# DIFFERENT APPROACHES IN EUCLIDIAN GEOMETRY 

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#### Abstract

Usually the evaluations at the end of the geometry education are, aimed to measure geometric knowledge and ability. But in this study we tried to investigate how the geometric thinking should be, which is more important than knowledge and ability. Teaching at this direction we aim; improving attention, imagine, creative and criticizing thought, with additional geometric drawings agility, different perspectives and relating between forms. The main objective is focused on geometric thoughts improvement in geometric knowledge and ability. At this direction some geometry questions have been solved, without changing their data's, but putting them to different geometric thought base. With these solutions some activities were presented about degree, length, domain and circle to find answers of the following questions "How should the geometric thought be? How should it be improved?"


## INTRODUCTION

When we look at the TIMIS-1999 (International Mathematic and Science Research) geometry results, the ÖSS and likely exams results, we see Turkey's national and international success rate is far away from the world (1).

We can think its reason is, the geometry program is usually at the end, because of this its importance lacks, or the exam systems (LGS, ÖSS). This is only one of the reasons of failure. It's wrong to see the failure because of this reason or its neighborhood. For us, the main reason of the failure is; the teachers' behavior of leading the students to memorizing in geometric knowledge and ability obtaining process. Because of this many students think the geometry is a chain of formulas. Because the researches show that for many students the geometry is shown as a bunch of formulas, memorizing rules or memorizing contents unconsciously. Because of this, in application level, for given problems the aim has to be solving and making the student solve the problem away from formula, generalization, memorizing and contents, by different ways. In this direction by the following applications we tried to find answers of these questions; How can the geometric thought be? How can the geometric thought improve?


## Application 1 (Degree application)

In the figure if;
$|\mathrm{CD}| \perp|\mathrm{AB}|,|\mathrm{BF}| \perp|\mathrm{AC}|$,
$|\mathrm{BE}|=|\mathrm{EC}|$ and $\mathrm{mBÂ} \mathrm{C}=80^{\circ}$
How much is DEF degree?
Solution.1.1: A geometry educated student writes $|\mathrm{DE}|=|\mathrm{EF}|=1 / 2|\mathrm{BC}|$ equation, because for BDC and BFC right triangles respectively $|\mathrm{DE}|$ and $|\mathrm{EF}|$ are the border intermediates of hypotenuse.
Then by using
$\Delta \quad \Delta$
BDE and FEC isosceles triangles the student finds $\mathrm{x}=2 \mathrm{a}+2 \mathrm{~b}-180$.
$\Delta$
From ABC, writes $a+b=100$ in previous equation, finds $x=20^{\circ}$.
It's impossible that a student can earn judgment from this way of solution. Because, the student only applies his own knowledge.

## Solution.1.2



This solution is, a geometry educated students solving the problem by combining one chapter with another. In this direction the student can combine the triangle with circle. Here; because of
$\mathrm{mBDC}=\mathrm{mBFC}=90^{\circ}$ there is a circle, which has the diameter $|\mathrm{BC}|$ and has D and F points. From the definitions and theorems of exterior angle and centre angle in circle; from

$$
80=\frac{180-x}{2}
$$

we find $\mathrm{x}=20^{\circ}$. In this way the student used his knowledge as solution 1.1. The student didn't earn any judgment and different geometric thought.

Let's get a different interpretation by Solution 3 below.

## Solution.1.3 (geometric thinking - different approach)

In this solution the student can prefer B and C angles, which the sum of them is $100^{\circ}$. For example, take angle $\mathrm{B}, 40^{\circ}$ and angle C, $60^{\circ}$ :


With this geometric thought:
$m(B \hat{E} D)=100^{\circ}$ and $m(F E \hat{C})=60^{\circ}$, we find $x=20^{\circ}$.

Application 2 (Length)
In the figure;


If $|A B|=7, \quad|A C|=6, \quad|K C|=5 \quad$ and
$|\mathrm{AH}| \perp|\mathrm{BC}|$, then $\quad|\mathrm{BK}|=$ ?

## Solution 2.1.1 (traditional solution)


$\Delta$
in BKH and CKH right triangles

$$
\begin{gather*}
\mathrm{x}^{2}-5^{2}=\mathrm{a}^{2}-\mathrm{b}^{2} .  \tag{1}\\
\Delta
\end{gather*}
$$

in ABH and ACH right triangles
$7^{2}-a^{2}=6^{2}-b^{2}$
from (1) and (2) $x^{2}-5^{2}=7^{2}-6^{2}$
we find $x=\sqrt{38}$

## Solution 2.1.2 (traditional solution)



When we take the symmetrical of BKC triangle for $|\mathrm{BC}|$ border, we obtain a quadrangle, which has right diagonals.
As in solution 2.1.1 if we use the Pythagorean equation for
$\Delta \quad \Delta \quad \Delta \quad \Delta$
$\mathrm{ADH}, \mathrm{AHC}, \mathrm{KCH}, \mathrm{KCB}$ right triangles, we have
$7^{2}+5^{2}=x^{2}+6^{2}$ and then find $x=\sqrt{38}$.
Let's solve this question by a new approach different than these usual solutions:

## Solution.2.2



Let's put $|\mathrm{BC}|$ and $|\mathrm{AH}|$ line parts static rings and $|\mathrm{AC}|,|\mathrm{CK}|,|\mathrm{KB}|,|\mathrm{AB}|$ line parts, at the tips, moving rings: If we collide $|\mathrm{BK}|$ and $|\mathrm{KC}|$ edges with $|\mathrm{BC}|$ line part we obtain figure 2.5.
$\Delta \quad \Delta$
With ABH and AHC right triangles, using Pythagorean equation;
From $7^{2}-x^{2}=6^{2}-5^{2}$ we find $x=\sqrt{38}$.
Let's carry the different approach, which we formed in application 1.1 and application 2.1, to domain concept by application 3 .

## Application 3 (Domain)


$4|\mathrm{AE}|=|\mathrm{EC}|$
$3|\mathrm{BD}|=|\mathrm{BC}|$
if $|\mathrm{AF}|=|\mathrm{FB}|$ then $\mathrm{A}(\mathrm{FED}) / \mathrm{A}(\mathrm{ABC})=$ ?

Solution.3.1 (traditional solution)


The domains of ECD, FBD and AFE triangles are respectively $S_{1} S_{2} S_{3}$ and $A B C$ triangles domain is $S$. In this condition

$$
\begin{aligned}
& \frac{A(F E D)}{A(A B C)}=\frac{S-\left(S_{1}+S_{2}+S_{3}\right)}{S} \\
& \downarrow \\
& S_{1}=s\left(\frac{3}{4} \cdot \frac{4}{5}\right) \\
& S_{2}=s\left(\frac{1}{4} \cdot \frac{1}{2}\right) \\
& S_{3}=s\left(\frac{1}{2} \cdot \frac{1}{5}\right)
\end{aligned}
$$

After these operations this ratio is $7 / 40$.

## Solution.3.2 (geometric thinking - different approach)

As long as the desired result is a ratio, we can use the data we want and the triangles we want like a constant function.


Under this thought, let ABC be an equilateral triangle, which has a border of 20 cm .

$$
\begin{aligned}
A(D E F) & =A(A B C)-\left(S_{1}+S_{2}+S_{3}\right) \\
& =200 \cdot \sin 60-165 \cdot \sin 60
\end{aligned}
$$

From this $\mathrm{A}(\mathrm{DEF}) / \mathrm{A}(\mathrm{ABC})=7 / 40$

## DISCUSSION, CONCLUSION AND SUGGESTIONS

1. To improve the students' geometric knowledge, ability and thoughts;
1.1. Teachers, who owned the geometric thought, are needed first. Because of this, at the teacher educating institution, geometry education must have a special importance. The existing mathematic teachers' self renewing must be interrogated.
1.2. The geometric figures' have to be classified, get new forms; have additional drawings, carryings or perceiving them in different environments.
2. The students must relate algebra and geometry first, then the other classes.
3. Through the education process, some applications have to be arranged, which is far from memorizing but making students think.
4. The questions or examples have to have more than one solution and they have to relate with the other subjects of the area.

## REFERENCES

1) Olkun, S. \& Aydoğdu, T. elementary-Online. Third international Mathematics and Science Research (TIMSS) 2003. pg.28-35
2) S. Hızarcı, A.S. İpek, Applied Mathematics and Computation. 2003
3) S. Hızarcı, A. Kaplan, C. Işık, A.S. İpek, Euclid Geometry and Private Educating, PEGEM A Broadcasting 2. Publish
