NONUNIFORM SHELL/FLOW VIBRATION ANALYSIS FOR A NUCLEAR PLANT

A. A. Lakis* A. Oulmane*.
*****Department of Mechanical Engineering, École Polytechnique de Montréal.
C.P. 6079, Succ. Centre-Ville, Montréal, Québec, Canada H3C 3A7.
Corresponding Author: A. A. Lakis (aouni.lakis@polymtl.ca, +1 514-340-4711 ext. 4906).

ABSTRACT

In this study a hybrid finite element method is applied to investigate the vibration and stability of a clamped-clamped cylindrical shell subjected to flowing fluid (considering the flow to be uniform and then cross-sectionally nonuniform). The structural model is based on a combination of thin shell theory and the classical finite element method. The method is basically a hybrid finite element technique in which the shell is subdivided into cylindrical elements. The displacement functions of an element are determined by exact solution of the equations of static equilibrium of a thin cylindrical shell (Sanders' shell theory) instead of the usual and more arbitrary interpolating polynomials. This proposed hybrid finite element method is very powerful for predicting a loss of stability by buckling and for analyzing the dynamic behavior of different structures at a lower computational cost than other methods.

1. INTRODUCTION

Cylindrical shells are used in many applications such as propellant tanks or gas deployed skirts of spacecraft, structures for aerospace, aircraft, marine, etc. In the design of shells, stability and vibration under various loading conditions are two of the most important parameters to consider. Buckling is a problem related to the stability of thin structures. Analysis of structural stability requires a dynamic approach to predict this phenomenon. Buckling is frequently observed when slender thin structures (low flexural stiffness) are subjected to compressive stresses. Beyond a certain value, the applied load causes a significant change in the shape of the structure, which results in sudden or progressive appearance of wrinkles or ripples [1]. In theory, there are several criteria of stability including; the Liapounov criterion, the minimum criterion of potential energy and the criterion of variation of the total potential energy. Liapounov [2] defines the first stability criterion; it is based on the existence of a boundary delimiting the stability zone around the equilibrium position. The second criterion is an energy criterion based on the variation of the total potential energy. It applies to conservative systems: "a structure is in a stable equilibrium configuration if and only if the increase of total potential energy for any sufficiently small permissible displacement is positive".
The field of "dynamic" instability of structures has been the subject of many studies for 30 years. The term dynamic stability [3] encompasses a large number of problems (impacts, periodic or non-periodic loadings, earthquakes ...). In this presentation we will classify the types of studies currently carried out. We will then give a definition of dynamic buckling applied to the case of harmonic excitation of a structure. In the case of a simple oscillator (mass - spring), resonance occurs when the structure is excited with a harmonic force at a natural frequency. The exciting force, which is collinear with the movement of the mass, causes oscillation of the mass with a continuous increase of displacements.

Theoretical [5] and experimental [6] studies on rectangular thin plates show that parametric resonance can lead to dynamic instability of thin structures.
A large number of industrial machines consist of a fixed part (stator) and a rotating part (rotor). It is therefore possible to have critical speeds of rotation which lead to unstable vibration modes. The presence of a liquid in the moving zone also modifies the stability of the assembly. Crandall [7] carried out a detailed study of this problem type.
Currently, one of the major design challenges for thin terrestrial structures is related to seismic loadings on storage tanks or nuclear power plants. Buckling design rules take into account the effect of a seismic load (possibly multiplied by a safety factor) by transforming the loads caused by an earthquake into equivalent static loads. These are added to the thermo-mechanical loadings and a computation for static buckling is carried out. However, an earthquake does not only create an increase in static loads, it is accompanied by a dynamic effect (frequency of loading, duration), which may include movement of fluid in reservoirs (sloshing) [8].
The presence of fluid changes the vibratory response of the structure, including the coupling zones of the modes during dynamic instability calculations. Chiba and Tani [9] carried out the first experimental approach. Mylar shells, filled with liquid, were subjected to horizontal excitation forces applied at constant amplitude or acceleration. Under these conditions, parametric resonance-type instability for a frequency that combines two vibration modes can occur (these modes differ only by one in their circumferential mode (for example; 12, 13)). The liquid height influences the positions of the instability zones.
The experimental tests of Shih et *al*. and Nakamura et *al* [10, 11] show that buckling is mainly influenced by the constraints associated with the lowest mode, the higher vibration modes playing only a secondary role. The first theoretical works on this type of instability are those of Liu and Uras [12], but their study does not allow determination of all the zones of instability obtained experimentally during a horizontal excitation. The flow of a fluid in contact with structures can cause them to vibrate. Indeed, the turbulence born from each singularity encountered by the flow creates hydrodynamic fluctuating forces. The purpose of this study is to determine the risk of vibrations caused by these forces in non-uniform shells subjected to flowing fluids. Our specific structural application is a fast liquid sodium reactor.
The set of calculations conducted for the system were confined to two cases: (*i*) calculation of the frequencies of oscillation of a straight cylindrical tube (shell) with uniform internal flow of varied flow velocity, and (*ii*) the same for a flow of cross-sectionally nonuniform flow velocity. The final aim of these calculations was to ascertain whether the system developed fluid-elastic instabilities (buckling or flutter) at the design flow velocities.

2. THE ANALYTICAL MODEL FOR UNIFORM FLOW

In this work, a combined formulation of shell theory and a hybrid finite element method is applied to model the shell structure. The shell is subdivided into cylindrical elements and the displacement functions of each element are determined by exact solution of the equations of static equilibrium of a thin cylindrical shell (Sanders' shell theory) instead of the usual and more arbitrary interpolating polynomials. In so doing, the accuracy of the formulation is less affected as the number of elements is decreased and as higher shell modes are computed (shell-mode wavenumber n, and axially-disposed wavenumber m (see Appendix A, Figure on p. 2)). This represents a

significant advantage over polynomial interpolation, in terms of accuracy as well as computational cost.
The finite element is a 2-node, 4-dof-per-node cylindrical frustum, for which no circumferential modelling is required (Figure 1). This allows full analytical shell treatment over the element. Moreover, this formulation allows convenient axial modelling in the case of axially nonuniform shells. As is customary for the finite element method in general, axial modelling (involving a number of elements *N*) ensures convergence to stationary values of the eigen-properties sought.
Once the mass and stiffness matrices are known for each shell element (m_s and k_s), the global mass and stiffness matrices for the whole shell (M_s and K_s) can be constructed by assembling the element matrices. The equation of motion may be written as follows:

$$M_s\ddot{\Delta} + K_s\Delta = 0 \quad (1)$$

where $\Delta = \{\delta_1; \delta_2; \ldots\ldots\ldots \delta_{N+1}\}^T$. δ_i is the displacement vector of the *i*th node of the assembly (δ each involving the axial, transverse, circumferential, and rotational displacements: $\delta_i = \left\{U_i; W_i, \left(\frac{dW_i}{dx}\right) i, V_i\right\}^T$, for each value of *n*). When the shell is filled with flowing fluid, it is subject to inertial forces, centrifugal forces and Coriolis-type forces coupled with elastic deformation of its walls. The mathematical model in this case leads to the following equation:

$$(M_s - M_f)\ddot{\Delta} - C_f\dot{\Delta} + (K_s - K_f)\Delta = 0 \quad (2)$$

where Δ is a displacement vector, M_s and K_s are, respectively, the mass and stiffness E matrices of the shell *in vacuo*, and M_f, C_f and K_f represent the inertia, Coriolis and centrifugal forces of the flowing fluid.
Consider now the way in which the shell interacts with the fluid. It is assumed that;
(*i*) the flow is potential and the fluid incompressible, considering further the limiting case of small vibrations; (*ii*) the pressure of the fluid on the walls is purely lateral and the velocity distribution throughout the fluid cross-section is constant; (*iii*) the internal pressure is not unduly high, so that pressurization of the shell is negligible; in any case, internal pressurization is known to stabilize the system.
The governing equation for the potential flow is given by:

$$\nabla^2\Phi = \left(\frac{1}{c^2}\right)[\ddot{\Phi} + 2U_x\dot{\Phi}' + U_x^2\Phi''] \quad (3)$$

where c is the velocity of sound in the fluid (here taken to be infinite for incompressible i flow) and U_x is the velocity of the fluid throughout the cross-section of the shell; the overdot and prime stand for $\partial(\ )/\partial\tau$ and $\partial(\ )/\partial x$, respectively, τ is the time, and Φ is the potential of the disturbances. The perturbation velocities are given by:

$$V_x = Ux + \partial(\Phi)/\partial x,$$

$$V_\emptyset = (1/r) + \partial(\Phi)/\partial\emptyset, \quad (4)$$

$$V_r = \partial(\Phi)/\partial r,$$

where V_x, $V_\emptyset$ and V_r, are the velocities in the axial, circumferential and radial directions.

The dynamic condition on the shell surface is given by Bernoulli's equation for disturbed motion, and it allows us to obtain the pressures of the fluid on the shell walls as follows:

$$P_i = -\rho_i\left(\dot{\Phi}_i + U_{xi}\Phi'_i\right)_{r=a}$$

$$P_e = \rho_e\left(\dot{\Phi}_e + U_{xe}\Phi'_e\right)_{r=a+t} \quad (5)$$

a and *t* are the internal radius and thickness of the shell element, respectively, while suffixes *i* and *e* indicate the internal and external region *vis-a-vis* the shell; [for the case of interest here, U_{xe}= 0 and the effect of the external fluid is neglected: $P_e = 0$]
The impermeability condition:

$$(V_r)_{r=a} = (\partial(\Phi)/\partial r)_{r=a} = (\dot{W} + U_x W') \quad (6)$$

must be satisfied at all points of contact between the shell surface and fluid, and this allows the pressures in (5) to be expressed in terms of the radial displacement *W*.

Assuming that the displacement functions are similar to those of the shell *in vacuo* and that:

$$\Phi = \sum_{k=1}^{8} R_k(r) S_k(x, \emptyset, \tau) \quad (7)$$

we obtain the internal and external pressure acting on the wall of the shell as follows [for details, the reader is referred to Lakis & Laveau (1991)]:

$$P = \sum[-\rho_i r_k + a\rho_e s_k]\ddot{W}_k + 2a[-\rho_i U_{xi} r_k + \rho_e U_{xe} s_k]\dot{W}'_k + a[-\rho_i U^2{}_{xi} r_k + \rho_e U^2{}_{xe} s_k]W''_k \quad (8)$$

where $W_k = C_k \exp\left[i\frac{\lambda_k x}{a+i\omega\tau}\right]\cos n\emptyset$; ρ_i and ρ_e are the densities of the fluid inside and outside the shell, respectively, ω is the circular frequency and τ, and s_k are given in terms of the Bessel functions of the first and second kind of order *n*, J_{n+1} and Y_{n+1}, respectively. By carrying out the necessary matrix operations of the finite element

8x8

method and integrating, we obtain the inertia, centrifugal (stiffness) and Coriolis (damping) forces of the moving fluid acting on the wall of the shell as follows:

$$[m_f] = [A^{-1}]^T [S_F][A^{-1}]$$

$$[k_f]_{8x8} = [A^{-1}]^T [G_F][A^{-1}] \quad (9)$$

$$[c_f]_{8x8} = [A^{-1}]^T [D_F][A^{-1}]$$

where the matrices S_F, D_F and G_F are given in terms of Bessel functions of the first and second kind, and in terms of the following dimensionless parameters:

$$\delta_i = (a_i/t_1)(\rho_i/\rho_1), \qquad \delta_e = (a_e/t_1)(\rho_e/\rho_1),$$
$$U_0^2 = p(1,1,1)/\rho_1 t_1, \qquad \bar{U}_i = U_{xi}/U_0,\ \bar{U}_e = U_{xe}/U_0, \qquad (10)$$
$$\gamma_i = a_i/r_1,\ \gamma_e = a_e/r_1$$

where ρ_1, $t_1$and r_1 are, respectively, the density, thickness and radius of the first shell. Element $p(1, 1, 1)$ is the first element of the elasticity matrix [P] given by Lakis & Trinh [13]. Since the mass, damping and stiffness matrices are known for each fluid element, the global mass, damping and stiffness matrices for whole column of fluid, M_f, C_f; and K_f, respectively, may be constructed by superposition in the normal manner. Between fluid elements, continuity is satisfied exactly by requiring an exact match of velocity normal to the element with the velocity of the adjacent element at all points on the interface. Each of these (square) matrices will be of order $4(N + 1)$, where N is the total number of finite elements.
The dimensionless parameters V and Ω are defined by:

$$\mathrm{V} = U/U_0,\ \Omega = \omega/\omega_0\ \varepsilon = (R/t)(\rho_f/\rho_s) \qquad (11)$$

Where $U_0 = [E/\rho_s(1-\upsilon^2)]^{1/2}$, $\omega_0 = U_0/R$. The physical characteristics of the shell are defined by the mean radius R, wall thickness t, Young's modulus E, Poisson ratio ν and density ρ_s. U and ω are the dimensional flow velocity and natural frequency, respectively.
In Appendix A, two nondimensional V and Ω are used for the reader's convenience. Those in (11) correspond to V_2 and Ω_2 .

3. THE ANALYTICAL MODEL FOR CROSS-SECTIONALLY NONUNIFORM FLOW

This analysis is presented in greater detail than in Section 2 because it is not readily found in any published paper. It is based on the work presented by Selmane & Lakis [14], wherein some details may be found.

Sanders' equations of motion may be written as:

$$L_k(U, V, W, P_{ij}) = 0, \qquad k = 1,2,3 \tag{12}$$

where U, V, W are the axial, tangential and radial displacements of the mean shell surface and P_{ij} are elements of the matrix of elasticity $[P]$, which is generally anisotropic.

The strain-displacement relation is given by:

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ 2\bar{\varepsilon}_{x\theta} \\ \mathrm{K}_x \\ \mathrm{K}_\theta \\ 2\bar{\mathrm{K}}_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial U}{\partial x} \\ \frac{1}{R}\frac{\partial V}{\partial \theta} + \frac{W}{R} \\ \frac{\partial V}{\partial x} + \frac{1}{R}\frac{\partial U}{\partial \theta} \\ -\frac{\partial^2 V}{\partial x^2} \\ -\frac{1}{R^2}\frac{\partial^2 W}{\partial \theta^2} + \frac{1}{R^2}\frac{\partial V}{\partial \theta} \\ -\frac{2}{R}\frac{\partial^2 W}{\partial x \partial \theta} + \frac{3}{2R}\frac{\partial V}{\partial \theta} - \frac{1}{2R^2}\frac{\partial U}{\partial \theta} \end{Bmatrix} \tag{13}$$

The finite element used is shown in Figure 2. It is a cylindrical panel segment defined by two line-nodes, i and j, within which an averaged flow velocity is defined. Each node has four degrees of freedom: three displacements (axial, radial and circumferential) and one rotation. The panels are assumed to be freely simply supported (V = W = 0) along their curved edges and to have arbitrary straight edge boundary conditions.
For motions associated with the mth axial wave number, we may write:

$$\{U, V, W\} = \{A, B, C\}\exp(\mathrm{i}mx + \eta\theta) \tag{14}$$

then, by substituting Equation (14) into (12), we obtain:

$$\begin{Bmatrix} U(x,\theta) \\ W(x,\theta) \\ V(x,\theta) \end{Bmatrix} = [T_m][R]\{C\} \tag{15}$$

where $[R]$ is a (3 x 8) matrix given by:

$$R(1,j) = \alpha_j e^{\eta_j \theta}, R(2,j) = e^{\eta_j \theta}, R(3,j) = \beta_j e^{\eta_j \theta} ; j = 1,2, \dots \dots 8 \quad (16)$$

η_j $(j = 1,2, \dots \dots 8)$ are the roots of the characteristic equation of an empty shell. Since A, B and C are not independent, we may write $A = \alpha C$ and $B = \beta C$, which determine α_i and β_i.$\{C\}$ is a vector of eight constants which are linear combinations of the C_j. The eight C_j are the only free constants, which must be determined from eight boundary conditions, four at each straight edge of the finite element.

For the case of clamped-clamped shells, Equation (14) becomes:

$$\begin{Bmatrix} U(x,\theta) \\ W(x,\theta) \\ V(x,\theta) \end{Bmatrix} = \begin{bmatrix} \sin m\pi x/L & 0 & 0 \\ 0 & \sin m\pi x/L & 0 \\ 0 & 0 & \sin m\pi x/L \end{bmatrix} \begin{Bmatrix} U_m(\theta) \\ W_m(\theta) \\ V_m(\theta) \end{Bmatrix} = [T_m] \begin{Bmatrix} U_m(\theta) \\ W_m(\theta) \\ V_m(\theta) \end{Bmatrix}$$
(17)

We now express the nodal displacement vectors as follows:

$$\{\delta_i\} = \left\{ U_{mi};\ W_{mi}; \left(\frac{dW_m}{d\theta}\right); V_{mi} \right\}^T \quad (18)$$

Each $\{\delta_i\}$ may be determined from Equation (15), where θ in $[R]$ now has a definite value, $\theta = 0$ or $\theta = \emptyset$, as the case may be; hence we obtain:

$$\begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix} = [A]\{c\} \quad (19)$$

where the elements of matrix $[A]$ are determined from those of matrix $[R]$ and given by:

$$A(1,j) = \alpha_j \qquad A(2,j) = 1 \quad A(3,j) = \eta_j$$

$$A(4,j) = \beta_j \qquad A(5,j) = \alpha_j e^{\eta_j \emptyset} \quad A(6,j) = e^{\eta_j \emptyset}$$

$$A(7,j) = \eta_j e^{\eta_j \emptyset} \qquad A(8,j) = \beta_j e^{\eta_j \emptyset} \quad j = 1, \dots \dots 8$$

Finally, combining Equations (15) and (19), we obtain:

$$\begin{Bmatrix} U(x,\theta) \\ W(x,\theta) \\ V(x,\theta) \end{Bmatrix} = [T_m][R][A^{-1}]\begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix} = [N]\begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix} \tag{20}$$

which defines the displacement functions.
The strains are related to the displacements through Equations (13). Accordingly, we now express $\{\varepsilon\}$ in terms of $\{\delta_i\}$ and $\{\delta_j\}$, and after lengthy manipulations we obtain:

$$\{\varepsilon\} = \begin{bmatrix} [T_m] & 0 \\ 0 & [T_m] \end{bmatrix} [Q][A^{-1}]\begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix} = [B]\begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix} \tag{21}$$

where $[Q]$ is a (6 x 8) matrix. The corresponding stresses may be related to the strains by the elasticity matrix $[P]$:

$$\{\sigma\} = [P]\{\varepsilon\} = [P][B]\begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix} \tag{22}$$

The matrix P of an anisotropic shell is given by:

$$[P] = \begin{bmatrix} p_{11} & p_{12} & 0 & p_{14} & p_{15} & 0 \\ p_{21} & p_{22} & 0 & p_{24} & p_{25} & 0 \\ 0 & 0 & p_{33} & 0 & 0 & p_{36} \\ p_{41} & p_{42} & 0 & p_{44} & p_{45} & 0 \\ p_{51} & p_{52} & 0 & p_{54} & p_{55} & 0 \\ 0 & 0 & p_{63} & 0 & 0 & p_{66} \end{bmatrix} \tag{23}$$

The elements p_{ij} of $[P]$ characterize the shell anisotropy, which depends on the mechanical properties of the material of the structure.
The mass and stiffness matrices, $[m_s]$ and $[k_s]$ respectively, for one finite element may be written as follows:

$$[ms] = \rho_s t \int_0^L \int_0^\phi [N]^T [N] dA \quad \text{and } [ks] = \int_0^L \int_0^\phi [B]^T [P][B] dA \tag{24}$$

where ρ_s is the density of the shell, t its thickness, dA a surface element, $[P]$ the elasticity matrix and the matrices $[N]$ and $[B]$ are obtained from Equations (20) and (21), respectively.
The matrices $[m_s]$ and $[k_s]$ were obtained analytically by carrying out the necessary matrix operations and integration over x and θ in Equation (24). The global matrices $[M_s]$ and $[k_s]$ may be obtained, respectively, by superimposing the mass $[m_s]$ and

stiffness $[k_s]$ matrices for each individual panel finite element. With the global matrices assembled, the total equation of motion has the same form as Equation (1).

Also in this case, the fluid is modelled by potential flow theory and hence Equations (3) to (6) apply.
To proceed, we take:

$$W(x,\theta,t) = \sum_{j=1}^{8} Ce^{\eta_j \theta} \sin\frac{m\pi x}{L} e^{i\omega t} \tag{25}$$

Assuming then,

$$\Phi\ (x,\theta,r,t) = \sum_{j=1}^{8} R_j(r) S_j(x,\theta,t) \tag{26}$$

and applying the impermeability condition, Equation (6), with the radial displacement given by relation (25), we determine the function $S_j(x,\theta,t)$. Introducing this explicit term $S_j(x,\theta,t)$ into Equation (26) and then into Equation (5), we find a relation for the dynamic pressure as a function of the displacement W_j and the function $R_j(r)$:

$$P_u = -\rho_f \sum_{j=1}^{8} \frac{R_j(r)}{\frac{d\left(R_j(r)\right)}{dr}} [\ddot{W}_j + 2U_{xu}\dot{W}'_j + U^2{}_{xu}W''_j] \tag{27}$$

where $\dot{()}$ and ( )' represent $\partial(\quad)/\partial t$ and $\partial(\quad)/\partial x$, respectively.
By using relation (3) with $c = 0$, we obtain the following Bessel equation:

$$r^2 \frac{d^2R_j(r)}{dr} + r\frac{dR_j(r)}{dr} + R_j(r)\left[\left(\frac{im\pi}{L}\right)^2 r^2 - \left(i\eta_j\right)^2\right] = 0 \tag{28}$$

where i is the complex number, $i^2 = -1$, and η_j is the complex solution of the characteristic equation.
The general solution of Equation (28) is given by:

$$R_j(r) = AJ_{i\eta_j}\left(\frac{im\pi}{L}r\right) + BY_{i\eta_j}\left(\frac{im\pi}{L}r\right) \tag{29}$$

where $J_{i\eta_j}$ and $Y_{i\eta_j}$ are, respectively, the Bessel functions of the first and second kind of order $i\eta_j$.

For internal flow, the solution (29) must be finite on the axis of the shell ($r = 0$); this means we have to set the constant B equal to zero.
Finally, we obtain the equation for the pressure on the wall as follows:

$$P_u = -\rho_u \sum_{j=1}^{8} Z_{uj}\left(\frac{im\pi R_u}{L}\right) [\ddot{W}_j + 2U_{xu}\dot{W}'_j + U^2{}_{xu}W''_j] \tag{30}$$

where $\dot{()}$ and ( )' represent $\partial(\quad)/\partial t$ and $\partial(\quad)/\partial x$, respectively and

$$Z_{uj}\left(\frac{im\pi R_u}{L}\right) = \frac{R_u}{i\eta_j - \frac{im\pi R_u}{L}\frac{J_{\eta_j+1}(\frac{im\pi R_u}{L})}{J_{\eta_j}(\frac{im\pi R_u}{L})}} \text{ if } u = i \quad (31)$$

$$Z_{uj}\left(\frac{im\pi R_u}{L}\right) = \frac{R_u}{i\eta_j - \frac{im\pi R_u}{L}\frac{Y_{\eta_j+1}(\frac{im\pi R_u}{L})}{Y_{\eta_j}(\frac{im\pi R_u}{L})}} \text{ if } u = i \quad (32)$$

where $\eta_j(j = 1,...,8)$ are the roots of the characteristic equation of the empty shell, m is the axial mode number, R the mean radius of the shell, and L its length. The subscript u is equal to i for internal flow and is equal to e for external flow.

By introducing the displacement function (20), into the dynamic pressure expression (30) and performing the matrix operation required by the finite element method, the mass, damping and stifiness matrices for fluid are obtained by evaluating the following integral:

$$\int_A [N]^T \{P_u\} dA \quad (33)$$

leading to

$$[m_f] = [A^{-1}]^T[S_f][A^{-1}], [c_f] = [A^{-1}]^T[D_f][A^{-1}], [k_f] = [A^{-1}]^T[G_f][A^{-1}] \quad (34)$$

The matrix $[A]$ is given by Equation (19) and the elements of $[S_f]$, $[D_f]$ and $[G_f]$ are:

$$S_f(r,s) = -\frac{RL}{2} I_{rs}(\rho_i Z_{is} - \rho_e Z_{es}) \quad (35)$$

$$D_f(r,s) = \frac{Rm^2\pi^2}{4L} I_{rs}(\rho_i U_{xi} Z_{is} - \rho_e U_{xe} Z_{es}) \quad (36)$$

$$G_f(r,s) = \frac{Rm^2\pi^2}{2L} I_{rs}(\rho_i U^2{}_{xi} Z_{is} - \rho_e U^2{}_{xe} Z_{es}) \quad (37)$$

where $r, s = 1, \ldots, 8$, ρ is the density of the fluid, and U_x, its velocity. Z is defined by relations (31) and (32), the subscript i means internal flow and e means external flow. I_{rs} is defined by:

$$I_{rs} = \frac{1}{\eta_r+\eta_s}[e^{(\eta_r+\eta_s)\emptyset} - 1] \quad \text{for } \eta_r + \eta_s \neq 0 \quad (38)$$

$$I_{rs} = \emptyset \quad \text{for } \eta_r + \eta_s = 0$$

in which $r, s = 1, \ldots, 8$, η is the root of the characteristic equation of the empty shell and $\emptyset$ is the angle for one finite element.

Finally, the global matrices $[M_f]$, $[C_f]$ and $[K_f]$ may be obtained, respectively, by superimposing the mass $[m_f]$, damping $[c_f]$ and stiffness $[k_f]$ matrices for each individual fluid finite element. The equations of motion are then reduced to a standard first-order eigenvalue problem, from which the eigenfrequencies may be computed.

4. CALCULATIONS, RESULTS AND DISCUSSION

The calculations conducted and the results obtained are presented in Appendices *A* and *B*. Here, we shall discuss the results in these two appendices.

4.1 Uniform flow case

The results are given in Appendix *A*.

The geometric and material data for the shell and the fluid are given on p. 2. Also shown is a graphical representation of the wavenumbers n and m (though the graphs for the axial mode shapes shown are for simply supported shells - purely because it was easier to draw).

The frequencies of the empty and liquid-filled shell are given on pp. 3 and 4a (zero flow velocity) in dimensional terms, and the latter on p. 4 in dimensionless form.

It is seen that for $m = 1$ and $m = 2$ the minimum frequency corresponds to $n = 2$ and $n = 3$, respectively.

Page 5 details the two nondimensionalization schemes used to present the results that follow (with flow). Both these schemes have been used in the literature of shells with flow by different sets of researchers.

The results with flow are given on pp. 6-15. The flow velocity ranges to $U = 200$ m/s, well beyond the design value.

As can be seen in the results for $n = 2$ for example (p. 8), the loci of the modes do not lie on the Imag(Ω_1) axis. Real (Ω_1) decreases as V_1 decreases. Imag(Ω_1) is greater than zero, but small. The key to understanding the predicted dynamic behaviour is by recognizing that the ratio Imag(Ω_1)/Real(Ω_1) is of the order of 10^{-6} to 10^{-4}. Hence Imag(Ω_1) ≈ 0.

It is recalled that, in the absence of dissipation, this conservative system should lose stability by divergence (buckling) when Rea1(Ω_1) = 0 (see Paidoussis & Denise [15], Weaver & Myklatun [16], Amabili, Pellicano & Paidoussis [17]).

Taking Imag(Ω_1) ≈ 0, it is clear that divergence for this system would probably occur in the $n = 2$ mode for $V_1 >> 6.53$, corresponding to $U >> 200$ m/s, which is much larger than the design $U = 9.2$ m/s [(1)]. Even taking into account imperfections and nonlinear effects to the very maximum of a factor of 3, the results presented here show that, without a doubt, this system is absolutely stable at the design flow, with a large margin of safety.

One may wonder why we decided to use a complex finite element method for such a simple system instead of one of the simpler classical methods cited in the foregoing?"

The answer is this. If the system had been found to be unstable, or even close to instability, then a more sophisticated analysis would have been necessary, involving the spatially bent cold and hot legs of the target system, with varying properties. This would only be feasible if the hybrid finite element method was used in these calculations.

[(1)] Note that the figure for $n = 5$, pp. 14 and 15, begins at Real (Ω_1) = 225, not zero.

4.2 Cross-sectionally nonuniform flow

Results for the following case are given in Appendix B.
In this case, each cross-section was subdivided into a number of segments, for example the 12 segments shown on p. 3 of Appendix B. Within each segment, an average velocity is calculated from information (distributions) such as shown on p. 4, in the manner detailed in the diagram on p. 2. Thus, for the segment shown,

$$U_{av} = \left(\sum_i v_i A_i\right) / \left(\sum_i A_i\right)$$

This, for the example shown, results in the segment flow rates given in the Table on p. 3.
As in the previous case, these calculations required the development of a new formulation of the hybrid finite element method. Furthermore, these calculations could only be completed using this new method.
The results show that, again in this case the system is stable at the design flow with a very large margin of safety.

Comparing the uniform and nonuniform flow cases, e. g. for n = 2, we see that the frequencies Real (Ω_1) are higher for nonuniform flow. This means that the system is even more stable under these flow conditions, although for n = 4 the frequency values are very slightly lower (at U = 9.2 m/s, Real(Ω_1) = 138 versus 141).
The mode shapes are shown on pp. 9 and 10. Those for nonuniform flow look deformed compared to a uniform flow distribution, but they still retain the characteristics of one, two, three and four nodal diameters for n = 1, 2, 3 and 4.

5. CONCLUSION

It has been shown, first considering the flow to be uniform and then cross-sectionally nonuniform, that the clamped-clamped shell under study is stable by a very large margin at the design flow. Even at an average flow velocity of 200 m/s, this system is stable. Additionally, even considering the effect of imperfections and nonlinear softening effects, this system is stable at the design flow velocity of 9.2 m/s with a large safety margin.
It should be noted that in the analysis it is assumed that the boundary conditions correspond to a clamped shell at both ends, and hence loss of stability can only be by divergence (buckling). This is not necessarily so if the system is cantilevered or if it is flexibly supported at one or both ends, although the critical velocities for divergence or flutter in such cases should be of the same order of magnitude as for clamped ends.
Finally, although the system is stable, it will still be subject to vibration induced by boundary-layer (near-field) and far-field noise. Even low-amplitude vibration can eventually lead to fatigue. A successful model for predicting this flow-induced vibration in the shell modes of the system is available (Lakis & Kerboua 2016 [18]). Moreover, the authors have the tools for predicting low-amplitude high-cycle fatigue.

REFERENCES

1) KOITER W. T. *A consistent first approximation in general theory of buckling of structures. IUTAM Symposium, Harvard University, June 1974, p. 133-147.*

2) PIGNATARO M., RIZZI N., LUONGO A. *Stability, Bifurcation and post-critical behavior of elastic structures. Amsterdam: editions Elsevier, 1991, 358p.*
3) SIMITSES G.J. *Instability of dynamically loaded structures. Appl. Mech. Rev., 1987, Vol. 40, N° 10, p. 1403-1408.*
4) FARADAY M. *On a peculiar class of acoustical figures and on certain forms assumed by a group of particles upon vibrating elastic surfaces. Phil Trans R. Soc London, 1831, vol. 121, p. 229-318.*
5) OSTIGUY G., NGUYEN H. *Dynamic stability and resonance of rectangular plates. Mécanique Matériaux Électricité, Octobre/Novembre 1982, N° 394-395, p. 465-471.*
6) RAVINGER J. *Dynamic post-buckling behaviour of plate girders. Journal of Construct. Steel Research, 1992, N° 21, p. 175-194.*
7) CRANDALL S. H. *The physical nature of rotor instability in Rotor dynamical instability, M. L. ADAMS, ASME-AMD, 1983, Vol. 55, p. 1-18.*
8) OHAYON R., MORAND H.J.-P. *Interactions fluides-structures. Recherches en Mathématiques Appliquées, RMA N° 23, Paris : Editions MASSON, 1992, 212 p.*
9) CHIBA M., TANI J. *Experimental study on the dynamic stability of nuclear containment vessels under horizontal excitation. SMIRT9, Lausanne, 1987, Vol. K, p. 447-451.*
10) SHIH C.F., BABCOCK C.D. *Buckling of oil storage tanks in SSPL tank farm during the 1979 Imperial Valley earthquake. Journal of Pressure Vessel Technology, May 1987, Vol. 109, p. 249-255.*
11) KOKUBO K., MATSUURA S., NAKAMURA H., OGISI S., OHTSUBO H. *Shear-bending buckling analyses of fast breeder reactor main vessels. Nuclear Engineering and Design, 1993, Vol. 36, n° 3, p. 259-266.*
12) LIU W.K., URAS R.A. *Dynamic buckling of liquid-filled shells under horizontal excitation. Journal of sound and vibration, 1990, N° 141, p. 389-408.*
13) Lakis, A.A. & Trinh, N.D. Liquid phase velocity in turbulent flow of a water/dispersed bubble mixture in large diameter horizontal pipes. European Journal of Engineering and Technology, (2017), 5(1), 62-89.
14) Selmane, A. & A.A. Lakis. Vibration analysis of anisotropic open cylindrical shells subjected to a flowing fluid. Journal of Fluids and Structures, 1997, 11, 111-134.
15) Païdoussis, MP, and Denise, J.-P. Flutter of thin cylindrical shells conveying fluid. Journal of Sound and Vibration, 1972, 20, 9-26.
16) Weaver, D.S. & Myklatun, B. On the stability of thin pipes with an internal flow. Journal of Sound and Vibration, 1973, 31, 399-410.
17) Amabili, M., Pellicano, F and Païdoussis, MP. Nonlinear dynamics and stability of circular cylindrical shells containing flowing fluid. I. Stability. Journal of Sound and Vibration, 1999, 225, 655-699.
18) Kerboua, Y. & Lakis, A.A. Numerical model to analyze the aerodynamic behavior of a combined conical cylindrical shell. Aerospace Science and Technology, (2016), 58, 601-617.

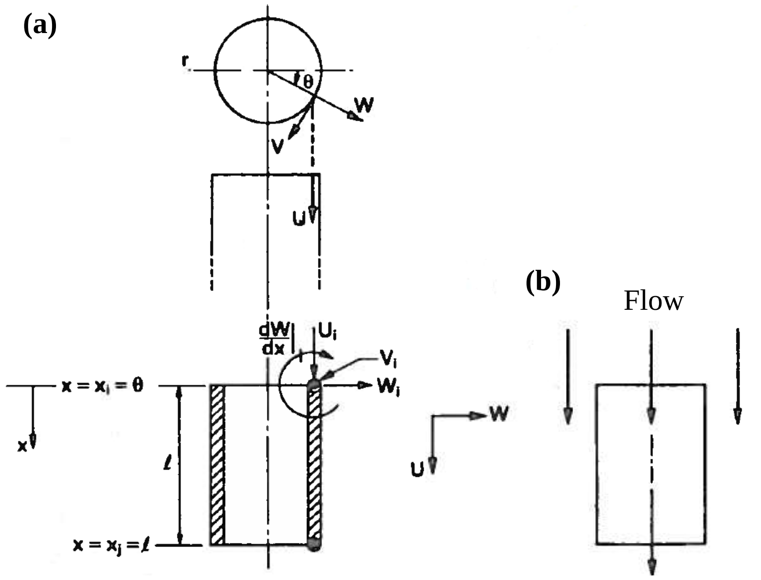


Figure 1. Displacement and degrees of freedom at a node. (a) Cylindrical hybrid finite element in *vacuo*. (b) Cylindrical hybrid finite element with flowing fluid (internal and/or external flow).

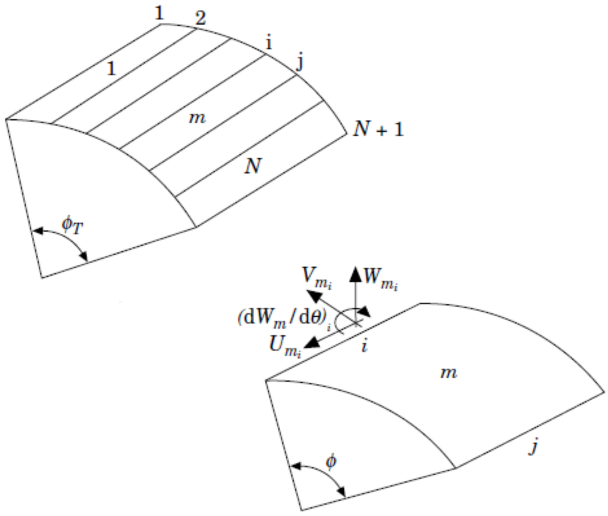


Figure 2. (a) Finite element idealization; (b) Nodal displacements at node *i* (*N* is the number of finite elements).

ISSN 2059-9951

INPUT DATA

Shell characteristics:

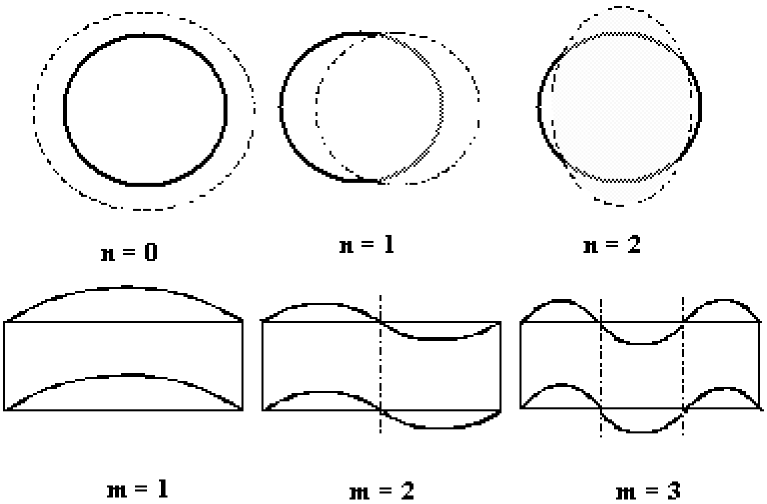
R	= 627.05	mm	= 24.687 in
t	=15.9	mm	= 0.626 in
ρ_s	= 7.86	gf/cm^3	$=7.354802\ e\text{-}4\ lb*sec^2/\ in^4$
ρ_l	= 0.85	gf/cm^3	$=7.95366\ e\text{-}5\ lb*sec^2/\ in^4$
E	= 1.694 e+4	kgf/mm^2	$=1.66124651\ e+11\ Pa=2.40943459\ e+7\ lb/in^2$
L	= 7130	mm	=280.7086 in
ν	= 0.3		

B.C.: clamped - clamped

In this development: **m** = axial wave number;

and

n = circumferential wave number.



ISSN 2059-9951

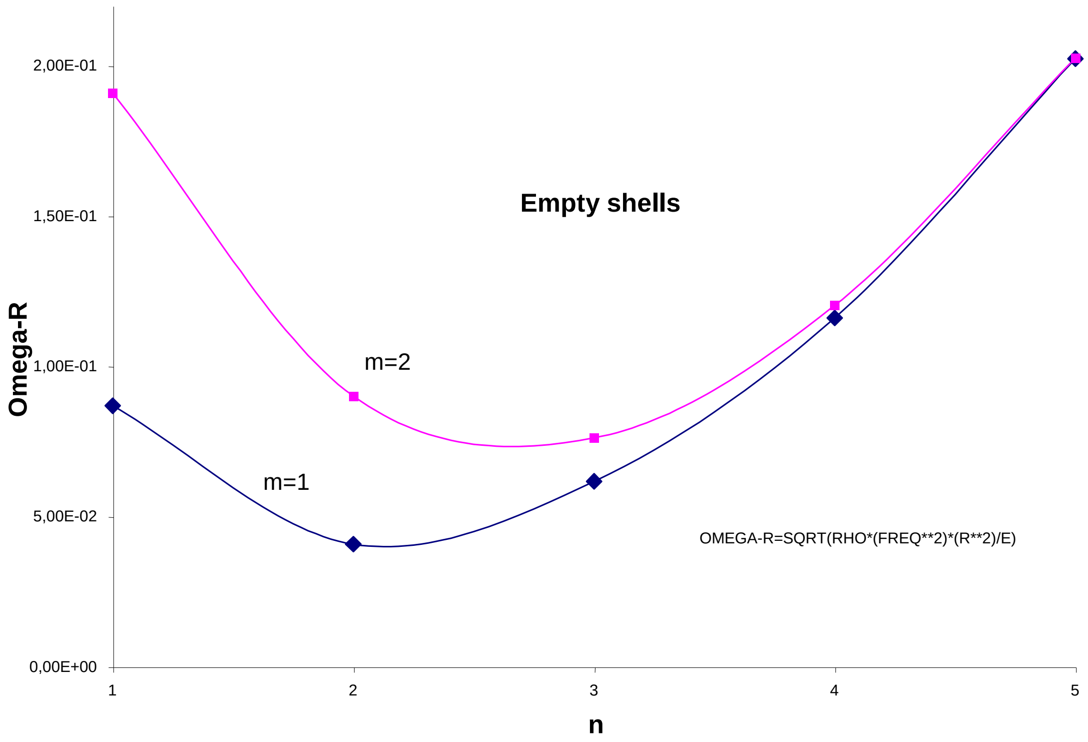
EMPTY SHELLS

R/t = 39,437 L/R = 11,37

B.C.: clamped – clamped

Frequencies [Hz]

m\n	1	2	3	4	5
1	101,29	47,56	71,93	135,46	236,18
2	222,73	105,10	88,85	140,30	236,41
3	369,64	183,82	124,14	152,51	238,93
4	531,70	277,13	174,85	174,51	246,35
5	703,70	382,62	237,64	206,84	260,17
6	811,04	497,71	310,59	248,83	280,91
7	873,50	616,34	391,74	299,33	308,43
8	988,30	725,09	475,91	356,04	341,87
9	1013,80	803,16	548,48	410,82	377,09
10	1098,40	1530,90	1049,00	615,47	461,31


Empty shells
m=2
m=1
OMEGA-R=SQRT(RHO*(FREQ**2)*(R**2)/E)
Omega-R
2,00E-01
1,50E-01
1,00E-01
5,00E-02
0,00E+00
1
2
3
4
5
n

SHELL FILLED WITH LIQUID

R/t = 39,437 L/R = 11,37

B.C.: clamped – clamped

Capital Omega1

m\n	1		2		3		4		5	
	Real	**Imaginary**	**Real**	**Imaginary**	**Real**	**Imaginary**	**Real**	**Imaginary**	**Real**	**Imaginary**
1	8,55E+01	-7,66E-28	4,28E+01	-4,63E-31	7,03E+01	-3,28E-31	1,41E+02	-1,12E-31	2,61E+02	7,04E-32
2	1,92E+02	-7,12E-27	9,53E+01	-8,73E-30	8,71E+01	-2,41E-31	1,46E+02	-7,31E-32	2,57E+02	6,95E-32
3	3,25E+02	-3,24E-26	1,68E+02	-5,13E-29	1,22E+02	1,38E-31	1,59E+02	-2,03E-32	2,57E+02	8,72E-32
4	4,74E+02	-9,57E-26	2,56E+02	-1,74E-28	1,73E+02	1,31E-30	1,83E+02	4,62E-32	2,69E+02	6,14E-32
5	6,33E+02	-2,16E-25	3,57E+02	-4,46E-28	2,37E+02	4,05E-30	2,18E+02	1,27E-31	2,85E+02	4,99E-32
6	7,94E+02	-4,00E-25	4,71E+02	-9,55E-28	3,12E+02	9,49E-30	2,63E+02	2,16E-31	3,09E+02	3,35E-32
7	9,46E+02	-6,24E-25	5,91E+02	-1,79E-27	3,96E+02	1,92E-29	3,18E+02	2,80E-31	3,40E+02	6,06E-33
8	1,07E+03	-8,31E-25	7,07E+02	-2,90E-27	4,85E+02	3,49E-29	3,80E+02	2,39E-31	3,79E+02	-5,31E-32
9	1,15E+03	-9,68E-25	7,94E+02	-3,97E-27	5,63E+02	5,52E-29	4,40E+02	-5,44E-32	4,18E+02	-2,09E-31

$\boldsymbol{\Omega_1}$ = CAPITAL OMEGA1= OMEGA/OMEGA.ZERO1 ; OMEGA [RAD/SEC]

OMEGA.ZERO1=U.ZERO1/TOTAL.LENGTH= .42947609640665E+01

AND U.ZERO1= (PI**2/TOTAL.LENGTH)*SQRT {K/ [RHO (1)*TH (1)]} = .12055394026135E+04

WHERE:

K= (E (1)*TH (1) **3)/ (12.*(1.-NU (1) **2))

ISSN 2059-9951

SHELL SUBJECTED TO A FLOWING FLUID

The following results are given in function of two non-dimensional terms:

1

$$V_1 = \frac{U}{\left(\frac{\pi^2}{L}\right)\sqrt{\frac{E\,t^2}{12\,\rho_S\left(1-\upsilon^2\right)}}}$$

$$\Omega_1 = \frac{\Omega\,(\text{rad/sec})}{\left(\frac{\pi^2}{L^2}\right)\sqrt{\frac{E\,t^2}{12\,\rho_S\left(1-\upsilon^2\right)}}}$$

2

$$V_2 = \frac{U}{\sqrt{\frac{E}{\rho_S\left(1-\upsilon^2\right)}}}$$

$$\Omega_2 = \frac{\Omega\,(\text{rad/sec})}{\left(\frac{1}{R}\right)\sqrt{\frac{E}{\rho_S\left(1-\upsilon^2\right)}}}$$

n = 1

R / t = 40.43, L/R=11.09, ν = 0.3

B.C. : clamped - clamped

DIMENSIONLESS PARAMETERS NUMBER 1:

Ω_1 = CAPITAL OMEGA1= OMEGA/OMEGA.ZERO1 ; OMEGA [RAD/SEC]

OMEGA.ZERO1=U.ZERO1/TOTAL.LENGTH= .42947609640665E+01

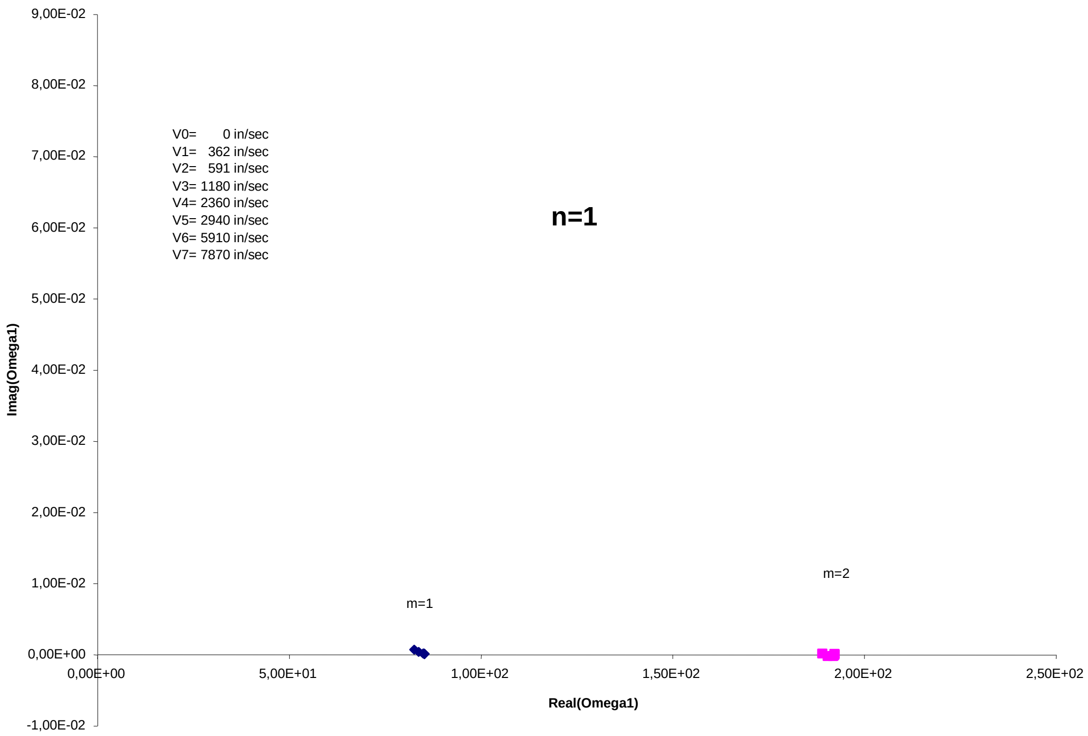
AND U.ZERO1=(PI**2/TOTAL.LENGTH)*SQRT{K/[RHO(1)*TH(1)]}= .12055394026135E+04

WHERE:

K=(E(1)*TH(1)**3)/(12.*(1.-NU(1)**2))

V1= U/U.ZERO1, U [m/sec]

U [m/sec]	V1	Ω_1			
		m=1		**m=2**	
		Real	**Imaginary**	**Real**	**Imaginary**
0,00E+00	0,00E+00	8,55E+01	-7,66E-28	1,92E+02	-7,12E-27
9,20E+00	3,00E-01	8,55E+01	4,24E-06	1,92E+02	-4,29E-05
1,50E+01	4,90E-01	8,55E+01	7,04E-06	1,92E+02	-6,97E-05
3,00E+01	9,80E-01	8,54E+01	1,53E-05	1,92E+02	-1,37E-04
6,00E+01	1,96E+00	8,52E+01	4,02E-05	1,92E+02	-2,54E-04
7,46E+01	2,44E+00	8,51E+01	5,87E-05	1,92E+02	-2,98E-04
1,50E+02	4,90E+00	8,40E+01	2,76E-04	1,91E+02	-2,84E-04
2,00E+02	6,53E+00	8,28E+01	5,99E-04	1,89E+02	6,22E-05


n=1
V0= 0 in/sec
V1= 362 in/sec
V2= 591 in/sec
V3= 1180 in/sec
V4= 2360 in/sec
V5= 2940 in/sec
V6= 5910 in/sec
V7= 7870 in/sec
m=1
m=2
Imag(Omega1)
Real(Omega1)
9,00E-02
8,00E-02
7,00E-02
6,00E-02
5,00E-02
4,00E-02
3,00E-02
2,00E-02
1,00E-02
0,00E+00
-1,00E-02
0,00E+00
5,00E+01
1,00E+02
1,50E+02
2,00E+02
2,50E+02

n = 1

R / t = 40.43, L/R=11.09, ν = 0.3

B.C. : clamped - clamped

DIMENSIONLESS PARAMETERS NUMBER 2:

Ω_2 = CAPITAL OMEGA2= OMEGA/OMEGA.ZERO2 ; OMEGA [RAD/SEC]

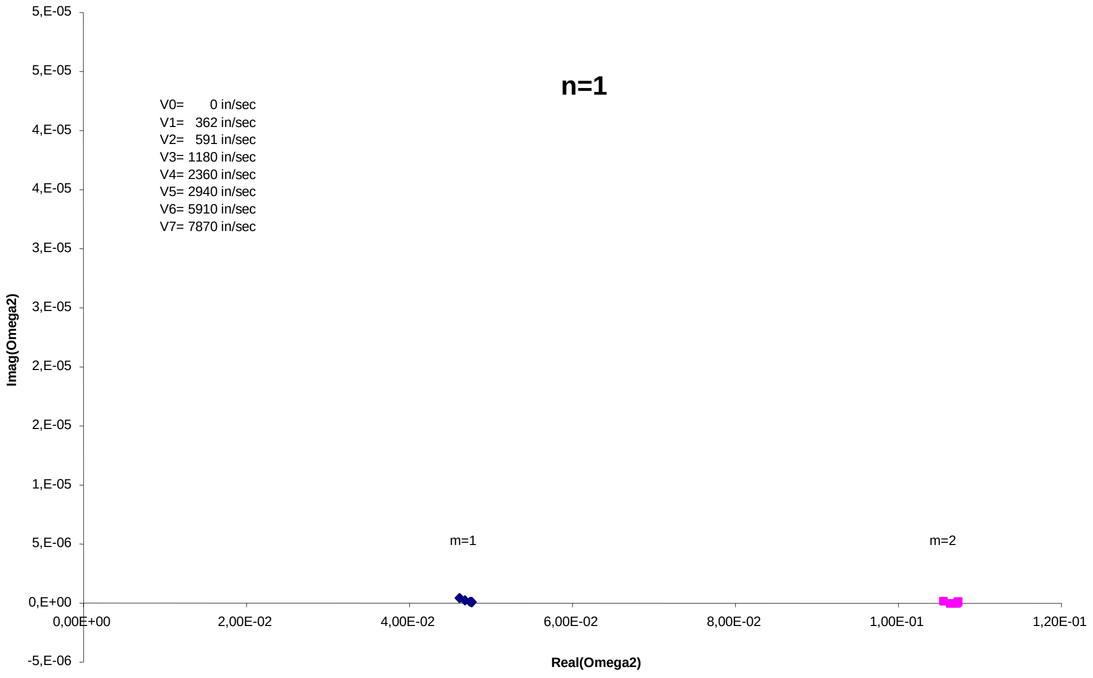
OMEGA.ZERO2=U.ZERO2/RA(1)= .76856926179968E+04

AND U.ZERO2=SQRT{E(1)/[RHO(1)*(1-NU(1)**2)]} = .18973669366049E+06

EPSILON=[RA(1)/TH(1)]*[RHOL/RHO(1)]= .42648259249989E+01

V2= U/U.ZERO2, U [m/sec]

U [m/sec]	V2	Ω_2			
		m=1		**m=2**	
		Real	**Imaginary**	**Real**	**Imaginary**
0,00E+00	0,00E+00	4,78E-02	-4,28E-31	1,08E-01	-3,98E-30
9,20E+00	1,91E-03	4,78E-02	2,37E-09	1,08E-01	-2,40E-08
1,50E+01	3,11E-03	4,78E-02	3,93E-09	1,08E-01	-3,90E-08
3,00E+01	6,22E-03	4,77E-02	8,53E-09	1,08E-01	-7,65E-08
6,00E+01	1,24E-02	4,76E-02	2,24E-08	1,07E-01	-1,42E-07
7,46E+01	1,55E-02	4,76E-02	3,28E-08	1,07E-01	-1,67E-07
1,50E+02	3,11E-02	4,69E-02	1,54E-07	1,07E-01	-1,59E-07
2,00E+02	4,15E-02	4,63E-02	3,35E-07	1,06E-01	3,47E-08


n=1
V0= 0 in/sec
V1= 362 in/sec
V2= 591 in/sec
V3= 1180 in/sec
V4= 2360 in/sec
V5= 2940 in/sec
V6= 5910 in/sec
V7= 7870 in/sec
m=1
m=2
Imag(Omega2)
Real(Omega2)
5,E-05
5,E-05
4,E-05
4,E-05
3,E-05
3,E-05
2,E-05
2,E-05
1,E-05
5,E-06
0,E+00
-5,E-06
0,00E+00
2,00E-02
4,00E-02
6,00E-02
8,00E-02
1,00E-01
1,20E-01

n = 2

R / t = 40.43, L/R=11.09, ν = 0.3

B.C.: clamped - clamped

DIMENSIONLESS PARAMETERS NUMBER 1:

Ω_1 = CAPITAL OMEGA1= OMEGA/OMEGA.ZERO1 ; OMEGA [RAD/SEC]

OMEGA.ZERO1=U.ZERO1/TOTAL.LENGTH= .42947609640665E+01

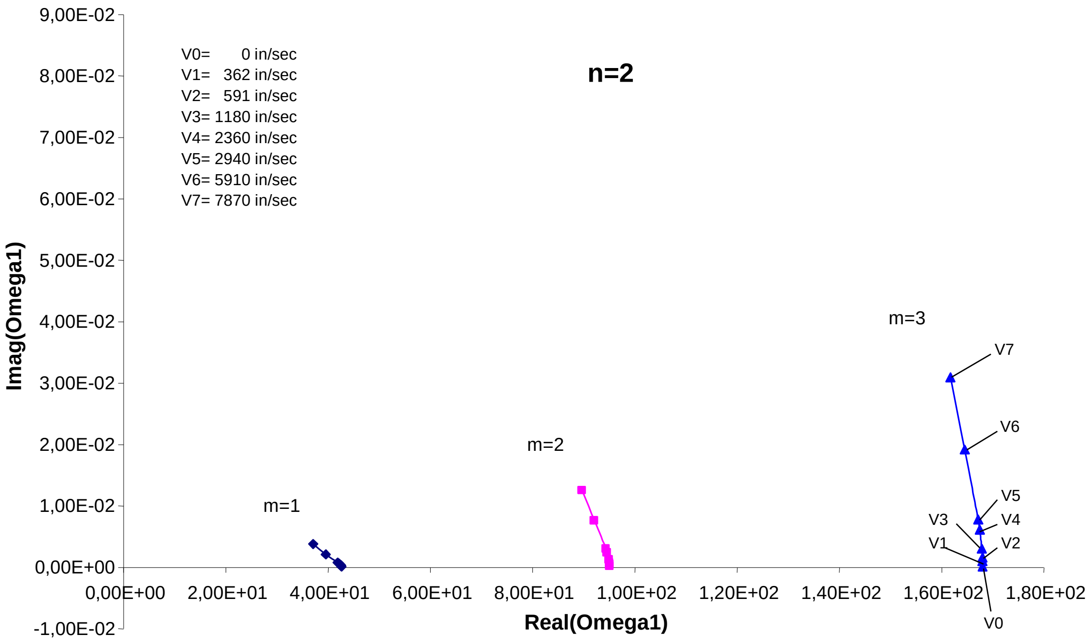
AND U.ZERO1=(PI**2/TOTAL.LENGTH)*SQRT{K/[RHO(1)*TH(1)]}= .12055394026135E+04

WHERE:

K=(E(1)*TH(1)**3)/(12.*(1.-NU(1)**2))

V1= U/U.ZERO1, U [m/sec]

U [m/sec]	V1	Ω_1					
		m=1		**m=2**		**m=3**	
		Real	**Imaginary**	**Real**	**Imaginary**	**Real**	**Imaginary**
0,00E+00	0,00E+00	4,28E+01	-4,63E-31	9,53E+01	-8,73E-30	1,68E+02	-5,13E-29
9,20E+00	3,00E-01	4,28E+01	6,87E-05	9,53E+01	3,19E-04	1,68E+02	8,76E-04
1,50E+01	4,90E-01	4,28E+01	1,12E-04	9,53E+01	5,21E-04	1,68E+02	1,43E-03
3,00E+01	9,80E-01	4,27E+01	2,30E-04	9,52E+01	1,05E-03	1,68E+02	2,89E-03
6,00E+01	1,96E+00	4,23E+01	4,98E-04	9,48E+01	2,21E-03	1,68E+02	6,00E-03
7,46E+01	2,44E+00	4,20E+01	6,56E-04	9,46E+01	2,84E-03	1,67E+02	7,66E-03
1,50E+02	4,90E+00	3,97E+01	1,98E-03	9,23E+01	7,38E-03	1,65E+02	1,91E-02
2,00E+02	6,53E+00	3,73E+01	3,65E-03	8,99E+01	1,23E-02	1,62E+02	3,08E-02


n=2
V0= 0 in/sec
V1= 362 in/sec
V2= 591 in/sec
V3= 1180 in/sec
V4= 2360 in/sec
V5= 2940 in/sec
V6= 5910 in/sec
V7= 7870 in/sec
m=1
m=2
m=3
V0
V1
V2
V3
V4
V5
V6
V7
Imag(Omega1)
Real(Omega1)
9,00E-02
8,00E-02
7,00E-02
6,00E-02
5,00E-02
4,00E-02
3,00E-02
2,00E-02
1,00E-02
0,00E+00
-1,00E-02
0,00E+00
2,00E+01
4,00E+01
6,00E+01
8,00E+01
1,00E+02
1,20E+02
1,40E+02
1,60E+02
1,80E+02

ISSN 2059-9951

n = 2

R / t = 40.43, L/R=11.09, ν = 0.3

B.C. : clamped - clamped

DIMENSIONLESS PARAMETERS NUMBER 2:

$\mathbf{\Omega_2}$ = CAPITAL OMEGA2= OMEGA/OMEGA.ZERO2 ; OMEGA [RAD/SEC]

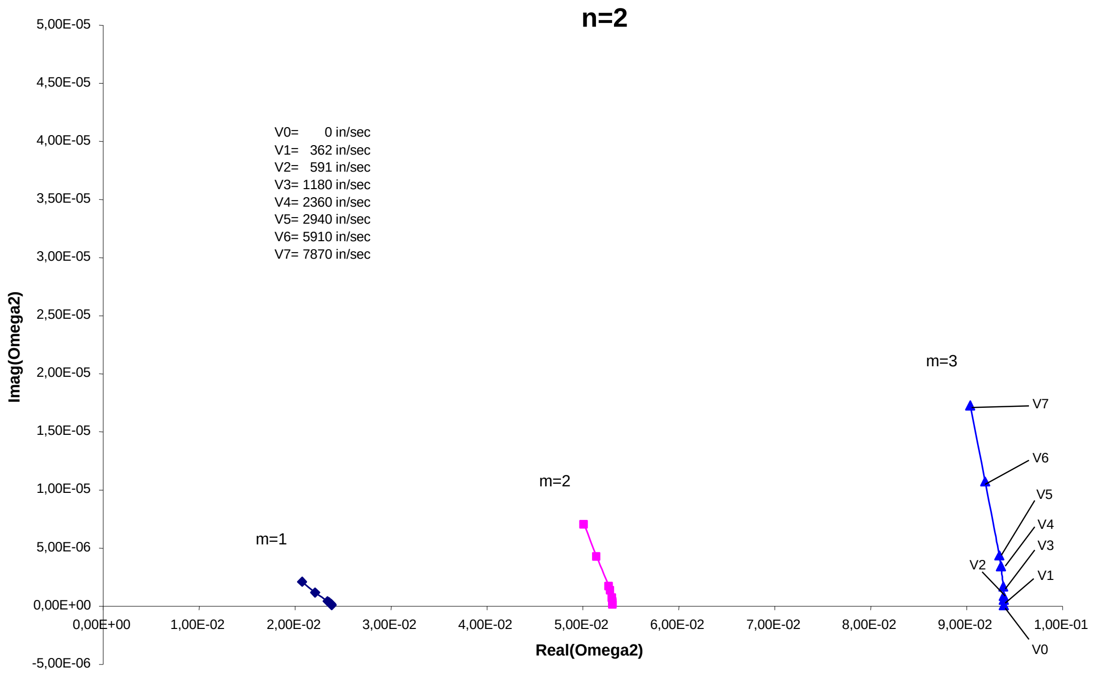
OMEGA.ZERO2=U.ZERO2/RA(1)= .76856926179968E+04

AND U.ZERO2= SQRT{E(1)/[RHO(1)*(1-NU(1)**2)]} = .18973669366049E+06

EPSILON=[RA(1)/TH(1)]*[RHOL/RHO(1)]= .42648259249989E+01

V2= U/U.ZERO2, U [m/sec]

U [m/sec]	V2	Ω_2					
		m=1		**m=2**		**m=3**	
		Real	**Imaginary**	**Real**	**Imaginary**	**Real**	**Imaginary**
0,00E+00	0,00E+00	2,39E-02	-2,59E-34	5,33E-02	-4,88E-33	9,40E-02	-2,87E-32
9,20E+00	1,91E-03	2,39E-02	3,84E-08	5,32E-02	1,78E-07	9,40E-02	4,89E-07
1,50E+01	3,11E-03	2,39E-02	6,28E-08	5,32E-02	2,91E-07	9,40E-02	7,99E-07
3,00E+01	6,22E-03	2,38E-02	1,28E-07	5,32E-02	5,89E-07	9,39E-02	1,61E-06
6,00E+01	1,24E-02	2,36E-02	2,79E-07	5,30E-02	1,23E-06	9,37E-02	3,35E-06
7,46E+01	1,55E-02	2,35E-02	3,66E-07	5,28E-02	1,59E-06	9,35E-02	4,28E-06
1,50E+02	3,11E-02	2,22E-02	1,10E-06	5,16E-02	4,13E-06	9,20E-02	1,07E-05
2,00E+02	4,15E-02	2,08E-02	2,04E-06	5,03E-02	6,89E-06	9,05E-02	1,72E-05


n=2
V0= 0 in/sec
V1= 362 in/sec
V2= 591 in/sec
V3= 1180 in/sec
V4= 2360 in/sec
V5= 2940 in/sec
V6= 5910 in/sec
V7= 7870 in/sec
m=1
m=2
m=3
V7
V6
V5
V4
V3
V2
V1
V0
Imag(Omega2)
Real(Omega2)
5,00E-05
4,50E-05
4,00E-05
3,50E-05
3,00E-05
2,50E-05
2,00E-05
1,50E-05
1,00E-05
5,00E-06
0,00E+00
-5,00E-06
0,00E+00
1,00E-02
2,00E-02
3,00E-02
4,00E-02
5,00E-02
6,00E-02
7,00E-02
8,00E-02
9,00E-02
1,00E-01

n = 3

R / t = 40.43, L/R=11.09, ν = 0.3

B.C. : clamped - clamped

DIMENSIONLESS PARAMETERS NUMBER 1:

Ω_1 = CAPITAL OMEGA1= OMEGA/OMEGA.ZERO1 ; OMEGA [RAD/SEC]

OMEGA.ZERO1=U.ZERO1/TOTAL.LENGTH= .42947609640665E+01

AND U.ZERO1=(PI**2/TOTAL.LENGTH)*SQRT{K/[RHO(1)*TH(1)]}= .12055394026135E+04

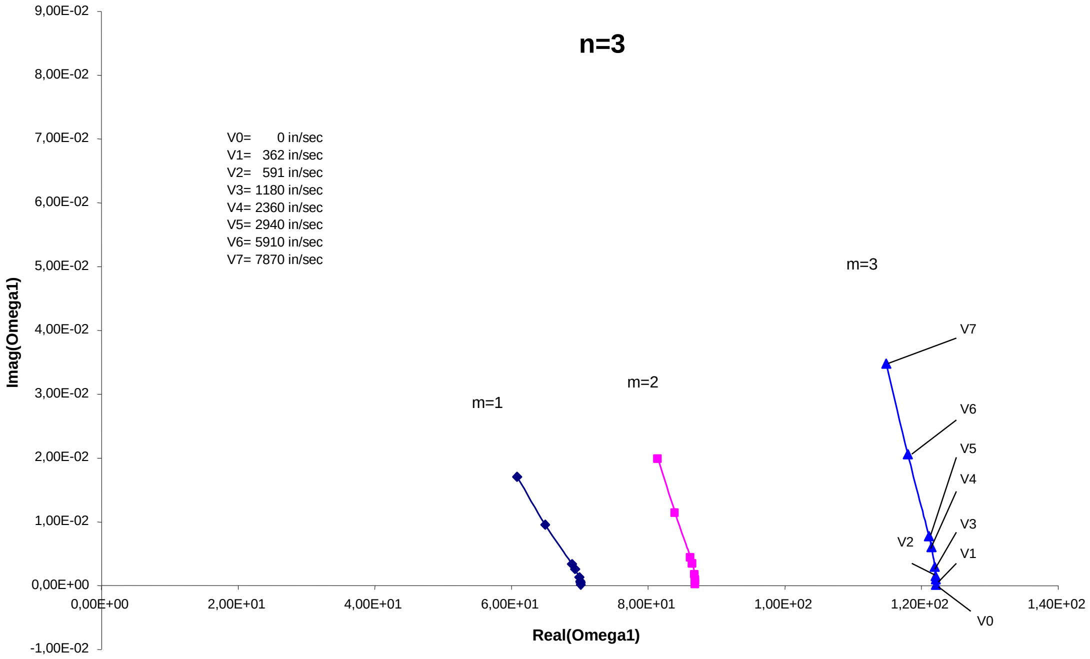
WHERE:

K=(E(1)*TH(1)**3)/(12.*(1.-NU(1)**2))

V1= U/U.ZERO1, U [m/sec]

U [m/sec]	V1	Ω_1					
		m=1		**m=2**		**m=3**	
		Real	**Imaginary**	**Real**	**Imaginary**	**Real**	**Imaginary**
0,00E+00	0,00E+00	7,03E+01	-3,28E-31	8,71E+01	-2,41E-31	1,22E+02	1,38E-31
9,20E+00	3,00E-01	7,03E+01	3,45E-04	8,71E+01	4,65E-04	1,22E+02	8,42E-04
1,50E+01	4,90E-01	7,02E+01	5,65E-04	8,71E+01	7,60E-04	1,22E+02	1,38E-03
3,00E+01	9,80E-01	7,01E+01	1,15E-03	8,70E+01	1,54E-03	1,22E+02	2,79E-03
6,00E+01	1,96E+00	6,95E+01	2,48E-03	8,66E+01	3,23E-03	1,22E+02	5,90E-03
7,46E+01	2,44E+00	6,90E+01	3,24E-03	8,64E+01	4,17E-03	1,21E+02	7,62E-03
1,50E+02	4,90E+00	6,51E+01	9,42E-03	8,41E+01	1,12E-02	1,18E+02	2,04E-02
2,00E+02	6,53E+00	6,10E+01	1,69E-02	8,16E+01	1,97E-02	1,15E+02	3,47E-02

ISSN 2059-9951



n = 3

R / t = 40.43, L/R=11.09, ν = 0.3

B.C. : clamped - clamped

DIMENSIONLESS PARAMETERS NUMBER 2:

Ω_2 = CAPITAL OMEGA2= OMEGA/OMEGA.ZERO2 ; OMEGA [RAD/SEC]

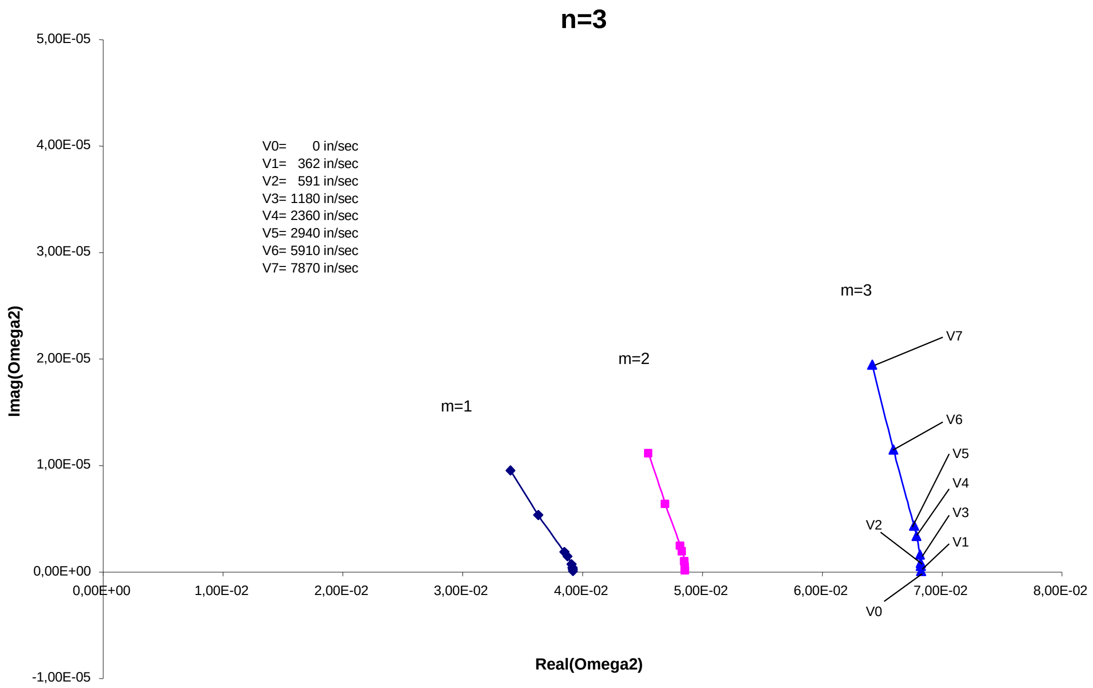
OMEGA.ZERO2=U.ZERO2/RA(1)= .76856926179968E+04

AND U.ZERO2=SQRT{E(1)/[RHO(1)*(1-NU(1)**2)]} = .18973669366049E+06

EPSILON=[RA(1)/TH(1)]*[RHOL/RHO(1)]= .42648259249989E+01

V2= U/U.ZERO2, U [m/sec]

U [m/sec]	V2	Ω_2					
		m=1		**m=2**		**m=3**	
		Real	**Imaginary**	**Real**	**Imaginary**	**Real**	**Imaginary**
0,00E+00	0,00E+00	3,93E-02	-1,83E-34	4,87E-02	-1,35E-34	6,83E-02	7,69E-35
9,20E+00	1,91E-03	3,93E-02	1,93E-07	4,87E-02	2,60E-07	6,83E-02	4,71E-07
1,50E+01	3,11E-03	3,92E-02	3,16E-07	4,87E-02	4,25E-07	6,83E-02	7,70E-07
3,00E+01	6,22E-03	3,92E-02	6,44E-07	4,86E-02	8,60E-07	6,82E-02	1,56E-06
6,00E+01	1,24E-02	3,88E-02	1,39E-06	4,84E-02	1,81E-06	6,79E-02	3,30E-06
7,46E+01	1,55E-02	3,86E-02	1,81E-06	4,83E-02	2,33E-06	6,77E-02	4,26E-06
1,50E+02	3,11E-02	3,64E-02	5,27E-06	4,70E-02	6,25E-06	6,60E-02	1,14E-05
2,00E+02	4,15E-02	3,41E-02	9,46E-06	4,56E-02	1,10E-05	6,42E-02	1,94E-05


n=3
5,00E-05
4,00E-05
3,00E-05
2,00E-05
1,00E-05
0,00E+00
-1,00E-05
Imag(Omega2)
V0= 0 in/sec
V1= 362 in/sec
V2= 591 in/sec
V3= 1180 in/sec
V4= 2360 in/sec
V5= 2940 in/sec
V6= 5910 in/sec
V7= 7870 in/sec
m=1
m=2
m=3
V7
V6
V5
V4
V3
V2
V1
V0
0,00E+00
1,00E-02
2,00E-02
3,00E-02
4,00E-02
5,00E-02
6,00E-02
7,00E-02
8,00E-02
Real(Omega2)

n = 4

R / t = 40.43, L/R=11.09, ν = 0.3

B.C. : clamped - clamped

DIMENSIONLESS PARAMETERS NUMBER 1:

Ω_1 = CAPITAL OMEGA1= OMEGA/OMEGA.ZERO1 ; OMEGA [RAD/SEC]

OMEGA.ZERO1=U.ZERO1/TOTAL.LENGTH= .42947609640665E+01

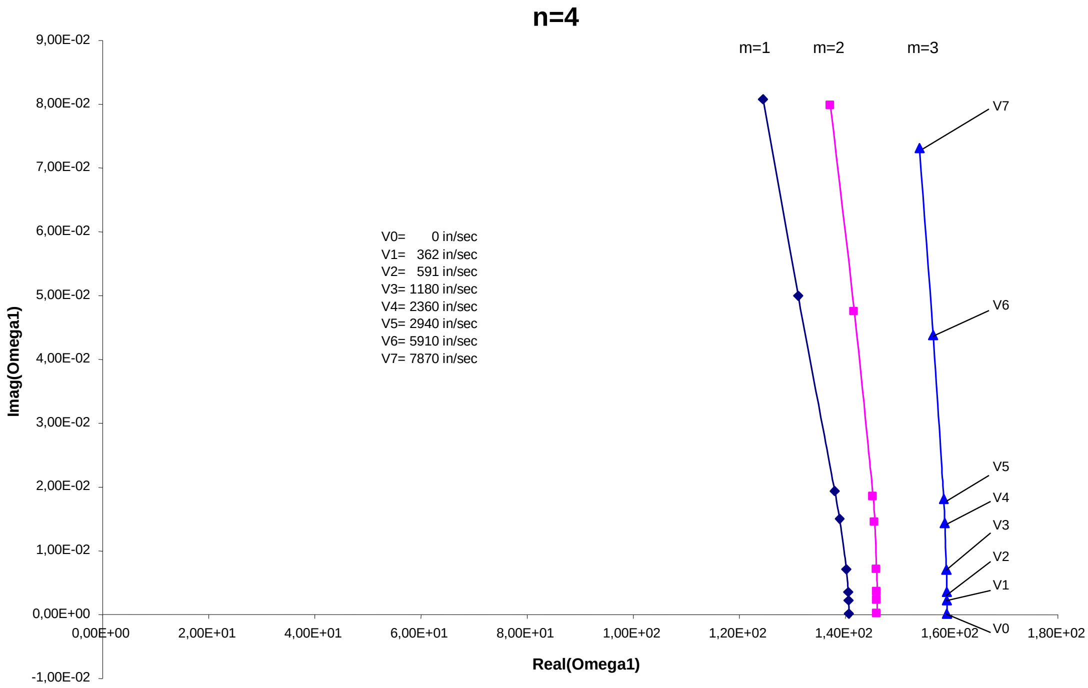
AND U.ZERO1=(PI**2/TOTAL.LENGTH)*SQRT{K/[RHO(1)*TH(1)]}= .12055394026135E+04

WHERE:

K=(E(1)*TH(1)**3)/(12.*(1.-NU(1)**2))

V1= U/U.ZERO1, U [m/sec]

U [m/sec]	V1	Ω_1					
		m=1		**m=2**		**m=3**	
		Real	**Imaginary**	**Real**	**Imaginary**	**Real**	**Imaginary**
0,00E+00	0,00E+00	1,41E+02	-1,12E-31	1,46E+02	-7,31E-32	1,59E+02	-2,03E-32
9,20E+00	3,00E-01	1,41E+02	2,09E-03	1,46E+02	2,11E-03	1,59E+02	2,11E-03
1,50E+01	4,90E-01	1,41E+02	3,43E-03	1,46E+02	3,45E-03	1,59E+02	3,44E-03
3,00E+01	9,80E-01	1,40E+02	6,97E-03	1,46E+02	6,94E-03	1,59E+02	6,91E-03
6,00E+01	1,96E+00	1,39E+02	1,49E-02	1,46E+02	1,43E-02	1,59E+02	1,42E-02
7,46E+01	2,44E+00	1,38E+02	1,92E-02	1,45E+02	1,83E-02	1,59E+02	1,80E-02
1,50E+02	4,90E+00	1,31E+02	4,98E-02	1,42E+02	4,73E-02	1,57E+02	4,37E-02
2,00E+02	6,53E+00	1,25E+02	8,06E-02	1,37E+02	7,96E-02	1,54E+02	7,30E-02


n=4
m=1
m=2
m=3
V0= 0 in/sec
V1= 362 in/sec
V2= 591 in/sec
V3= 1180 in/sec
V4= 2360 in/sec
V5= 2940 in/sec
V6= 5910 in/sec
V7= 7870 in/sec
V7
V6
V5
V4
V3
V2
V1
V0
Imag(Omega1)
Real(Omega1)
9,00E-02
8,00E-02
7,00E-02
6,00E-02
5,00E-02
4,00E-02
3,00E-02
2,00E-02
1,00E-02
0,00E+00
-1,00E-02
0,00E+00
2,00E+01
4,00E+01
6,00E+01
8,00E+01
1,00E+02
1,20E+02
1,40E+02
1,60E+02
1,80E+02

n = 4

R / t = 40.43, L/R=11.09, ν = 0.3

B.C. : clamped - clamped

DIMENSIONLESS PARAMETERS NUMBER 2:

$\boldsymbol{\Omega_2}$ = CAPITAL OMEGA2= OMEGA/OMEGA.ZERO2 ; OMEGA [RAD/SEC]

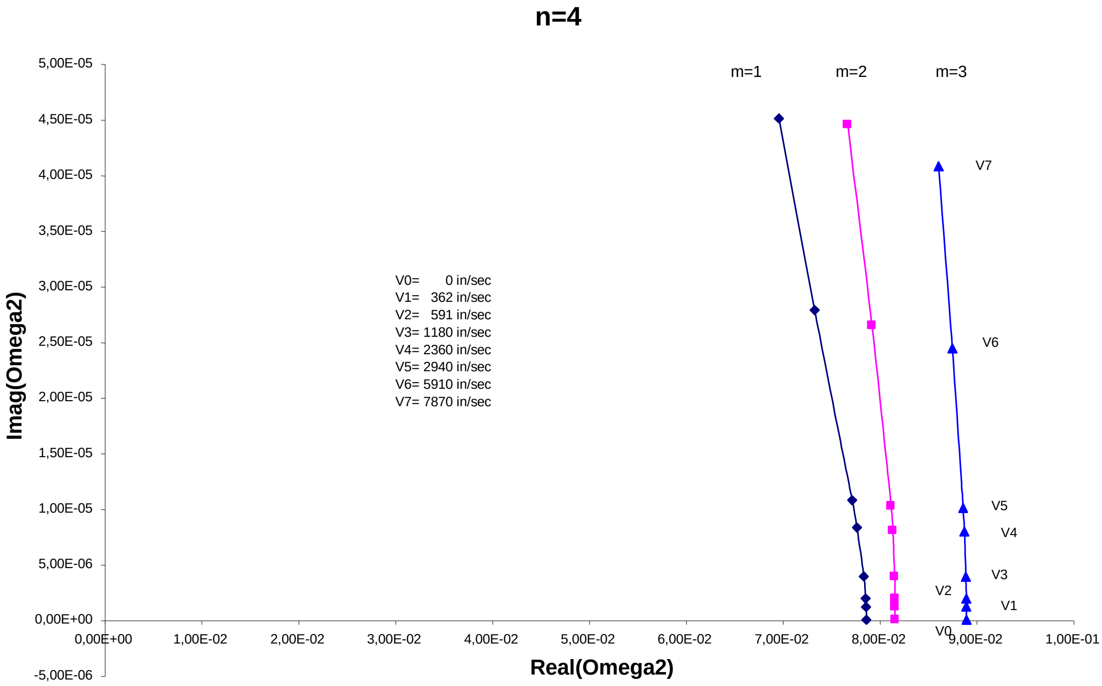
OMEGA.ZERO2=U.ZERO2/RA(1)= .76856926179968E+04

AND U.ZERO2=SQRT{E(1)/[RHO(1)*(1-NU(1)**2)]} = .18973669366049E+06

EPSILON=[RA(1)/TH(1)]*[RHOL/RHO(1)]= .42648259249989E+01

V2= U/U.ZERO2, U [m/sec]

U [m/sec]	V2	Ω_2					
		m=1		**m=2**		**m=3**	
		Real	**Imaginary**	**Real**	**Imaginary**	**Real**	**Imaginary**
0,00E+00	0,00E+00	7,87E-02	-6,28E-35	8,17E-02	-4,08E-35	8,90E-02	-1,14E-35
9,20E+00	1,91E-03	7,87E-02	1,17E-06	8,16E-02	1,18E-06	8,90E-02	1,18E-06
1,50E+01	3,11E-03	7,86E-02	1,91E-06	8,16E-02	1,93E-06	8,90E-02	1,92E-06
3,00E+01	6,22E-03	7,84E-02	3,90E-06	8,16E-02	3,88E-06	8,89E-02	3,86E-06
6,00E+01	1,24E-02	7,77E-02	8,30E-06	8,14E-02	8,01E-06	8,88E-02	7,92E-06
7,46E+01	1,55E-02	7,72E-02	1,07E-05	8,12E-02	1,02E-05	8,87E-02	1,00E-05
1,50E+02	3,11E-02	7,33E-02	2,78E-05	7,93E-02	2,64E-05	8,76E-02	2,44E-05
2,00E+02	4,15E-02	6,96E-02	4,51E-05	7,67E-02	4,45E-05	8,61E-02	4,08E-05


n=4
Imag(Omega2)
Real(Omega2)
m=1
m=2
m=3
V0= 0 in/sec
V1= 362 in/sec
V2= 591 in/sec
V3= 1180 in/sec
V4= 2360 in/sec
V5= 2940 in/sec
V6= 5910 in/sec
V7= 7870 in/sec
V7
V6
V5
V4
V3
V2
V1
V0
5,00E-05
4,50E-05
4,00E-05
3,50E-05
3,00E-05
2,50E-05
2,00E-05
1,50E-05
1,00E-05
5,00E-06
0,00E+00
-5,00E-06
0,00E+00
1,00E-02
2,00E-02
3,00E-02
4,00E-02
5,00E-02
6,00E-02
7,00E-02
8,00E-02
9,00E-02
1,00E-01

n = 5

R / t = 40.43, L/R=11.09, ν = 0.3

B.C. : clamped - clamped

DIMENSIONLESS PARAMETERS NUMBER 1:

Ω_1 = CAPITAL OMEGA1= OMEGA/OMEGA.ZERO1 ; OMEGA [RAD/SEC]

OMEGA.ZERO1=U.ZERO1/TOTAL.LENGTH= .42947609640665E+01

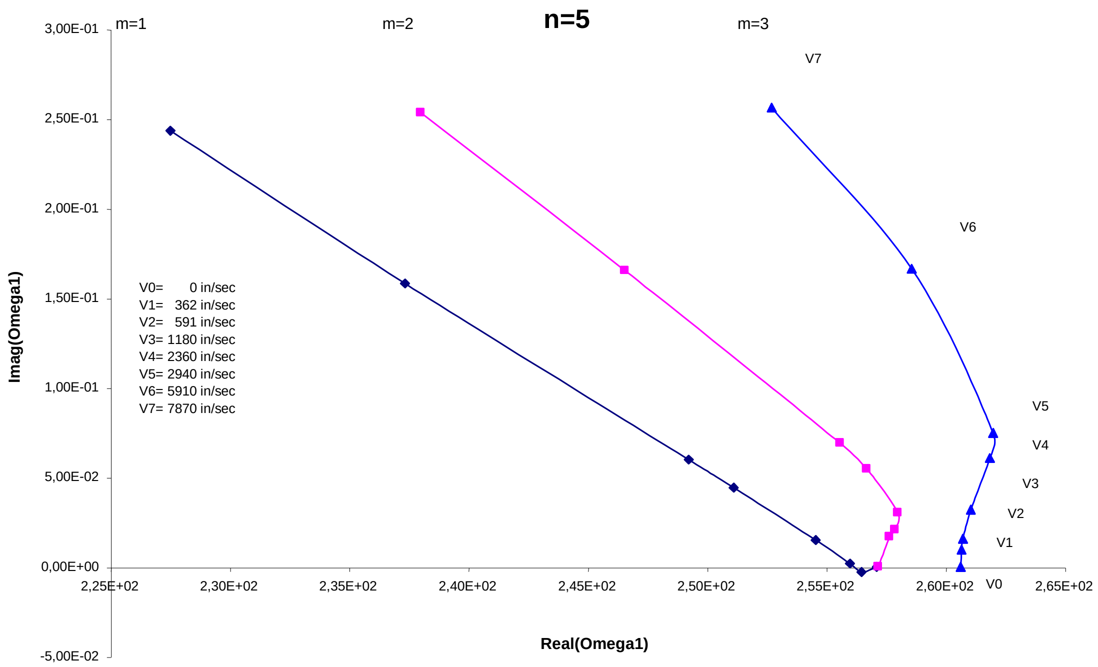
AND U.ZERO1=(PI**2/TOTAL.LENGTH)*SQRT{K/[RHO(1)*TH(1)]}= .12055394026135E+04

WHERE:

K=(E(1)*TH(1)**3)/(12.*(1.-NU(1)**2))

V1= U/U.ZERO1, U [m/sec]

U [m/sec]	V1	Ω_1					
		m=1		**m=2**		**m=3**	
		Real	**Imaginary**	**Real**	**Imaginary**	**Real**	**Imaginary**
0,00E+00	0,00E+00	2,57E+02	8,72E-32	2,57E+02	6,95E-32	2,61E+02	7,04E-32
9,20E+00	3,00E-01	2,56E+02	-3,01E-03	2,58E+02	1,67E-02	2,61E+02	9,56E-03
1,50E+01	4,90E-01	2,56E+02	1,78E-03	2,58E+02	2,05E-02	2,61E+02	1,57E-02
3,00E+01	9,80E-01	2,55E+02	1,50E-02	2,58E+02	3,01E-02	2,61E+02	3,18E-02
6,00E+01	1,96E+00	2,51E+02	4,42E-02	2,57E+02	5,46E-02	2,62E+02	6,07E-02
7,46E+01	2,44E+00	2,49E+02	5,98E-02	2,56E+02	6,90E-02	2,62E+02	7,46E-02
1,50E+02	4,90E+00	2,37E+02	1,58E-01	2,47E+02	1,65E-01	2,59E+02	1,66E-01
2,00E+02	6,53E+00	2,28E+02	2,43E-01	2,38E+02	2,53E-01	2,53E+02	2,56E-01


n=5
m=1
m=2
m=3
V7
V6
V5
V4
V3
V2
V1
V0
V0= 0 in/sec
V1= 362 in/sec
V2= 591 in/sec
V3= 1180 in/sec
V4= 2360 in/sec
V5= 2940 in/sec
V6= 5910 in/sec
V7= 7870 in/sec
Imag(Omega1)
Real(Omega1)
3,00E-01
2,50E-01
2,00E-01
1,50E-01
1,00E-01
5,00E-02
0,00E+00
-5,00E-02
2,25E+02
2,30E+02
2,35E+02
2,40E+02
2,45E+02
2,50E+02
2,55E+02
2,60E+02
2,65E+02

n = 5

R / t = 40.43, L/R=11.09, ν = 0.3

B.C. : clamped - clamped

DIMENSIONLESS PARAMETERS NUMBER 2:

Ω_2 = CAPITAL OMEGA2= OMEGA/OMEGA.ZERO2 ; OMEGA [RAD/SEC]

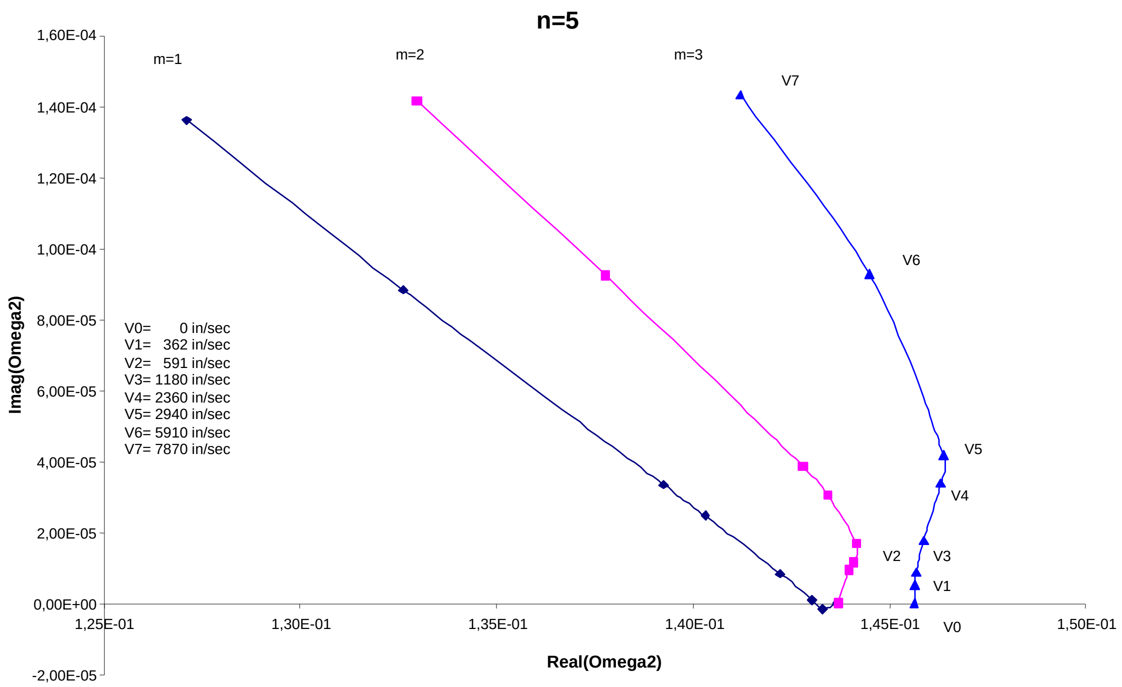
OMEGA.ZERO2=U.ZERO2/RA(1)= .76856926179968E+04

AND U.ZERO2=SQRT{E(1)/[RHO(1)*(1-NU(1)**2)]} = .18973669366049E+06

EPSILON=[RA(1)/TH(1)]*[RHOL/RHO(1)]= .42648259249989E+01

V2= U/U.ZERO2, U [m/sec]

U [m/sec]	V2	Ω_2					
		m=1		**m=2**		**m=3**	
		Real	**Imaginary**	**Real**	**Imaginary**	**Real**	**Imaginary**
0,00E+00	0,00E+00	1,44E-01	4,88E-35	1,44E-01	3,89E-35	1,46E-01	3,93E-35
9,20E+00	1,91E-03	1,43E-01	-1,68E-06	1,44E-01	9,36E-06	1,46E-01	5,34E-06
1,50E+01	3,11E-03	1,43E-01	9,97E-07	1,44E-01	1,15E-05	1,46E-01	8,79E-06
3,00E+01	6,22E-03	1,42E-01	8,37E-06	1,44E-01	1,68E-05	1,46E-01	1,78E-05
6,00E+01	1,24E-02	1,40E-01	2,47E-05	1,43E-01	3,05E-05	1,46E-01	3,39E-05
7,46E+01	1,55E-02	1,39E-01	3,34E-05	1,43E-01	3,86E-05	1,46E-01	4,17E-05
1,50E+02	3,11E-02	1,33E-01	8,83E-05	1,38E-01	9,23E-05	1,45E-01	9,29E-05
2,00E+02	4,15E-02	1,27E-01	1,36E-04	1,33E-01	1,42E-04	1,41E-01	1,43E-04


n=5
m=1
m=2
m=3
V7
V6
V5
V4
V3
V2
V1
V0
V0= 0 in/sec
V1= 362 in/sec
V2= 591 in/sec
V3= 1180 in/sec
V4= 2360 in/sec
V5= 2940 in/sec
V6= 5910 in/sec
V7= 7870 in/sec
Imag(Omega2)
Real(Omega2)
1,60E-04
1,40E-04
1,20E-04
1,00E-04
8,00E-05
6,00E-05
4,00E-05
2,00E-05
0,00E+00
-2,00E-05
1,25E-01
1,30E-01
1,35E-01
1,40E-01
1,45E-01
1,50E-01

R/t = 39,437 L/R = 11,37 ν = 0.3

B.C.: clamped - clamped

m	U		v1	v2	n=1				n=2			
	[in/sec]	[m/sec]			Ω1		Ω2		Ω1		Ω2	
1	0	0	0	0	8,55E+01	-7,66E-28	4,78E-02	-4,28E-31	4,28E+01	-4,63E-31	2,39E-02	-2,59E-34
	3,62E+02	9,20E+00	3,00E-01	1,91E-03	8,55E+01	4,24E-06	4,78E-02	2,37E-09	4,28E+01	6,87E-05	2,39E-02	3,84E-08
	5,91E+02	1,50E+01	4,90E-01	3,11E-03	8,55E+01	7,04E-06	4,78E-02	3,93E-09	4,28E+01	1,12E-04	2,39E-02	6,28E-08
	1,18E+03	3,00E+01	9,80E-01	6,22E-03	8,54E+01	1,53E-05	4,77E-02	8,53E-09	4,27E+01	2,30E-04	2,38E-02	1,28E-07
	2,36E+03	6,00E+01	1,96E+00	1,24E-02	8,52E+01	4,02E-05	4,76E-02	2,24E-08	4,23E+01	4,98E-04	2,36E-02	2,79E-07
	2,94E+03	7,46E+01	2,44E+00	1,55E-02	8,51E+01	5,87E-05	4,76E-02	3,28E-08	4,20E+01	6,56E-04	2,35E-02	3,66E-07
	5,91E+03	1,50E+02	4,90E+00	3,11E-02	8,40E+01	2,76E-04	4,69E-02	1,54E-07	3,97E+01	1,98E-03	2,22E-02	1,10E-06
	7,87E+03	2,00E+02	6,53E+00	4,15E-02	8,28E+01	5,99E-04	4,63E-02	3,35E-07	3,73E+01	3,65E-03	2,08E-02	2,04E-06
2	0	0	0	0	1,92E+02	-7,12E-27	1,08E-01	-3,98E-30	9,53E+01	-8,73E-30	5,33E-02	-4,88E-33
	3,62E+02	9,20E+00	3,00E-01	1,91E-03	1,92E+02	-4,29E-05	1,08E-01	-2,40E-08	9,53E+01	3,19E-04	5,32E-02	1,78E-07
	5,91E+02	1,50E+01	4,90E-01	3,11E-03	1,92E+02	-6,97E-05	1,08E-01	-3,90E-08	9,53E+01	5,21E-04	5,32E-02	2,91E-07
	1,18E+03	3,00E+01	9,80E-01	6,22E-03	1,92E+02	-1,37E-04	1,08E-01	-7,65E-08	9,52E+01	1,05E-03	5,32E-02	5,89E-07
	2,36E+03	6,00E+01	1,96E+00	1,24E-02	1,92E+02	-2,54E-04	1,07E-01	-1,42E-07	9,48E+01	2,21E-03	5,30E-02	1,23E-06
	2,94E+03	7,46E+01	2,44E+00	1,55E-02	1,92E+02	-2,98E-04	1,07E-01	-1,67E-07	9,46E+01	2,84E-03	5,28E-02	1,59E-06
	5,91E+03	1,50E+02	4,90E+00	3,11E-02	1,91E+02	-2,84E-04	1,07E-01	-1,59E-07	9,23E+01	7,38E-03	5,16E-02	4,13E-06
	7,87E+03	2,00E+02	6,53E+00	4,15E-02	1,89E+02	6,22E-05	1,06E-01	3,47E-08	8,99E+01	1,23E-02	5,03E-02	6,89E-06
3	0	0	0	0	3,25E+02	-3,24E-26	1,82E-01	-1,81E-29	1,68E+02	-5,13E-29	9,40E-02	-2,87E-32
	3,62E+02	9,20E+00	3,00E-01	1,91E-03	3,25E+02	-5,63E-04	1,82E-01	-3,14E-07	1,68E+02	8,76E-04	9,40E-02	4,89E-07
	5,91E+02	1,50E+01	4,90E-01	3,11E-03	3,25E+02	-9,17E-04	1,82E-01	-5,13E-07	1,68E+02	1,43E-03	9,40E-02	7,99E-07
	1,18E+03	3,00E+01	9,80E-01	6,22E-03	3,25E+02	-1,83E-03	1,82E-01	-1,02E-06	1,68E+02	2,89E-03	9,39E-02	1,61E-06
	2,36E+03	6,00E+01	1,96E+00	1,24E-02	3,25E+02	-3,64E-03	1,81E-01	-2,04E-06	1,68E+02	6,00E-03	9,37E-02	3,35E-06
	2,94E+03	7,46E+01	2,44E+00	1,55E-02	3,24E+02	-4,51E-03	1,81E-01	-2,52E-06	1,67E+02	7,66E-03	9,35E-02	4,28E-06
	5,91E+03	1,50E+02	4,90E+00	3,11E-02	3,22E+02	-8,79E-03	1,80E-01	-4,91E-06	1,65E+02	1,91E-02	9,20E-02	1,07E-05
	7,87E+03	2,00E+02	6,53E+00	4,15E-02	3,20E+02	-1,14E-02	1,79E-01	-6,37E-06	1,62E+02	3,08E-02	9,05E-02	1,72E-05

R/t = 39,437 L/R = 11,37 ν = 0.3

B.C. : clamped - clamped

m	U		n=3				n=4				n=5			
	[in/sec]	[m/sec]	Ω1		Ω2		Ω1		Ω2		Ω1		Ω2	
1	0	0	7,03E+01	-3,28E-31	3,93E-02	-1,83E-34	1,41E+02	-1,12E-31	7,87E-02	-6,28E-35	2,57E+02	8,72E-32	1,44E-01	4,88E-35
	3,62E+02	9,20E+00	7,03E+01	3,45E-04	3,93E-02	1,93E-07	1,41E+02	2,09E-03	7,87E-02	1,17E-06	2,56E+02	-3,01E-03	1,43E-01	-1,68E-06
	5,91E+02	1,50E+01	7,02E+01	5,65E-04	3,92E-02	3,16E-07	1,41E+02	3,43E-03	7,86E-02	1,91E-06	2,56E+02	1,78E-03	1,43E-01	9,97E-07
	1,18E+03	3,00E+01	7,01E+01	1,15E-03	3,92E-02	6,44E-07	1,40E+02	6,97E-03	7,84E-02	3,90E-06	2,55E+02	1,50E-02	1,42E-01	8,37E-06
	2,36E+03	6,00E+01	6,95E+01	2,48E-03	3,88E-02	1,39E-06	1,39E+02	1,49E-02	7,77E-02	8,30E-06	2,51E+02	4,42E-02	1,40E-01	2,47E-05
	2,94E+03	7,46E+01	6,90E+01	3,24E-03	3,86E-02	1,81E-06	1,38E+02	1,92E-02	7,72E-02	1,07E-05	2,49E+02	5,98E-02	1,39E-01	3,34E-05
	5,91E+03	1,50E+02	6,51E+01	9,42E-03	3,64E-02	5,27E-06	1,31E+02	4,98E-02	7,33E-02	2,78E-05	2,37E+02	1,58E-01	1,33E-01	8,83E-05
	7,87E+03	2,00E+02	6,10E+01	1,69E-02	3,41E-02	9,46E-06	1,25E+02	8,06E-02	6,96E-02	4,51E-05	2,28E+02	2,43E-01	1,27E-01	1,36E-04
2	0	0	8,71E+01	-2,41E-31	4,87E-02	-1,35E-34	1,46E+02	-7,31E-32	8,17E-02	-4,08E-35	2,57E+02	6,95E-32	1,44E-01	3,89E-35
	3,62E+02	9,20E+00	8,71E+01	4,65E-04	4,87E-02	2,60E-07	1,46E+02	2,11E-03	8,16E-02	1,18E-06	2,58E+02	1,67E-02	1,44E-01	9,36E-06
	5,91E+02	1,50E+01	8,71E+01	7,60E-04	4,87E-02	4,25E-07	1,46E+02	3,45E-03	8,16E-02	1,93E-06	2,58E+02	2,05E-02	1,44E-01	1,15E-05
	1,18E+03	3,00E+01	8,70E+01	1,54E-03	4,86E-02	8,60E-07	1,46E+02	6,94E-03	8,16E-02	3,88E-06	2,58E+02	3,01E-02	1,44E-01	1,68E-05
	2,36E+03	6,00E+01	8,66E+01	3,23E-03	4,84E-02	1,81E-06	1,46E+02	1,43E-02	8,14E-02	8,01E-06	2,57E+02	5,46E-02	1,43E-01	3,05E-05
	2,94E+03	7,46E+01	8,64E+01	4,17E-03	4,83E-02	2,33E-06	1,45E+02	1,83E-02	8,12E-02	1,02E-05	2,56E+02	6,90E-02	1,43E-01	3,86E-05
	5,91E+03	1,50E+02	8,41E+01	1,12E-02	4,70E-02	6,25E-06	1,42E+02	4,73E-02	7,93E-02	2,64E-05	2,47E+02	1,65E-01	1,38E-01	9,23E-05
	7,87E+03	2,00E+02	8,16E+01	1,97E-02	4,56E-02	1,10E-05	1,37E+02	7,96E-02	7,67E-02	4,45E-05	2,38E+02	2,53E-01	1,33E-01	1,42E-04
3	0	0	1,22E+02	1,38E-31	6,83E-02	7,69E-35	1,59E+02	-2,03E-32	8,90E-02	-1,14E-35	2,61E+02	7,04E-32	1,46E-01	3,93E-35
	3,62E+02	9,20E+00	1,22E+02	8,42E-04	6,83E-02	4,71E-07	1,59E+02	2,11E-03	8,90E-02	1,18E-06	2,61E+02	9,56E-03	1,46E-01	5,34E-06
	5,91E+02	1,50E+01	1,22E+02	1,38E-03	6,83E-02	7,70E-07	1,59E+02	3,44E-03	8,90E-02	1,92E-06	2,61E+02	1,57E-02	1,46E-01	8,79E-06
	1,18E+03	3,00E+01	1,22E+02	2,79E-03	6,82E-02	1,56E-06	1,59E+02	6,91E-03	8,89E-02	3,86E-06	2,61E+02	3,18E-02	1,46E-01	1,78E-05
	2,36E+03	6,00E+01	1,22E+02	5,90E-03	6,79E-02	3,30E-06	1,59E+02	1,42E-02	8,88E-02	7,92E-06	2,62E+02	6,07E-02	1,46E-01	3,39E-05
	2,94E+03	7,46E+01	1,21E+02	7,62E-03	6,77E-02	4,26E-06	1,59E+02	1,80E-02	8,87E-02	1,00E-05	2,62E+02	7,46E-02	1,46E-01	4,17E-05
	5,91E+03	1,50E+02	1,18E+02	2,04E-02	6,60E-02	1,14E-05	1,57E+02	4,37E-02	8,76E-02	2,44E-05	2,59E+02	1,66E-01	1,45E-01	9,29E-05
	7,87E+03	2,00E+02	1,15E+02	3,47E-02	6,42E-02	1,94E-05	1,54E+02	7,30E-02	8,61E-02	4,08E-05	2,53E+02	2,56E-01	1,41E-01	1,43E-04

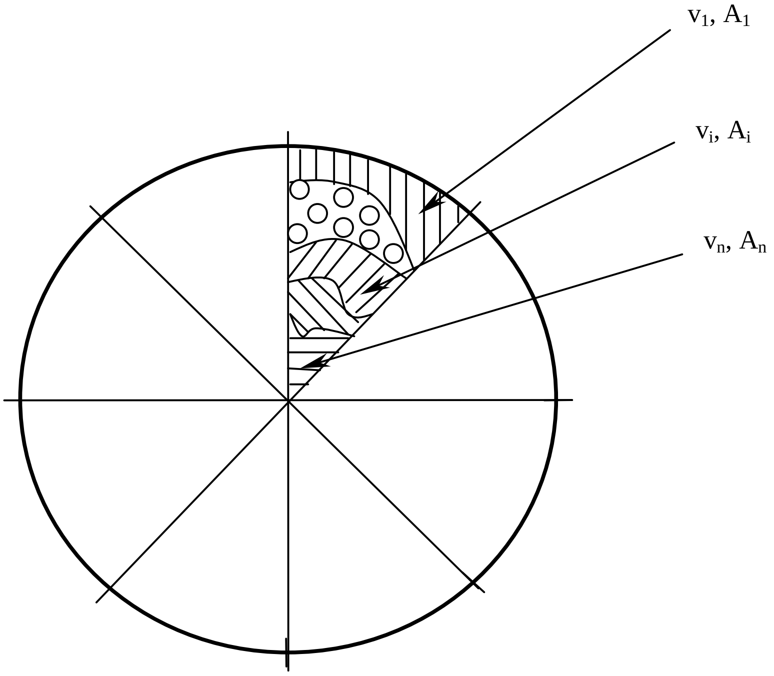
SHELL SUBJECTED TO A FLOWING FLUID CASE N^{O} 2: CROSS SECTION VELOCITY DISTRIBUTION

The following results are given as a function of two non-dimensional terms:

$$V_1 = \frac{U}{\left(\frac{\pi^2}{L}\right)\sqrt{\frac{E\,t^2}{12\,\rho_S\left(1-\upsilon^2\right)}}}$$

$$\Omega_1 = \frac{\Omega\,(\text{rad/sec})}{\left(\frac{\pi^2}{L^2}\right)\sqrt{\frac{E\,t^2}{12\,\rho_S\left(1-\upsilon^2\right)}}}$$

CROSS SECTION FLOW RATE - CALCULATION METHOD

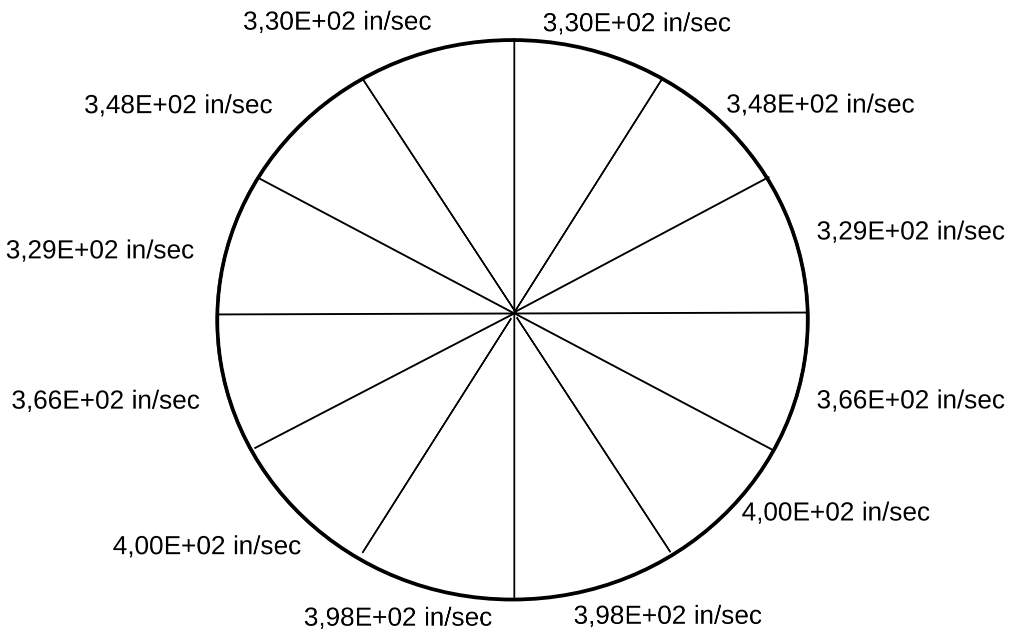


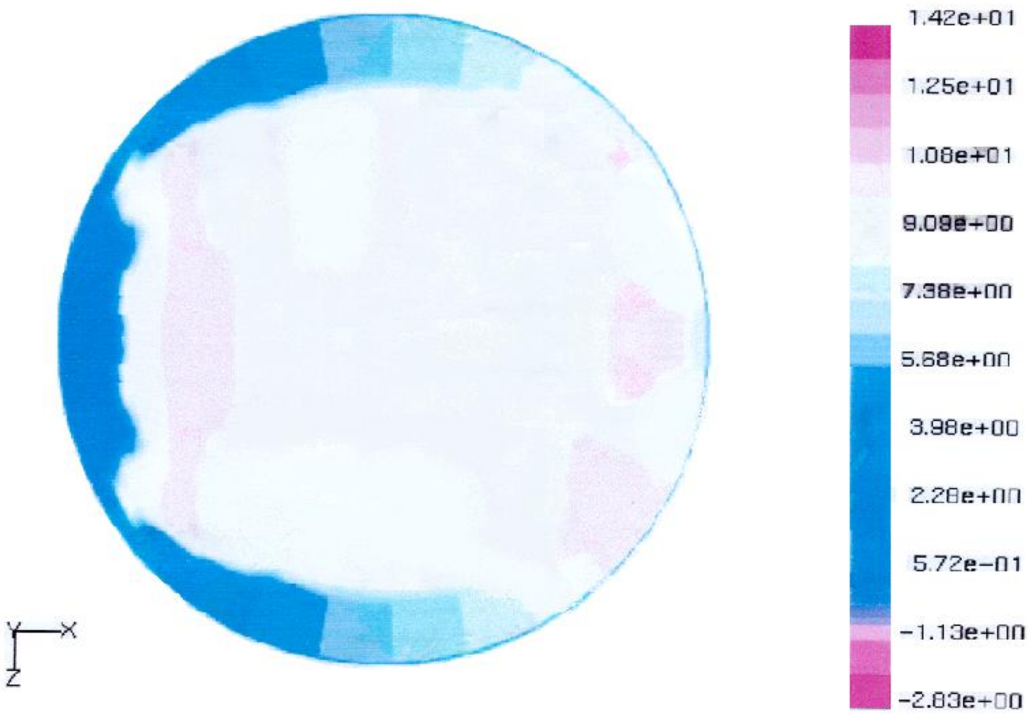
ISSN 2059-9951

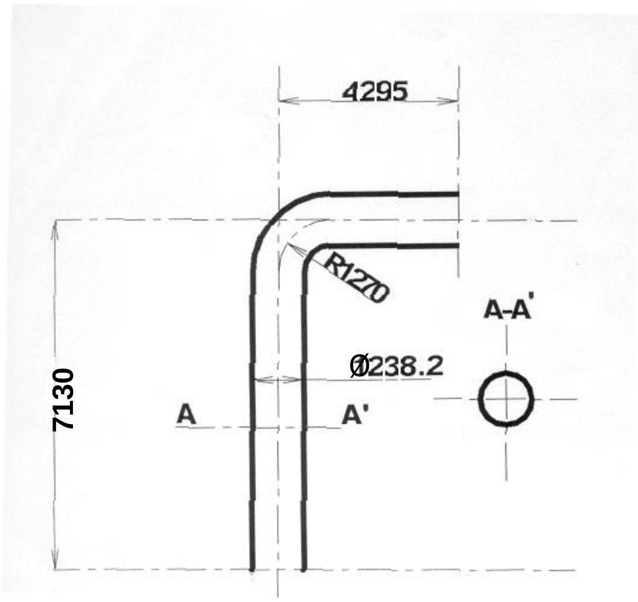
CROSS SECTION DISTRIBUTION OF THE VELOCITY

Example: (For 12 elements)

Element number	Per element cross rate	Average cross section flow rate
		[distribution per element]
1	9,115511E-01	3,301681E+02
2	9,629954E-01	3,488015E+02
3	9,095836E-01	3,294555E+02
4	1,010817E+00	3,661226E+02
5	1,104592E+00	4,000885E+02
6	1,100461E+00	3,985922E+02
7	1,100461E+00	3,985922E+02
8	1,104592E+00	4,000885E+02
9	1,010817E+00	3,661226E+02
10	9,095836E-01	3,294555E+02
11	9,629954E-01	3,488015E+02
12	9,115511E-01	3,301681E+02
Average	1.0	3,622047E+02 in/sec
Non-dimensional flow rate V_1= 3,00E-01		




1.42e+01
1.25e+01
1.08e+01
9.09e+00
7.38e+00
5.68e+00
3.98e+00
2.28e+00
5.72e-01
-1.13e+00
-2.83e+00
Y
X
Z


4295
R1270
A-A'
Ø238.2
7130
A
A'

CASE N^{O} 2

n = 1

R / t = 40.43, L/R=11.09, ν = 0.3

B.C. : clamped - clamped

DIMENSIONLESS PARAMETERS NUMBER 1:

Ω_1 = CAPITAL OMEGA1= OMEGA/OMEGA.ZERO1 ; OMEGA [RAD/SEC]

OMEGA.ZERO1=U.ZERO1/TOTAL.LENGTH= .42947609640665E+01

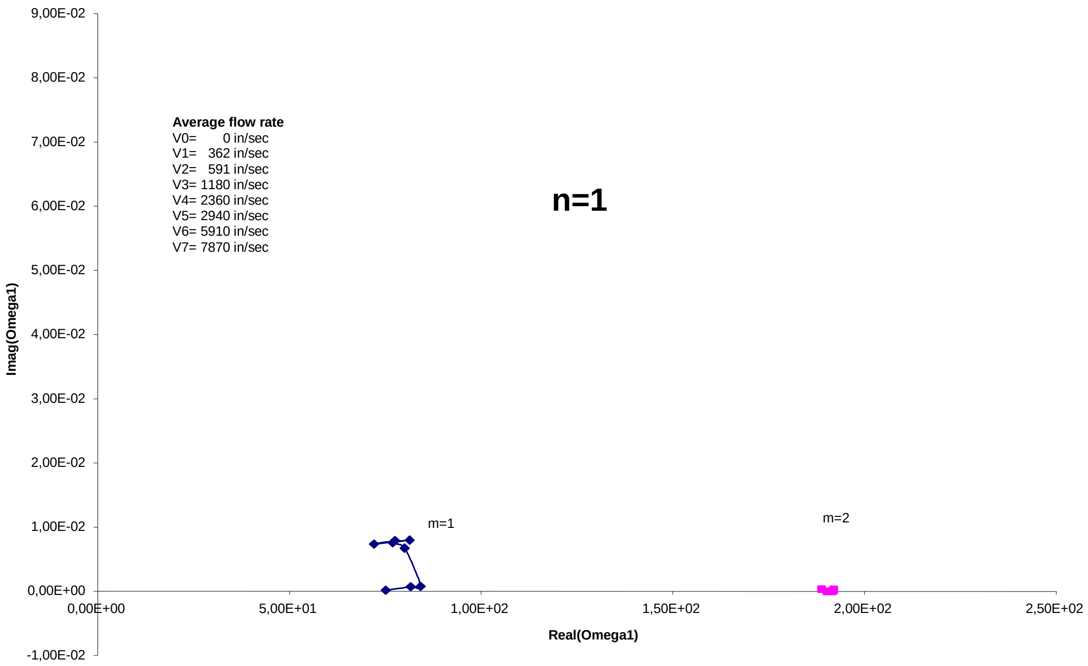
AND U.ZERO1=(PI**2/TOTAL.LENGTH)*SQRT{K/[RHO(1)*TH(1)]}= .12055394026135E+04

WHERE:

K=(E(1)*TH(1)**3)/(12.*(1.-NU(1)**2))

V1= U/U.ZERO1, U [m/sec]

U [m/sec]	V1	Ω_1			
		m=1		**m=2**	
		Real	**Imaginary**	**Real**	**Imaginary**
0,00E+00	0,00E+00	7,54E+01	-6,32E-12	1,46E+02	-9,14E-12
9,20E+00	3,00E-01	8,19E+01	5,48E-04	1,91E+02	-1,43E-04
1,50E+01	4,90E-01	8,45E+01	5,95E-04	1,93E+02	-3,23E-04
3,00E+01	9,80E-01	8,03E+01	6,58E-03	1,74E+02	-1,05E-03
6,00E+01	1,96E+00	7,72E+01	7,40E-03	1,60E+02	-2,24E-03
7,46E+01	2,44E+00	8,16E+01	7,81E-03	1,69E+02	-2,31E-03
1,50E+02	4,90E+00	7,23E+01	7,19E-03	1,90E+02	-1,53E-03
2,00E+02	6,53E+00	7,78E+01	7,74E-03	1,44E+02	1,33E-04


Average flow rate
V0= 0 in/sec
V1= 362 in/sec
V2= 591 in/sec
V3= 1180 in/sec
V4= 2360 in/sec
V5= 2940 in/sec
V6= 5910 in/sec
V7= 7870 in/sec
n=1
m=1
m=2
Imag(Omega1)
Real(Omega1)
9,00E-02
8,00E-02
7,00E-02
6,00E-02
5,00E-02
4,00E-02
3,00E-02
2,00E-02
1,00E-02
0,00E+00
-1,00E-02
0,00E+00
5,00E+01
1,00E+02
1,50E+02
2,00E+02
2,50E+02

CASE N^{O} 2

n = 2

R / t = 40.43, L/R=11.09, ν = 0.3

B.C. : clamped - clamped

DIMENSIONLESS PARAMETERS NUMBER 1:

Ω_1 = CAPITAL OMEGA1= OMEGA/OMEGA.ZERO1 ; OMEGA [RAD/SEC]

OMEGA.ZERO1=U.ZERO1/TOTAL.LENGTH= .42947609640665E+01

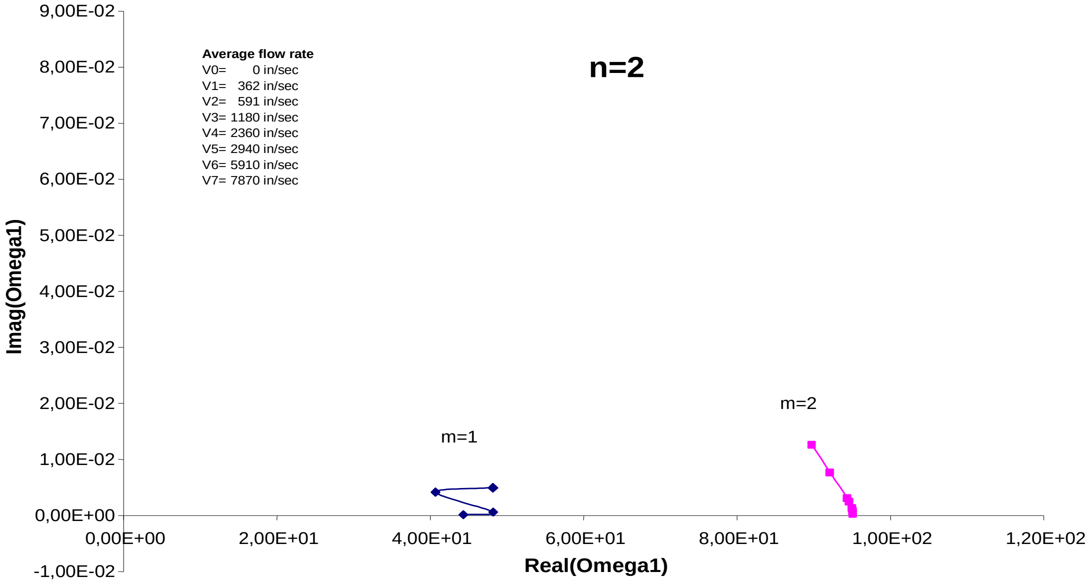
AND U.ZERO1=(PI**2/TOTAL.LENGTH)*SQRT{K/[RHO(1)*TH(1)]}= .12055394026135E+04

WHERE:

K=(E(1)*TH(1)**3)/(12.*(1.-NU(1)**2))

V1= U/U.ZERO1, U [m/sec]

U [m/sec]	V1	Ω_1			
		m=1		**m=2**	
		Real	**Imaginary**	**Real**	**Imaginary**
0,00E+00	0,00E+00	4,44E+01	6,43E-12	1,11E+02	2,77E-12
9,20E+00	3,00E-01	4,83E+01	4,63E-04	1,00E+02	1,46E-04
1,50E+01	4,90E-01	4,08E+01	4,04E-03	1,07E+02	8,74E-03
3,00E+01	9,80E-01	4,83E+01	4,80E-03	1,10E+02	8,02E-03
6,00E+01	1,96E+00	4,83E+01	4,80E-03	1,07E+02	9,85E-03
7,46E+01	2,44E+00	4,83E+01	4,80E-03	1,04E+02	1,04E-03
1,50E+02	4,90E+00	4,83E+01	4,80E-03	1,23E+02	1,22E-03
2,00E+02	6,53E+00	4,83E+01	4,80E-03	1,04E+02	1,00E-02


n=2
Average flow rate
V0= 0 in/sec
V1= 362 in/sec
V2= 591 in/sec
V3= 1180 in/sec
V4= 2360 in/sec
V5= 2940 in/sec
V6= 5910 in/sec
V7= 7870 in/sec
m=1
m=2
Imag(Omega1)
Real(Omega1)
9,00E-02
8,00E-02
7,00E-02
6,00E-02
5,00E-02
4,00E-02
3,00E-02
2,00E-02
1,00E-02
0,00E+00
-1,00E-02
0,00E+00
2,00E+01
4,00E+01
6,00E+01
8,00E+01
1,00E+02
1,20E+02

CASE N^O 2

n = 3

R / t = 40.43, L/R=11.09, ν = 0.3

B.C. : clamped - clamped

DIMENSIONLESS PARAMETERS NUMBER 1:

Ω_1 = CAPITAL OMEGA1= OMEGA/OMEGA.ZERO1 ; OMEGA [RAD/SEC]

OMEGA.ZERO1=U.ZERO1/TOTAL.LENGTH= .42947609640665E+01

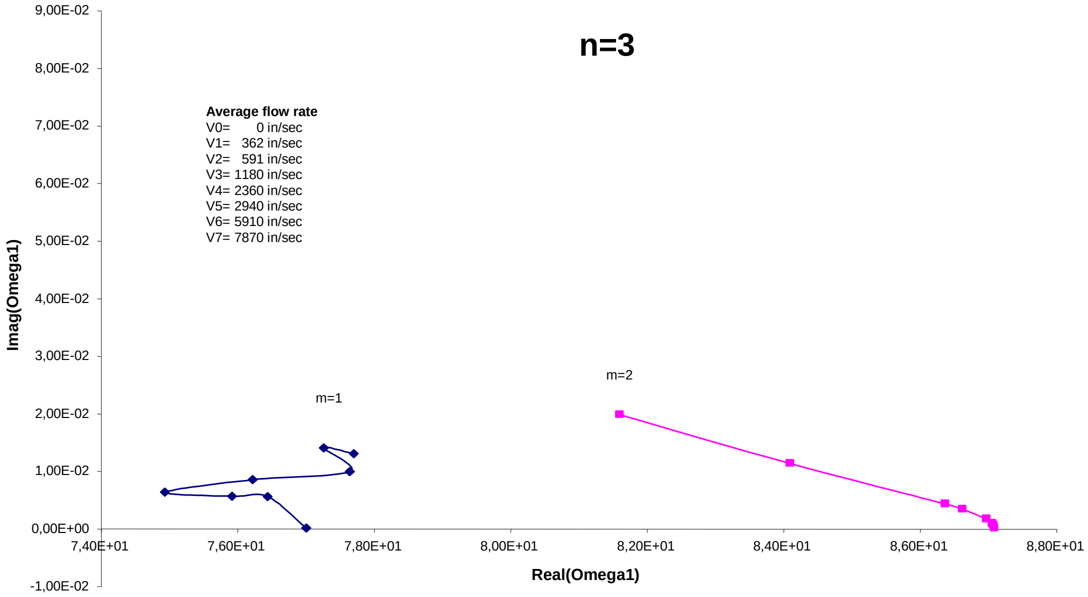
AND U.ZERO1=(PI**2/TOTAL.LENGTH)*SQRT{K/[RHO(1)*TH(1)]}= .12055394026135E+04

WHERE:

K=(E(1)*TH(1)**3)/(12.*(1.-NU(1)**2))

V1= U/U.ZERO1, U [m/sec]

U [m/sec]	V1	Ω_1			
		m=1		**m=2**	
		Real	**Imaginary**	**Real**	**Imaginary**
0,00E+00	0,00E+00	7,70E+01	2,17E-13	9,29E+01	1,92E-12
9,20E+00	3,00E-01	7,65E+01	5,48E-03	9,82E+01	1,46E-03
1,50E+01	4,90E-01	7,59E+01	5,51E-03	1,07E+02	8,74E-03
3,00E+01	9,80E-01	7,49E+01	6,22E-03	8,97E+01	9,28E-03
6,00E+01	1,96E+00	7,62E+01	8,40E-03	9,87E+01	1,02E-02
7,46E+01	2,44E+00	7,77E+01	9,81E-03	1,04E+02	9,78E-03
1,50E+02	4,90E+00	7,73E+01	1,39E-02	1,05E+02	9,78E-03
2,00E+02	6,53E+00	7,77E+01	1,29E-02	1,06E+02	9,78E-03


n=3
Average flow rate
V0= 0 in/sec
V1= 362 in/sec
V2= 591 in/sec
V3= 1180 in/sec
V4= 2360 in/sec
V5= 2940 in/sec
V6= 5910 in/sec
V7= 7870 in/sec
m=1
m=2
Imag(Omega1)
Real(Omega1)
9,00E-02
8,00E-02
7,00E-02
6,00E-02
5,00E-02
4,00E-02
3,00E-02
2,00E-02
1,00E-02
0,00E+00
-1,00E-02
7,40E+01
7,60E+01
7,80E+01
8,00E+01
8,20E+01
8,40E+01
8,60E+01
8,80E+01

CASE N^{O} 2

n = 4

R / t = 40.43, L/R=11.09, ν = 0.3

B.C. : clamped - clamped

DIMENSIONLESS PARAMETERS NUMBER 1:

Ω_1 = CAPITAL OMEGA1= OMEGA/OMEGA.ZERO1 ; OMEGA [RAD/SEC]

OMEGA.ZERO1=U.ZERO1/TOTAL.LENGTH= .42947609640665E+01

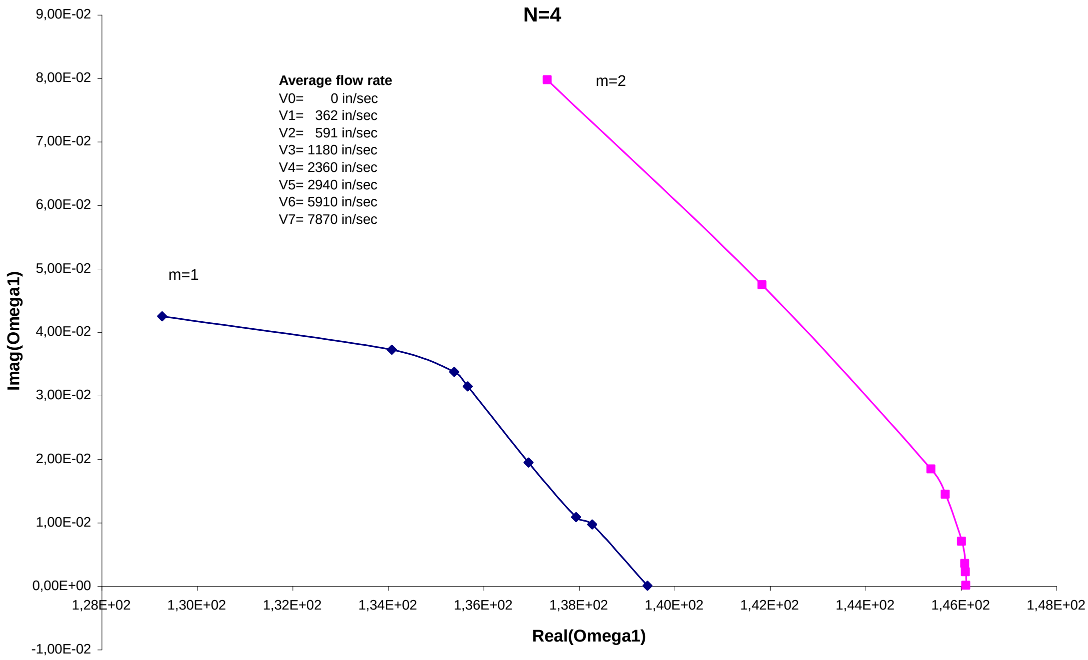
AND U.ZERO1=(PI**2/TOTAL.LENGTH)*SQRT{K/[RHO(1)*TH(1)]}= .12055394026135E+04

WHERE:

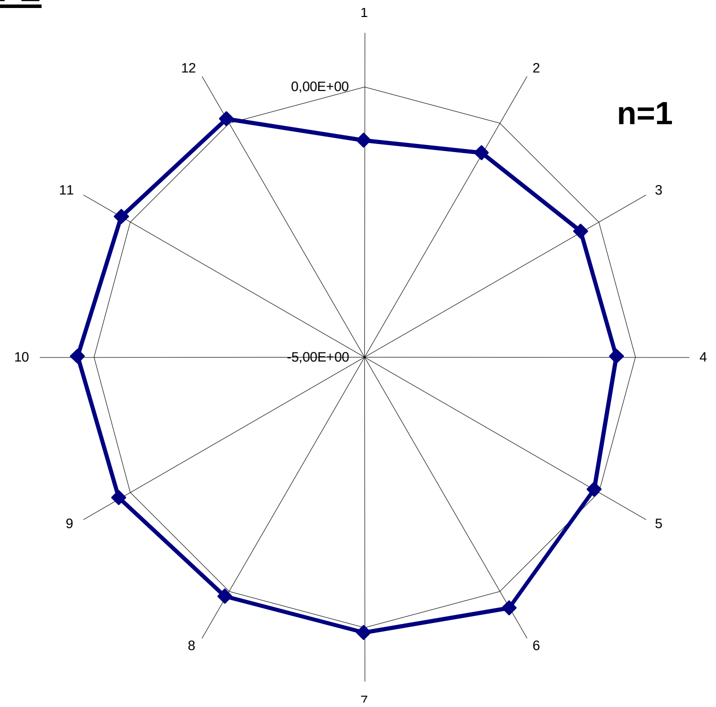
K=(E(1)*TH(1)**3)/(12.*(1.-NU(1)**2))

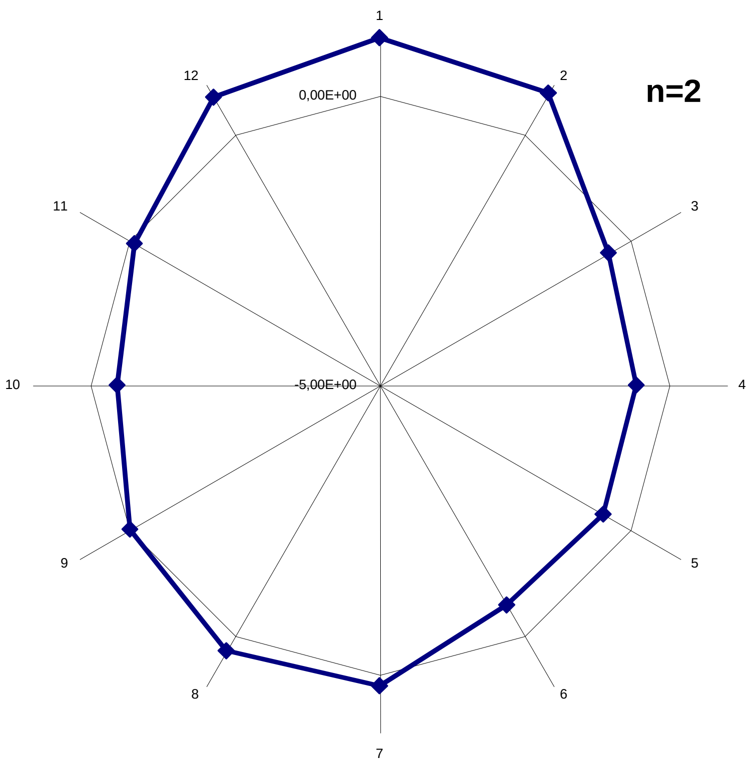
V1= U/U.ZERO1, U [m/sec]

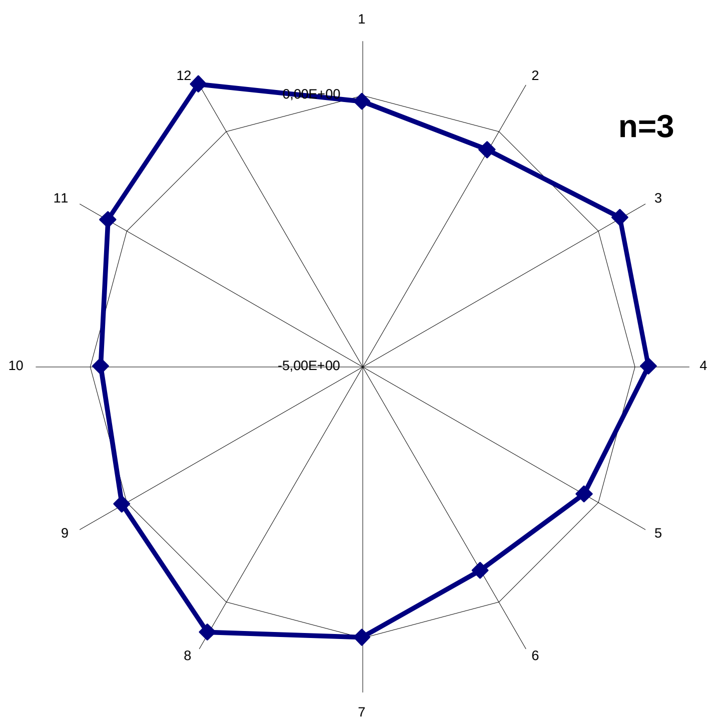
U [m/sec]	V1	Ω_1			
		m=1		m=2	
		Real	Imaginary	Real	Imaginary
0,00E+00	0,00E+00	1,39E+02	2,50E-13	1,48E+02	-1,77E-12
9,20E+00	3,00E-01	1,38E+02	9,66E-03	1,30E+02	9,22E-03
1,50E+01	4,90E-01	1,38E+02	1,08E-02	1,47E+02	1,13E-02
3,00E+01	9,80E-01	1,37E+02	1,94E-02	1,35E+02	5,14E-02
6,00E+01	1,96E+00	1,36E+02	3,14E-02	1,22E+02	9,71E-03
7,46E+01	2,44E+00	1,35E+02	3,37E-02	1,35E+02	4,23E-02
1,50E+02	4,90E+00	1,34E+02	3,72E-02	1,34E+02	4,17E-02
2,00E+02	6,53E+00	1,29E+02	4,25E-02	1,32E+02	3,17E-02

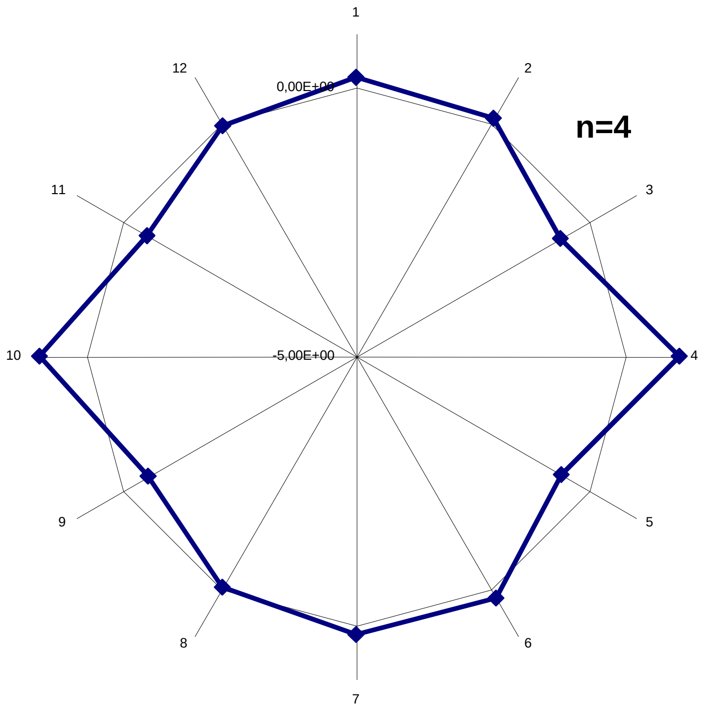

N=4
Average flow rate
V0= 0 in/sec
V1= 362 in/sec
V2= 591 in/sec
V3= 1180 in/sec
V4= 2360 in/sec
V5= 2940 in/sec
V6= 5910 in/sec
V7= 7870 in/sec
m=1
m=2
Imag(Omega1)
Real(Omega1)
9,00E-02
8,00E-02
7,00E-02
6,00E-02
5,00E-02
4,00E-02
3,00E-02
2,00E-02
1,00E-02
0,00E+00
-1,00E-02
1,28E+02
1,30E+02
1,32E+02
1,34E+02
1,36E+02
1,38E+02
1,40E+02
1,42E+02
1,44E+02
1,46E+02
1,48E+02

MODAL SHAPE


n=1
0,00E+00
-5,00E+00
1
2
3
4
5
6
7
8
9
10
11
12


n=2
1
2
3
4
5
6
7
8
9
10
11
12
0,00E+00
-5,00E+00


n=3
1
2
3
4
5
6
7
8
9
10
11
12
0,00E+00
-5,00E+00


n=4
1
2
3
4
5
6
7
8
9
10
11
12
0,00E+00
-5,00E+00