value of the asset. *a* ($a \geq 1$)is an amplifier which is decided by the risk preference of investors. The bigger *a* means that investors are more sensitive to the return and the risk of the risky asset. When the price of the risky asset rises ($S_i > S_{i-1}$), $\ln(S_i > S_{i-1})$ is positive, the dynamic risk multiplier becomes bigger with the amplifier *a*. So the investors get the more profits with the rising price of the risky asset. When stock price decreases ($S_i < S_{i-1}$), $\ln(S_i < S_{i-1})$ is negative, the dynamic risk multiplier becomes smaller to avoid the downside risk.

CPPI and TIPP with the dynamic risk multiplier

We introduce the trading process of CPPI and TIPP with the dynamic risk multiplier and name them the D-CPPI and D-TIPP strategy. Let us take the D-CPPI for example. The investors choose m_0 and *a* according to their risk preference at the initial. To achieve the insurance goal they set the floor and decide the positions of the risky asset and the riskless assets. In the next adjustment, when the price of the risk asset rises, the dynamic risk multiplier increases based on the mathematical model; when the price of the risk falls, the dynamic risk multiplier decreases. Then, the positions of risky asset and riskless asset are rebalanced. Repeat the steps until the end of investment.

The D-CPPI is mathematically described as:

$$\begin{cases} V_0 = E_0 + R_0 = C_0 + F_0 \\ E_0 = m_0 C_0 \end{cases} \qquad (11)$$

When $T = 0$, V_0 is the initial portfolio, E_0 is the initial position of risky asset, R_0 is the initial position of riskless asset, C_0 is the initial cushion, F_0 is the floor, C_0 is the initial risk multiplier, S_0 is the initial price of the risky asset.

When $T=1$, if $S_i > S_{i-1}$, the multiplier m_i increases to $m_{i-1}+a\ln(S_i/S_{i-1})$ according to (10);if $S_i = S_{i-1}$, m_i equals to m_{i-1};if $S_i < S_{i-1}$, the multiplier m_i decreases to $m_{i-1}+a\ln(S_i/S_{i-1})$.

Rebalance the positions of risky asset and riskless asset by the new multiplier, then we can get:

$$\begin{cases} V_i = E_i + R_i = C_i + F_i \\ E_i = m_i C_i \end{cases} \quad (12)$$

All above steps will be repeated until the end of trading strategy. The only difference between the D-TIPP and the D-CPPI strategy is that the former has a dynamic floor. Comparing the initial floor and the calculated floor for $t=i$,if $F_{i-1} > \lambda \times V_i$, F_{i-1} is the new floor. While if $F_{i-1} < \lambda \times V_i$, $\lambda \times V_i$ is the new floor. Therefore:

$$\begin{cases} V_i = E_i + R_i = C_i + F_i \\ E_i = m_i C_i \\ F_i = \max\{F_{i-1}, \lambda \times V_i\} \end{cases} \quad (13)$$

Where λ is the proportion of guarantee. Again all above steps will be repeated until achieving trading goals.

Stimulation Analysis

We evaluate the performances of the D-CPPI and D-TIPP strategy by using the actual data of Shanghai Composite Index. We also compare their performances with the traditional CPPI and TIPP strategy.

The market data collection and basic hypothesis

We consider the daily closing price of Shanghai Composite Index as the risky asset's samples, and then choose three periods presenting bull market, bear market and deer market. In each period we finally select 240 daily observations to analyze the performances of D-CPPI and D-TIPP strategy.

We imply five basic assumptions during the simulation which are consistent with previous research:

i. There are no dividends;
ii. The risk-free interest rate is the interest rate of the deposit during the period, and interests are calculated every day;
iii. The daily closing price of Shanghai Composite Index presents the risky asset price. Do not consider the everyday volatility of the price and the trading volumes;
iv. Our strategies are self-financing;
v. The transaction cost is 3‰ of the amount of adjustment. For example, the initial value of risky asset is 50000 yuan(RMB) and40000 yuan (RMB) after adjusting, then the transaction cost is (50000-40000)×3‰,that is 30 yuan (RMB).

The stimulation design

The following stimulation design applies to both the D-CPPI and the D-TIPP strategy:

(a) There are bull, bear and deer markets. The deer market includes two kinds of periods. One is first rising then falling period, the other is first falling then rising period. The bull market is from November 29, 2005 to November 24, 2006, the bear market is from November 1,2007 to

October 24,2008. The first rising then falling of the deer market is from June 25.2009 to June 18,2010 and The first falling then rising of the deer market is form September 23,2004 to September 19,2005, which better reflect the performances of different strategies;
(b) The initial value of portfolio is one billion yuan (RMB). Shanghai Composite Index of Stocks represents the risk asset, the deposit of bank during the same period represents riskless asset. The interest rate of the deposit from September 23,2004 to November 24,2006 and November 1,2007 to October 24,2008 is 0.72%. The interest rate of the deposit during June 25,2009 to June 18,2010 is 0.36%. To simplify, the risk-free rates of the bull, bear and deer markets are considered as 0.72%. The daily rate of riskless asset daily is 0.72%⁄365=0.002%;
(c) The initial risk multiplier m is selected for 2, 3, 4 separately, the amplifier a is selected for 1,2,3 separately;
(d) The proportion of guarantee λ (the floor) is selected for 0.80,0.85, 0.90separately;
(e) The adjustment rule is fixed on a daily basis;
(f) The performance evaluation indicators of strategies include the rate of return and the transaction cost. The rate of return equals to the difference between the initial and final value of the portfolio divided by the initial value of the portfolio. To simplify, we only calculate the transaction cost of the risky asset.

Performance analysis of four strategies under different markets.

Table 1: The comparison of D-CPPI,D-TIPP, CPPI and TIPP under different periods

the return rates	bull	bear	first rising then falling	first falling then rising
Shanghai Composite Index of Stocks	86.95%	-68.90%	-14.08%	-16.67%
CPPI	71.99%	-14.32%	-7.07%	-7.12%
D-CPPI	99.11%	-13.27%	-7.52%	-6.87%
TIPP	31.76%	-14.32%	-2.98%	-7.12%
D-TIPP	37.79%	-13.27%	-3.04%	-6.87%

Note: m_0 is 3, a is 2, λ is 85%, the transaction cost is 3‰, the risk-free rate is 0.002%.

The Shanghai Composite Index increased from 1096.99 to 2050.81 in bull market. The index increased 86.95% from November 29,2005 to November 24,2006.Shown in table 1, the portfolio insurance strategies lose half profit because of the limitation of the floor when the price of stocks rises continually. Especially the return of the TIPP strategy is worse, which has a more conservative floor during the rising market. The return of the CPPI strategy is 71.99% and the TIPP strategy is 43.10%.In this paper, we adjust the multiplier by using the risky asset price, to make the multiplier dynamically link with the price of the risky asset. So the D-CPPI and D-TIPP strategy grasp the profit by the rising price. The D-CPPI strategy whose return is 99.11% is the most outstanding, the D-TIPP strategy whose return is 37.76% is better than the traditional TIPP strategy.

The Shanghai Composite Index fell from 5914.29 to1839.62 in bear market. The index decreased dramatically due to the impact of the world financial crisis from November 1,2007 to October 24,2008. During the period, the floor of the TIPP strategy is the bigger value of the previous floor and the product of current value of portfolio and λ .Therefore, the TIPP strategy protected the loss in the downside market and had the same return with the CPPI strategy.

When price dropped, the dynamic multipliers of the D-CPPI and D-TIPP strategy became smaller by the negative return of the risky asset price and reduced the position of the risky asset. But the multipliers of the traditional CPPI and TIPP are constant, which decease the profits of the portfolio.

In deer market, the Shanghai Composite Index first rise and then fell from June 25,2009 to June 18,2010. The peak is 3471.44 and the bottom is 2513.22 on June 18, 2010 during this period, dropping by 14.08%. The Shanghai Composite Index first fell and then raised from September 23,2004 to September 19,2005. The peak is 1464.78 and the bottom is 1011.5 on July 11,2005, dropping by16.67%. Whether in the former period or the latter period, the returns of four strategies are approximately the same. In the first falling then rising market the returns of the D-CPPI and D-TIPP strategy are higher than these of the CPPI and TIPP strategy. Because they are more sensitive to the market than the traditional strategies, when the price of the risky asset rises and the dynamic multiplier becomes bigger, they can obtain more profits than traditional strategies. In the first rising then falling market, the performance of the TIPP strategy is the best because it benefits from returns in the early rising market and has the cushion to avoid the loss risk of dropping later. Likewise, in a market with significant fluctuations, the D-CPPI and D-TIPP strategy face more risk than the traditional strategies because of their limited adjustments of the dynamic multiplier.

The impact of the initial multiplier on four strategies

Table 2: The comparison of D-CPPI,D-TIPP, CPPI and TIPP with different initial multipliers

the return rates		CPPI	D-CPPI	TIPP	D-TIPP
$m_0 = 2$	bull	35.63%	53.19%	20.85%	26.78%
	bear	-13.34%	-9.75%	-13.34%	-9.75%
	first rising then falling	-4.36%	-4.93%	-2.43%	-2.72%
	first falling then rising	-4.70%	-4.58%	-4.70%	-4.58%
$m_0 = 3$	bull	71.99%	99.11%	31.76%	37.79%
	bear	-14.32%	-13.27%	-14.32%	-13.27%
	first rising then falling	-7.07%	-7.52%	-2.98%	-3.11%
	first falling then rising	-7.12%	-6.87%	-7.12%	-6.87%
$m_0 = 4$	bull	129.00%	168.76%	43.03%	49.02%
	bear	-14.54%	-14.30%	-14.54%	-14.30%
	first rising then falling	-9.49%	-9.83%	-2.82%	-2.83%
	first falling then rising	-9.20%	-8.91%	-9.20%	-8.91%

Note: a is 2, λ is 85%, the transaction cost is 3‰, the risk-free rate is 0.002%.

Shown in table 2, the returns of four strategies all have improved with the rising initial multiplier. The ability of capturing upward profit of the D-CPPI and D-TIPP strategy becomes stronger and so the returns are higher. Specifically, the return of the D-CPPI strategy is 168.76% with $m_0 = 4$, which mainly thanks to the increasing dynamic multiplier with the rising of the risky asset price in bull market. Especially after October 26, 2006, the dynamic multiplier is above 5, and so the ability of grasping upward profit becomes much stronger.

In bear market, the investor has a greater loss with the rising initial multiplier. But the D-CPPI and D-TIPP strategy are still overcoming the CPPI and TIPP strategy. For example, when the initial multiplier is 2, the performances of D-CPPI and D-TIPP are less of 4% negative returns. Because the dynamic multiplier continuously becomes smaller, the investor must reduce the position of risky asset and increase the position of riskless asset, which therefore protects the investor's portfolio. The multipliers of the CPPI and TIPP strategy are fixed which lack the flexibility and make the investor face a greater loss when the market goes bad.

In deer market, the price fluctuation of the risky asset makes the dynamic multiplier negative or positive and fluctuate around the initial. So the returns of four strategies are almost same. The lower the initial multiplier, the better the performance shows. In the first rising then falling market, the performance of the TIPP strategy is the best with $m_0 = 4$. When the dynamic multiplier becomes bigger, the rising return is not apparent. Because of benefits from the early return, the TIPP strategy offers protections to avoid a greater loss when the price drops during June 25,2009 to June 18,2010.In the first falling then rising market, the returns of the D-CPPI and D-TIPP strategy are slightly higher than the CPPI and TIPP strategy. Therefore across all three different markets, the performances of the D-CPPI and D-TIPP strategy are better than the CPPI and TIPP strategy.

The effect of the initial floor to four strategies

Table 3: The comparison of D-CPPI,D-TIPP, CPPI and TIPP with different initial floors

the return rates		CPPI	D-CPPI	TIPP	D-TIPP
$\lambda = 80\%$	bull	95.65%	131.76%	44.06%	52.87%
	bear	-19.25%	-17.86%	-19.25%	-17.86%
	first rising then falling	-9.56%	-10.20%	-4.40%	-4.59%
	first falling then rising	-9.66%	-9.32%	-9.66%	-9.32%
$\lambda = 85\%$	bull	71.99%	99.11%	31.76%	37.79%
	bear	-14.32%	-13.27%	-14.32%	-13.27%
	first rising then falling	-7.07%	-7.52%	-2.98%	-3.11%
	first falling then rising	-7.12%	-6.87%	-7.12%	-6.87%
$\lambda = 90\%$	bull	48.33%	66.46%	20.46%	24.13%
	bear	-9.39%	-8.69%	-9.39%	-8.69%
	first rising then falling	-4.55%	-4.85%	-1.68%	-1.76%
	first falling then rising	-4.59%	-4.42%	-4.59%	-4.42%

Note: m_0 is 3, a is 2, the transaction cost is 3‰, the risk-free rate is 0.002%.

Shown in table 3, the final returns of four strategies gradually decrease with the rising multiplier, the performances of the D-CPPI and D-TIPP strategy are better than the CPPI and TIPP strategy. The return of the D-CPPI strategy is the highest and is three times as the TIPP strategy in bull market.

In bear market, the returns of all strategies increase gradually along with the floor rising from 80% to 90%. All strategies behave well. At the same time, the floor of the TIPP strategy grows

with the risk-free rate and takes the bigger value between the previous floor and the present floor - which is the product of the proportion of guarantee and the portfolio. So the returns of the CPPI and TIPP strategy are apparently the same and the D-CPPI and D-TIPP strategy are also apparently achieving the same returns.

In deer market, the final returns of four strategies gradually increase with the rising multiplier. Therefore the investor may choose a higher floor to protect portfolio better. In the first rising then falling market, the returns of the CPPI and TIPP strategy are slightly better than the D-CPPI and D-TIPP strategy. In addition the performance of the TIPP strategy behaves particularly well to protect the benefit, which is worth paying attention to by the investor. By contrast, in the first falling then rising market the returns of the D-CPPI and D-TIPP strategy are higher.

The effect of the amplifier to four strategies

Table 4: The comparison of D-CPPI,D-TIPP, CPPI and TIPP with different amplifier

the return rates		CPPI	D-CPPI	TIPP	D-TIPP
$a=1$	bull	71.99%	84.90%	31.76%	34.79%
	bear	-14.32%	-13.97%	-14.32%	-13.97%
	first rising then falling	-7.07%	-7.30%	-2.98%	-3.04%
	first falling then rising	-7.12%	-6.99%	-7.12%	-6.99%
$a=2$	bull	71.99%	99.11%	31.76%	37.79%
	bear	-14.32%	-13.27%	-14.32%	-13.27%
	first rising then falling	-7.07%	-7.52%	-2.98%	-3.11%
	first falling then rising	-7.12%	-6.87%	-7.12%	-6.87%
$a=3$	bull	71.99%	114.61%	31.76%	40.75%
	bear	-14.32%	-12.05%	-14.32%	-12.05%
	first rising then falling	-7.07%	-7.75%	-2.98%	-3.16%
	first falling then rising	-7.12%	-6.75%	-7.12%	-6.75%

Note: m_0 is 3, λ is 85%, the transaction cost is 3‰, the risk-free rate is 0.002%.

The amplifier determines the adjusting degree of the multiplier and the amount of investing in risky asset when the price of risky asset changes. The amplifier is determined by investors' risk preference. The bigger the amplifier is, the more sensitive the investor is to the returns and the risks of the price. Shown in Table 4, the returns of the D-CPPI and D-TIPP strategy increase gradually along with the amplifier rising from 1 to 3 in bull market. The protection ability of the D-CPPI and D-TIPP strategy increases gradually and the losses reduce along with the rising amplifier in bear market. The results in deer market are not consistent. In the first rising then falling market, the returns of the CPPI and TIPP strategy are worse than the traditional strategies with the rising amplifier. If the price increases continually, the adjustment range of the D-CPPI and D-TIPP strategy becomes bigger, so they may suffer a greater loss. In the first falling then rising market the returns of the D-CPPI and D-TIPP strategy are better along with the rising amplifier. Therefore the investor should choose the amplifier prudently to protect the portfolio with considerations of market conditions.

CONCLUSIONS AND SUGGESTIONS

In this paper, we propose a dynamic CPPI strategy with a dynamic multiplier (D-CPPI) and a dynamic TIPP strategy with a dynamic multiplier (D-TIPP), and compare their performance against traditional CPPI and TIPP strategy using real market data in Chinese stock market. The conclusions are:

a. The original purpose of the portfolio insurance is to protect the under any market conditions (bull, bear and deer market).The performances of the D-CPPI and D-TIPP strategy perform better than the CPPI and TIPP strategy in bull and bear markets. While in deer market the performances of four strategies are quite equivalent. Therefore the investor should choose the proper amplifier and the initial multiplier cautiously.
b. With the different initial multiplier, four strategies perform quite differently. When the initial multiplier increases gradually, the investor achieves more profits using the D-CPPI and D-TIPP strategy in bull market. In the bear market, four strategies all face more losses but the dynamic multiplier of the D-CPPI and D-TIPP strategy becomes smaller, so they can protect the investor to reduce losses properly. In the first rising then falling market, the returns of the TIPP strategy behaves best while in the first falling then rising market the performances of the D-CPPI and D-TIPP strategy are better again.
c. The rising floor means the risk aversion of the investors becomes stronger. In bull market, the performances of the D-CPPI and D-TIPP strategy are better than the traditional strategies although all returns reduce gradually. In bear market, all strategies have stronger protecting ability apparently while the performances of the D-CPPI and D-TIPP strategy are still overcoming traditional strategies. In the first rising then falling market, the performances of the D-CPPI and D-TIPP strategy are slightly lower than the traditional strategies however in the first falling then rising market the dynamic strategies achieve satisfied results again.
d. The investors choose the different amplifier according to their own risk preference. The bigger the amplifier is, the more sensitive the investor is to the returns and the risks of the price. Overall the performances of the D-CPPI and D-TIPP strategy are more satisfied than traditional strategies. Specifically, in bull market, the amplifier enlarges the position of the risky asset so as to make more profits; in bear market, the amplifier reduces the position of the risky asset to lessen the risk. By contrast in deer market with more price fluctuations, it is difficult to simply apply D-CPPI and D-TIPP strategy to make better profits and the investor should choose an amplifier prudently – not too big or too small, so as to protect the portfolio according to the risk preference and the market conditions. For example, shown in Table 4 of a=3, when the amplifier is too big, the large range of multiplier adjustment causes the loss of profits gained earlier. Similarly when a small amplifier is chosen (Table 4: a=1), the benefits from the rising market would diminish eventually.

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