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INVESTIGATING CONVERGENCE OF DIFFERENT TYPES OF SIGNED DECISION-DIRECTED LMS EQUALIZER

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ABSTRACT

When the receiver does not have knowledge of the transmitted signal, it must make use of the blind channel estimation techniques to reveal the source signal. A subclass of the techniques is called decision directed least mean square equalization. This technique utilizes the detected signal to reconstruct the transmitted signal and uses this signal in place of the original signal. The decision directed LMS equalizer uses the sign operator in decision making. The three types of sign operators which are; sign error LMS algorithm, sign data LMS algorithm and sign.sign LMS algorithms are investigated and their performance noted.

INTRODUCTION

For over twenty years, research has centered on developing new algorithms and formulating a theoretical justification for these algorithms. Blind channel equalization is also known as self-recovery equalization. The essence of blind equalization is to recover the unknown input sequence to the unknown channel based solely on the probabilistic and the statistical properties of the input sequence. The receiver can synchronize to the received signal and adjust the equalizer without the training sequence. The term blind is used in this argument because it performs the equalization on the data without reference signal. Instead the blind equalizer relies on knowledge of the signal structure and its statistics to perform the equalization.

(i) Blind signal is the unknown signal which would be identified in output signal with accommodated noise at the receiver.
(ii) Channel equalization uses the idea and knowledge of the training sequences for channel estimation whereas blind channel equalization does not utilize the characteristics of the training sequences for frequency and impulse response analysis of channel.
(iii) Blind channel equalization differs from channel equalization and without knowing the channel characteristics like transfer function and SNR it efficiently estimates the channel and reduces ISI by blind signal separation at receiver side by suppressing noise in the received signal.

The algorithms whose operations are based on the principle of blind equalization are the Decision Directed Equalization algorithm and the Dispersion Minimization Algorithm but only decision directed equalization is treated below. For the decision directed equalizer, the decision device at the output $y\{k\}$ is a simple threshold device which can be represented as a sign operator that computes the source recovery as $\text{sign}\{y(k)\}$. The sign of $y(k)$ gives the delayed source signal that is equivalent to $S(k - \delta)$. This work treats three types of sign operators. They are the sign data LMS algorithm, sign error LMS algorithm and sign.sign LMS algorithm (i.e., $\text{sign}\{e(k)\}.\text{Sign}\{x(k)\}$).The next section of this paper develops an algorithm for blind equalization and section III presents an algorithm for optimizing tap coefficients using gradient descent method. Section IV treats the signed LMS algorithm while V treats the sign.sign LMS algorithm. Section VI presents some simulation results for the three types of sign operators.

DECISION DIRECTED LMS ALGORITHM FOR BLIND EQUALIZATION

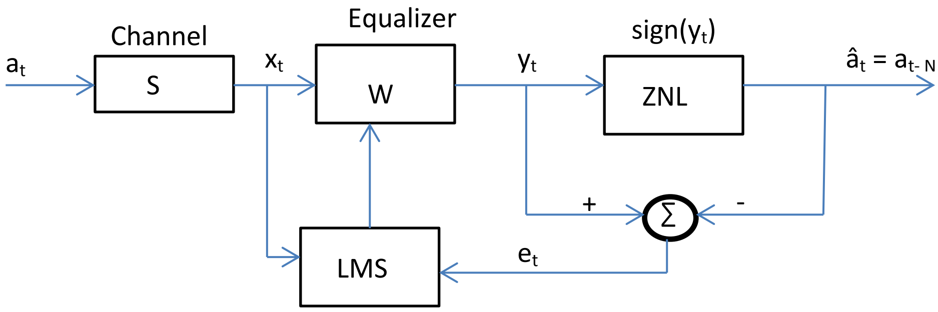

Fig 1: Decision directed LMS

Assume that a data sequence is transmitted through a linear channel with unknown impulse response S. The data may take on one of a small number of discrete values (e.g., ± a). The output of the unknown channel, x_t , is passed through a linear transversal filter W whose impulse response is approximately the delayed source sequence S^{-1}. The purpose of applying the equalizer W is to council out most of the distorting effects of the channel S so that the output sequence y_t is a new copy of the original transmitted sequence $\hat{a}_t$. The zero order memory less nonlinear decision process (ZNL) of fig (1) investigates each output y_t and replaces it with the closest value from the set of discrete input values producing the estimated sequence ($\hat{a}_t$). As a result of the delay introduced into the system by the equalizer W, the desired output generated by y_t when passed through the ZNL is given by the estimate of $\hat{a}_t = a_{t-N}$ where N is the unknown delay introduced by equalizer W [Steven J. Nowlan et al].

The system is frequently applied in digital communications where data must be converted into analog form for transmission before converting back into digital form at the receiver. The equalizer W is generally a filter with adjustable tap weights. The envisaged problem is how to adjust these tap weights so that the equalizer produces a good approximation of the source sequence S^{-1} . If the sequence (a_t)is known, the classical approach is to use an LMS or stochastic gradient descent procedure to minimize $E[(y_t - a_{t-N})^2]$. The usual practice is to assume stationarity of systems and processes and to substitute time averages for ensemble averages. But this expectation is technically evaluated with respect to ensemble averages of the signal. In this argument, one may assume that all expectations are evaluated as time rather than ensemble averages. In practical situations the sequence (a_t) is not known. Instead, a two-step procedure is used to adjust the equalizer; (i) an initial settling phase in which the transmitter sends a known initialization sequence and the receiver performs LMS adjustment of the equalizer and (ii) a phase in which the receiver uses the output of the ZNL ($\hat{a}_t$) as an estimate for (a_t) in performing LMS adjustment of the equalizer. It is this second step of the procedure that is called the decision-directed mode of equalization since the updates to the tap weights of the equalizer are controlled by the decisions made by the zero-order memory less nonlinear system.

In the classical decision directed LMS algorithm, the ZNL is a simple threshold device. For the binary channel we are considering, the output of the ZNL can be represented as a sign(y_t) and the decision directed LMS algorithm can be regarded as minimizing $E[(y_t - a\ \mathrm{sign}(y_t)^2]$.

In this paper, we discuss the different forms for operating LMS equalizer in decision-directed mode.

DEVELOPING ALGORITHM FOR OPTIMIZING TAP COEFFICIENTS

The adaptive filter works on the principle minimizing the mean square error between the filter output and target signal [Ude Anthony O., 2010]. Adaptive filters are used for estimation of non-stationary signals and systems or in application where a sample by sample adaptation of a process and/or low processing delay is required.

Least mean square (LMS), Recursive Least Square (RLS), and Steepest Descent Algorithms are based on finite impulse response (FIR) adaptive filtering where the filter coefficient corresponds to the weight vector of impinging signals on each array. The least mean squares (LMS) algorithms adjust the filter coefficients to minimize the cost function. Compared to recursive least squares (RLS) algorithms, the LMS algorithms do not involve any matrix operations. Therefore, the LMS algorithms require fewer computational resources and memory than the RLS algorithms. The implementation of the LMS algorithms also is less complicated than the RLS algorithms. However, the eigenvalue spread of the input correlation matrix, or the correlation matrix of the input signal, might affect the convergence speed of the resulting adaptive filter.

By adaptively varying the filter coefficients, the weight vectors are varied according to the changing channel condition and position of the mobile user (MS) [Debashre Mohapatra et al].

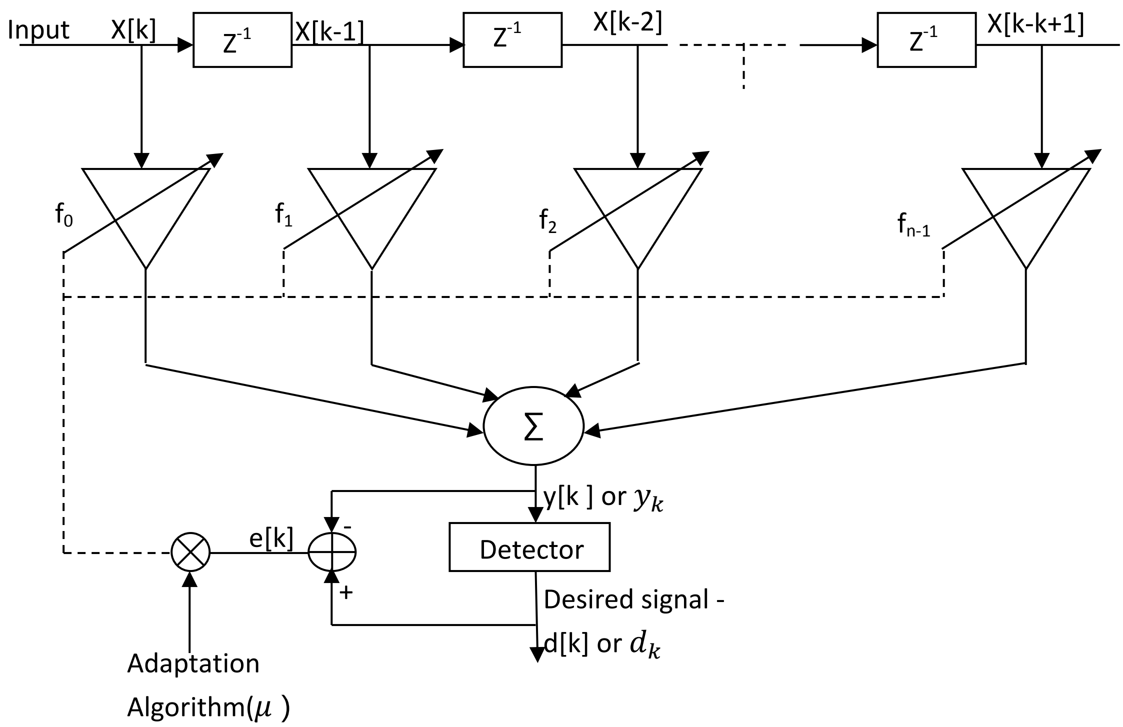

Figure 2: Block Diagram of Simple Adaptive Filter

The adaptive digital filter equations are summarized by [Proakis John G. et al, 2004]:
Output equation $y[k] = f[k].X[k]$ ... ...(1)
Where $f[k]$ is the filter coefficient or weight vector, while $X[n]$ is the input signal vector.

Error signal; $e[k] = d[k] - y[k] = d[k] - f[k].X[k] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2)$
Where $d[k]$ is the desired or reference signal.

Adaptive channel equalization is required for channels whose characteristics change with time. In this situation, the intersymbol interference (ISI) varies with time. Therefore the channel equalizer must track such time variations in the channel response and adapt its coefficients to reduce the intersymbol interference (ISI). In the context of this work, the optimum coefficient vector $\mathbf{f}_{opt}$ varies with time due to time variation in the matrix B [Proakis] and for the case of mean square error (MSE) criterion, time variations in the vector d as expressed in the set of linear equations in the general matrix form. Under this condition the iterative method described by the coefficient vector
$f_{k+1} = f_k - \mu g_k$ ……………………………….(3)
can be modified to use the estimates of the gradient components. Thus the algorithm for adjusting the equalizer tap coefficients may be expressed as
$\widehat{f_{k+1}} = \widehat{f_k} - \mu\widehat{g_k}$ ……………………………..(4)
Where $\widehat{g_k}$ denotes an estimate of the gradient vector g_k and $\widehat{f_k}$ denotes the estimate of the tap coefficient vector. In the case of the mean square error criterion, the gradient vector g_k can be expressed as
$g_k = -E(e_k y_k^*)$ ………………………………………………………………..(5)
An estimate $\widehat{g_k}$ of the gradient vector at the k^{th} iteration is computed as
$\widehat{g_k} = -(e_k y_k^*)$ …………………………………………………………….(6)
Where e_k denotes the difference between the desired output from the equalizer at the k^{th} time instant and the actual output $\boldsymbol{y}(kT)$ and y_k denotes the column vector of 2k +1 received signal values contained in the equalizer at time instant k. The error signal is expressed as ;
$e_k = d_k - y_k$ ……………………………………………………………(7)
Where $y_k = y(kT)$ is the equalizer output and d_k is the desired symbol. Hence by substituting (6) into (4), we obtain the adaptive algorithm for optimizing the tap coefficients (based on the MSE criterion) as
$\widehat{f_{k+1}} = \widehat{f_k} + \mu e_k y_k^*$ ……………………………………………………………(8)
Since the estimate of the gradient vector is used in (8), the algorithm is called the stochastic gradient algorithm. It is also known as the Least Mean Square (LMS) algorithm.
This adaptive equalizer based on the LMS algorithm can be implemented using MatLab. If the channel number of taps are selected for the equalizer as 2k+1=11, and the received signal plus noise power P_R is normalized to unity[Saeed V., 2006], the channel characteristic is given by the vector **x** as
x = [0.05 -0.063 0.088 -0.126 -0.25 0.9047 0.25 0 0.126 0.038 0.088].

In summary, the LMS algorithms adjust the filter coefficients to minimize the cost function. The LMS algorithms require fewer computational resources and memory. The implementation of the LMS algorithm is less complicated. The standard LMS algorithm performs the following operations to update the coefficients of the adaptive filter;

- Calculates the output signal y(k) from the adaptive filter as described in eqn(1).
- Calculates the error signal e(k) by using equation (2).
- Updates the coefficients by application of eqn(8).

SIGN LEAST MEAN SQUARE ALGORITHM

Some adaptive equalizer applications require implementing adaptive filter algorithms on hardware targets, such as digital signal processing (DSP) devices, FPGA targets and application specific integrated circuits (ASICs). In that respect, the targets require a simplified version of the standard LMS algorithm. The sign function as defined by the following equation, can simplify the standard LMS algorithm.

$$Sign(y) = \begin{cases} 1, & y > 0 \\ 0, & y = 0 \\ -1, & y < 0 \end{cases} \quad \ldots\ldots(9)$$

Applying the sign function to the standard LMS algorithm returns the following three types of sign LMS algorithms.

- Sign-error LMS algorithm; - it applies the sign function to the error signal e(k). This algorithm updates the coefficients of an adaptive filter using the following equation;

  $$f_{k+1} = f_k + \mu sign\big(e(k)\big)x(k) \ldots\ldots(10)$$

- Sign-data LMS algorithm;- it applies the sign function to the input signal vector $x(k)$. This algorithm updates the coefficients of an adaptive filter using the following equation,

  $$f_{k+1} = f_k + \mu.e(k).sign\big(x(k)\big) \ldots\ldots(11)$$

- Sign-Sign LMS algorithm;- this applies the sign function on both e(k) and $x(k)$. This algorithm updates the coefficients of an adaptive equalizer using the following equation;

  $$f_{k+1} = f_k + \mu.sign\big(e(k)\big).sign\big(x(k)\big) \ldots\ldots(12)$$

The sign LMS algorithms involves fewer computation operations than other algorithms. When the step-size μ equals a power of 2, the sign LMS algorithm can replace the multiplication operations with shift operations. Compared to the standard LMS algorithm, the sign LMS algorithm has a slower convergence speed and a greater steady state error. The sign-error LMS algorithm is used in this research and is applied at the decision directed LMS algorithm determination in section (II).

SIGN-SIGN LEAST MEAN SQUARE ALGORITHM

The equalizer coefficients are computed using the sign-sign least mean square (SS-LMS) method because it demonstrates the simplicity and robustness needed for realization in very high speed circuits (Gurpreet Kaur, Gurmeet Kuar). The flow chart for sign-sign least mean square algorithm shown in fig(3) is summarized as follows:

Step (i) – Initialize the filter weight for minimum mean square error.
Step (ii) – After that the i number of delayed versions of received signal using 100ps time delay was multiplied with these weights and got the actual output which was the summation of all these terms.
Step (iii) – The error signal was calculated as given in the following equation $e(k) = d(k) - y(k)$ … … … … … … … … … … … … ….(13)

where k number of inputs, d(k) is desired output signal, y(k) is the actual output signal and e(k) is the error signal.

Step (iv) – The filter weight was updated using sign-sign least mean square method as given below; $f(k) = f(k-1) + asign\big(e(k)\big)sign\big(u(k)\big)$ … … … … … … … (14)

where f(k) is the weight update, f(k-1) is the previous weight, u(k) is the actual input signal and a is the step-size which controls the convergence rate and stability of the algorithm. The value of the a is chosen from $0 < \alpha < \frac{2}{\sum_{i=1}^{K}\lambda_i}$ … … … … … … … … … … … … … … ….……….(15)

where λ_i is the i^th eigenvalue of the covariance matrix R_{kk}.

Step (v) – The procedure is repeated until the limit of minimum mean square error was achieved.

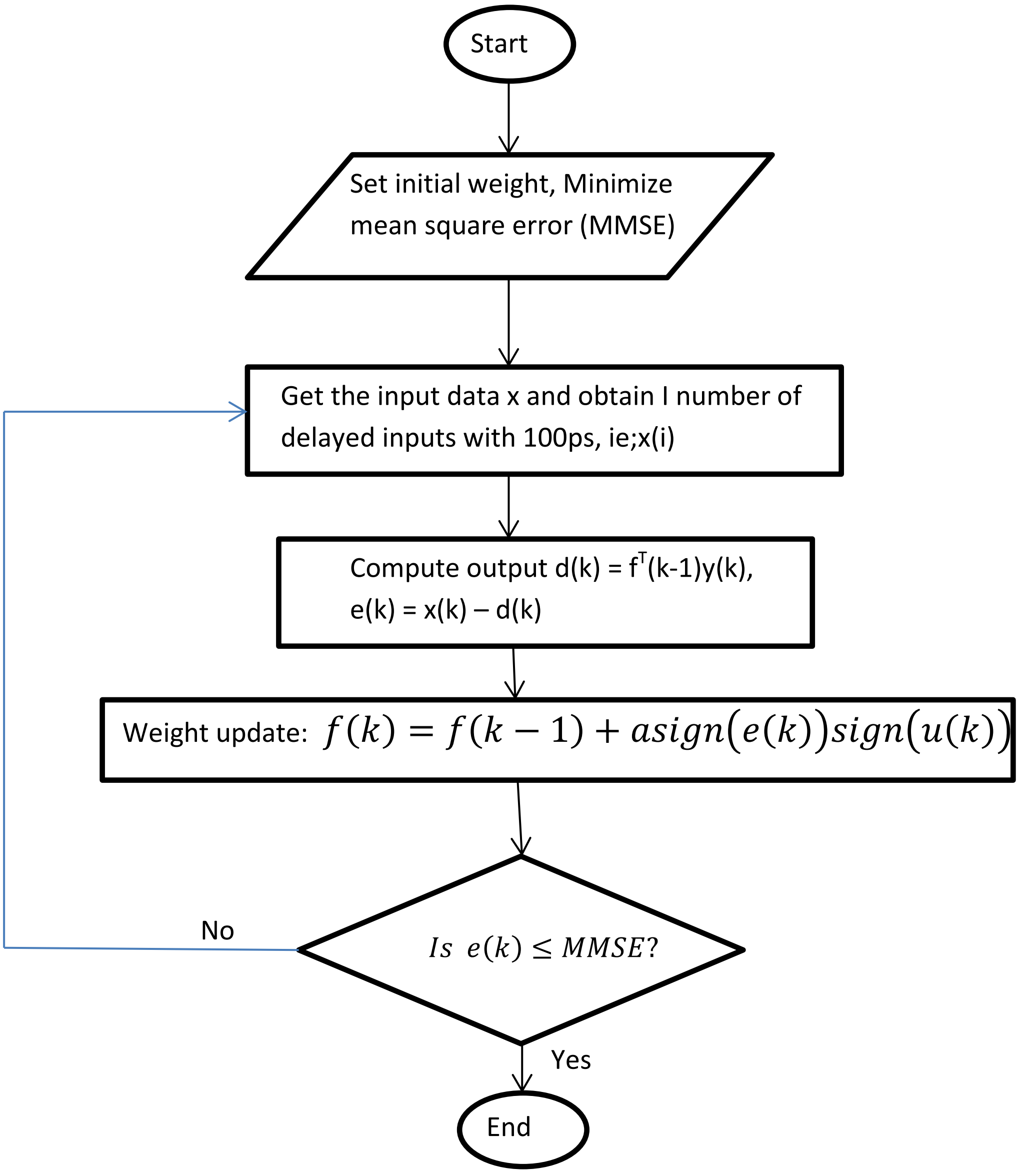

Fig 3: Flow chart of Sign-Sign LMS Algorithm

SIMULATION RESULTS

A simulation study has been conducted in order to evaluate the performance of the decision directed sign LMS operators. All the sign-algorithms are individually simulated using tap length of 10 at step-size of 0.0001. The number of iteration given is 5000. The input to the equalizer is a sinusoidal sign from a noisy channel. The signal graph is plotted against a time index value spanning up to 5000. The adaptive line enhancement is demonstrated using a 32-coefficient FIR filter to provide good introduction to the sign-sign algorithm. The power spectral density in the figure shows the sign-error LMS algorithm as giving an observed and enhanced signal with little or no noise ripples and this shows it to be better than the others. As it's PSD span between 0 to -40dB, that of sign-data and sign-sign LMS exceeds -40dB from zero value. The deviation of the enhanced signal from the original in both sign-data LMS algorithm, sign-sign LMS and the sign-error LMS algorithm is not much. Finally, the three have greater steady state error but slower convergence speed than the standard LMS algorithm.

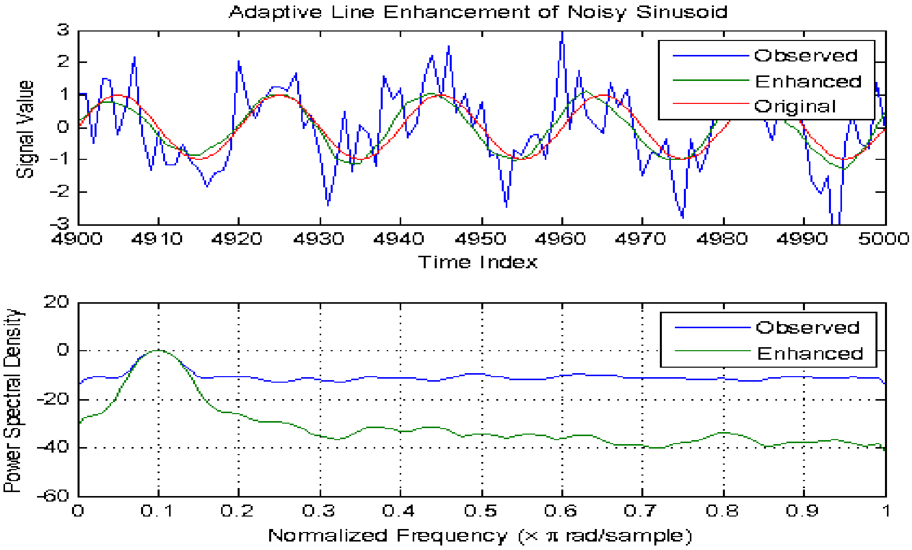

Fig 4: Sign Error LMS Algorithm

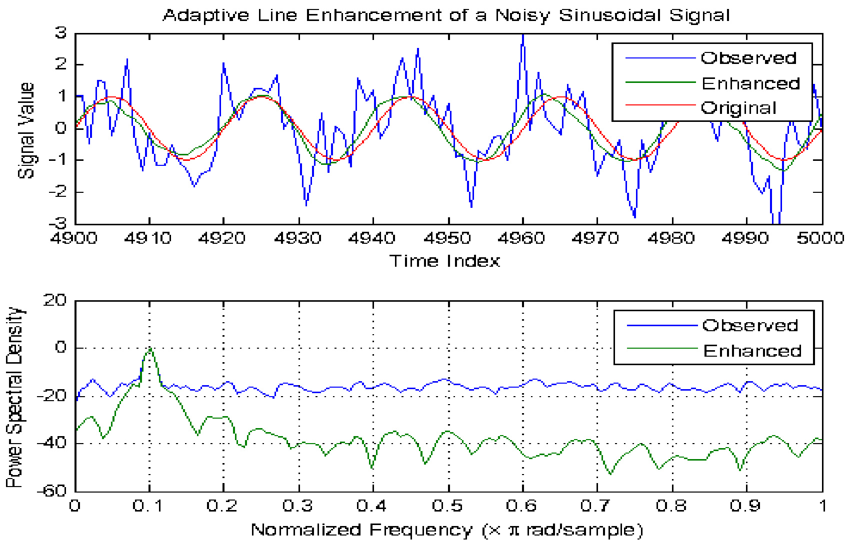

Fig 5: Sign data Algorithm

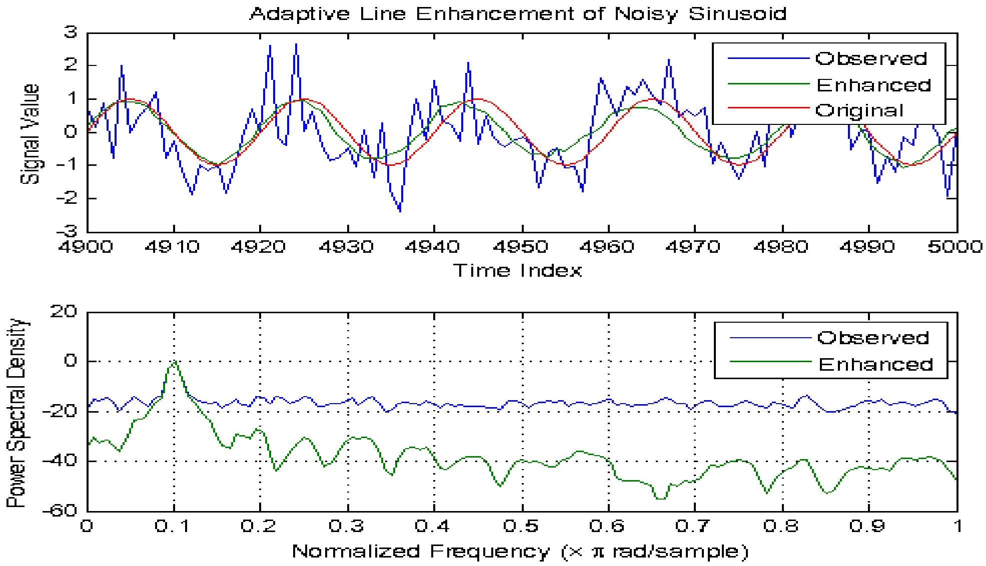

Fig 6: Sign-Sign LMS

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