GRAPH THEORY IN COMPUTER SCIENCE - AN OVERVIEW

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ABSTRACT

The field of mathematics plays vital role in various fields. One of the important areas in mathematics is graph theory which is used in structural models. We give a survey of graph theory used in computer sciences. The survey consists of a description of particular topics from the theory of graph of the areas of Computer science in which they are used. However, for each described theory we indicate the fields in which it is used (e.g. in modelling and searching Internet, in computer vision, pattern recognition, data mining, multiprocessor systems, statistical databases, and in several other areas). This paper gives an overview of the applications of graph theory in heterogeneous fields to some extent but mainly focuses on the computer science applications that uses graph theoretical concepts. Various papers based on graph theory have been studied related to scheduling concepts, computer science applications and an overview has been presented here.

Keywords: Graphs, network, application of graphs, graph algorithms, bipartite graph etc.

INTRODUCTION

Graph theory is an old subject, but one that has many fascinating modern applications. Graph theoretical ideas are highly utilized by computer science applications. Especially in research areas of computer science such data mining, image segmentation, clustering, image capturing, networking. These applications in turn have offered important stimulus to the development of the field, leading to generalizations of important graph theoretical concepts and challenging questions about them. The powerful combinatorial methods found in graph theory have also been used to prove significant and well-known results in a variety of areas in mathematics itself. Modeling of network topologies can be done using graph concepts. In the same way the most important concept of graph coloring is utilized in resource allocation, scheduling. This paper has been divided into two sections. First section gives the historical background of graph theory and some applications in scheduling. Second section emphasizes how graph theory is utilized in various computer applications.

History of Graph theory

The origin of graph theory started with the problem of Koinsber Bridge, in 1735. This problem lead to the concept of Eulerian Graph. Euler studied the problem of Koinsberg bridge and constructed a structure to solve the problem called Eulerian graph. In 1840, A.F Mobius gave the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems. The concept of tree, (a connected graph without cycles was implemented by Gustav Kirchhoff in 1845, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits. In 1852, Thomas Gutherie found the famous four color problem. Then in 1856, Thomas. P. Kirkman and William R. Hamilton studied cycles on polyhydra and invented the concept called Hamiltonian graph by studying trips that visited certain sites exactly once. In 1913, H. Dudeney mentioned a puzzle problem. Even though the four color problem was invented it

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was solved only after a century by Kenneth Appel and Wolfgang Haken. This time is considered as the birth of Graph Theory. Caley studied particular analytical forms from differential calculus to study the trees. This had many implications in theoretical chemistry. This lead to the invention of enumerative graph theory. Any how the term “Graph” was introduced by Sylvester in 1878 where he drew an analogy between “Quantic invariants” and covariants of algebra and molecular diagrams. In 1941, Ramsey worked on colorations which lead to the identification of another branch of graph theory called extreme graph theory. In 1969, the four color problem was solved using computers by Heinrich. The study of asymptotic graph connectivity gave rise to random graph theory.

Algorithms and graph theory

The major role of graph theory in computer applications is the development of graph algorithms. Numerous algorithms are used to solve problems that are modeled in the form of graphs. These algorithms are used to solve the graph theoretical concepts which intern used to solve the corresponding computer science application problems. Some algorithms are as follows:

1. Shortest path algorithm in a network 2. Finding a minimum spanning tree 3. Finding graph planarity 4. Algorithms to find adjacency matrices. 5. Algorithms to find the connectedness 6. Algorithms to find the cycles in a graph 7. Algorithms for searching an element in a data structure (DFS, BFS) and so on. Various computer languages are used to support the graph theory concepts. The main goal of such languages is to enable the user to formulate operations on graphs in a compact and natural manner. Some graph theoretic languages are:

1. SPANTREE – To find a spanning tree in the given graph.
2. GTPL – Graph Theoretic Language
3. GASP – Graph Algorithm Software Package
4. HINT – Extension of LISP
5. GRASPE – Extension of LISP
6. IGTS – Extension of FORTRAN
7. GEA – Graphic Extended ALGOL (Extension of ALGOL)
8. AMBIT – To manipulate digraphs
9. GIRL – Graph Information Retrieval Language

Vertex Coloring

Vertex coloring is one of the most important concepts in graph theory and is used in many real time applications in computer science. Various coloring methods are available and can be used on requirement basis. The proper coloring of a graph is the coloring of the vertices and edges with minimal number of colors such that no two vertices should have the same color. The minimum number of colors is called as the chromatic number and the graph is called properly colored graph.

Proper vertex coloring with Chromatic number 3 Proper edge coloring with Chromatic number 3

Figure 1.

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List coloring

In list coloring problem, each vertex v has a list of available colors and we have to find a coloring where the color of each vertex is taken from the list of available colors. This list coloring can be used to model situations where a job can be processed only in certain time slots or can be processed only by certain machines.

Minimum sum coloring

In minimum sum coloring, the sum of the colors assigned to the vertices is minimal in the graph. The minimum sum coloring technique can be applied to the scheduling theory of minimizing the sum of completion times of the jobs. The multicolor version of the problem can be used to model jobs with arbitrary lengths. Here, the finish time of a vertex is the largest color assigned to it and the sum of coloring is the sum of the finish time of the vertices. That is the sum of the finish times in a multicoloring is equal to the sum of completion times in the corresponding schedule.

Time table scheduling

Allocation of classes and subjects to the Teachers is one of the major issues if the constraints are complex. Graph theory plays an important role in this problem. For „t‟ Teachers with „n‟ subjects the available number of „p‟ periods timetable has to be prepared. This is done as follows. A bipartite graph (or bigraph is a graph whose vertices can be divided into two disjoint sets *U* and *V* such that every edge connects a vertex in *U* to one in *V*; that is, *U* and *V* are independent sets) G where the vertices are the number of Faculty say t_1, t_2, t_3, t_4, ……. t_k and n number of subjects say n_1, n_2, n_3, n_4, ……. n_m such that the vertices are connected by „pi‟ edges. It is presumed that at any one period each Teacher can teach at most one subject and that each subject can be taught by maximum one Teacher. Consider the first period. The timetable for this single period corresponds to a matching in the graph and conversely, each matching corresponds to a possible assignment of Teacher to subjects taught during that period. So, the solution for the timetabling problem will be obtained by partitioning the edges of graph G into minimum number of matching. Also the edges have to be colored with minimum number of colors. This problem can also be solved by vertex coloring algorithm. " The line graph L(G) of G has equal number of vertices and edges of G and two vertices in L(G) are connected by an edge iff the corresponding edges of G have a vertex in common. The line graph L(G) is a simple graph and a proper vertex coloring of L(G) gives a proper edge coloring of G by the same number of colors. So, the problem can be solved by finding minimum proper vertex coloring of L(G)." For example, Consider there are 4 Teachers namely t_1, t_2, t_3, t_4,. and 5 subjects say n_1, n_2, n_3, n_4, n_5 to be taught. The teaching requirement matrix p = [p_{ij}] is given as.

P	n_1	n_2	n_3	n_4	n_5
t_1	**2**	**0**	**1**	**1**	**0**
t_2	**0**	**1**	**0**	**1**	**0**
t_3	**0**	**1**	**1**	**1**	**0**
t_4	**0**	**0**	**0**	**1**	**1**

Figure – 2: The teaching requirement matrix for four Teachers and five subjects

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The bipartite graph is constructed as follows.

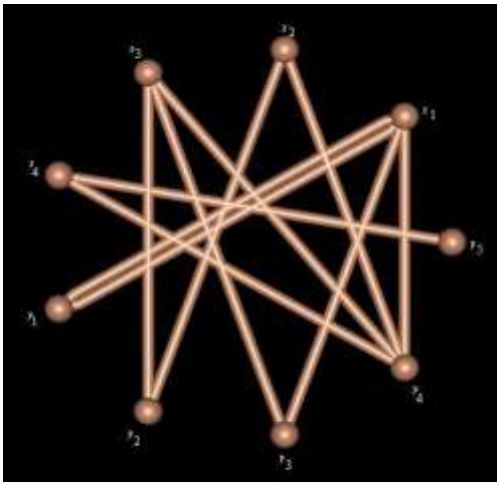

Figure 3. The bipartite multigraph G

Finally, the authors found that proper coloring of the above mentioned graph can be done by 4 colors using the vertex coloring algorithm which leads to the edge coloring of the bipartite multigraph G. Four colors are interpreted to four periods.

…	1	2	**3**	4
t_1	n_1	n_2	n_3	n_4

Figure 4: The schedule for the four subjects

Computer Network Security

A team of computer scientists led by Eric Filiol at the Virology and Cryptology Lab, ESAT, and the French Navy, ESCANSIC, have recently used the vertex cover algorithm to simulate the propagation of stealth worms on large computer networks and design optimal strategies for protecting the network against such virus attacks in real-time.

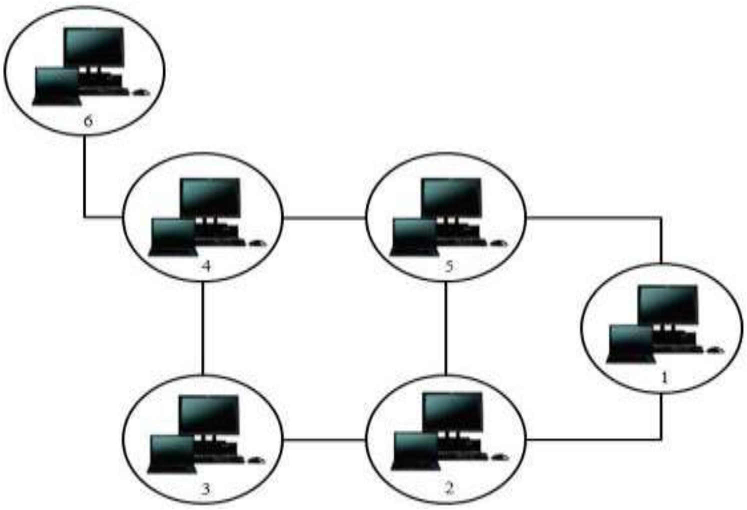

Figure 5. The set {2, 4, 5} *is a minimum vertex cover in this computer network*

The simulation was carried out on a large internet-like virtual network and showed that that the combinatorial topology of routing may have a huge impact on the worm propagation and thus some servers play a more essential and significant role than others. The real-time capability to identify them is essential to greatly hinder worm propagation. The idea is to find a minimum vertex cover in the graph whose vertices are the routing servers and whose edges are the (possibly dynamic) connections between routing servers. This is an optimal solution for worm propagation and an optimal solution for designing the network defense strategy. Figure 5 above shows a simple computer network and a corresponding minimum vertex cover {2, 4, 5}.

Automatic channel allocation for small wireless local area networks using graph coloring algorithm approach

In this section we focuses channel allocation issue in wireless LAN by means of modeling the network in the form of a graph and solving it using graph coloring methodology. The graph model is constructed and called as interference graph since the access points are interfering with some other access points in the same region. The graph is called as interference graph, which is constructed by the access points as nodes. An undirected edge is connecting these nodes if the nodes interfere with each other when using the same channel. Now, the channel allocation problem is converted into graph coloring problem. i.e vertex coloring problem. A vertex coloring function f :v(G)⟹C where C is the set of colors corresponds to the channels on the access points. These channels are preferably non overlapping edges. A coloring algorithm is developed by the authors called DSATUR (Degree of Saturation) for coloring purpose. The algorithm is a heuristic search. i.e It finds vertices with largest number of differently colored neighbours. If this subset contains only one vertex it is chosen for coloring. If the subset contains more than one vertex then the coloring is done based on the order of decreasing number of uncolored neighbours. If more than one candidate vertex is available then the final selection is replaced by a deterministic selection function to select the vertex. The protocol operation is done by identifying the neighbours by means of listening the messages generated by the access points. The protocol operation finishes when a message is rebroadcasted by the access points. After finishing this, the interference graph is constructed and the coloring algorithm is applied. The correspondence between the channels and the graph is that as the channels listen the messages in regular intervals as the same way the coloring algorithm should be kept running at regular intervals. A floorplan and the corresponding interference graph is given below.

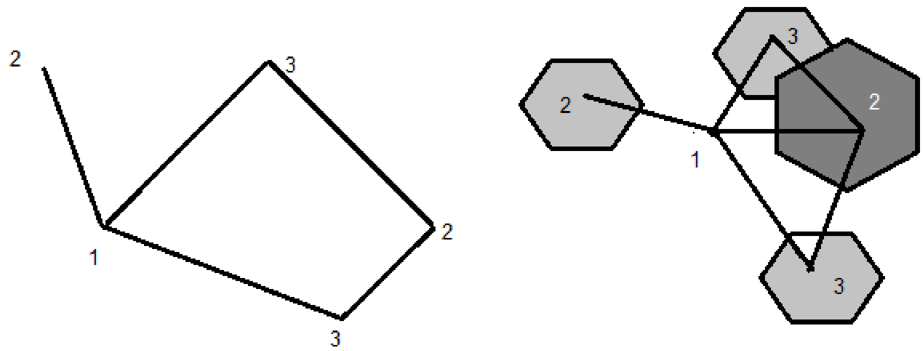

Figure 6. *Interference graph*

Finally we conclude that based on the refinement strategy the interference graph can be reconstructed by adding an edge in case of additional edges.

Graph theory relevant to ad-hoc networks

Here, the authors have discussed the role of graph theory related to the issues in Mobile Adhoc Networks (MANETS). In Adhoc networks, issues such as connectivity, scalability, routing, modeling the network and simulation are to be considered. Since a network can be modeled as a graph, the model can be used to analyze these issues. Graphs can be algebraically represented as matrices. Also, networks can be automated by means of algorithms. The issues such as node density, mobility among the nodes, link formation between the nodes and packet routing have to be simulated. To simulate these concepts random graph theory is sued. The connectivity issues are analyzed by using graph spanners, (A geometric spanner or a *k*-spanner graph or a *k*-spanner was initially introduced as a weighted graph over a set of points as its vertices and every pair of vertices has a path between them of weight at most *k* times the spatial distance between these points, for a fixed *k*.) proximity graphs,(A proximity graph is simply a graph in which two vertices are connected by an edge if and only if the vertices satisfy particular geometric requirements), sparsification and spectral graph theory. Various algorithms are also available to analyze the congestion in MANET's where these networks are modeled based on graph theoretical ideas.

A graph model for fault tolerant computing systems

This paper is based on graph theory where it is used to model the fault tolerant system. Here, the computer is represented as S and the algorithm to be executed by S is known as A. Both S and A are represented by means of graphs whose nodes represent computing facilities. It is shown that the algorithm A is executable by S if A is isomorphic to a sub graph of S. The authors have presented a graph model and algorithms for computing systems for fault tolerant systems. These graphs show the computing facility of a particular computation and the interconnection among them. This model is applied directly to the minimum configuration or structure required to achieve fault tolerance to a specified degree. The model is represented in the form of a facility graph. A facility graph is a graph G whose nodes represent system facilities and whose edges represent access links between facilities. A facility here is said to be a hardware or software components of any system that can fail independently. Hardware facilities include control units, arithmetic processors, storage units and input/output equipments. Software facilities include compilers, application programs, library routines, operating systems etc. Since each facility can access some other facilities, the real time systems are represented as a facility graph. The following is a labeled directed facility graph. Facility types are indicated by numbers in parentheses. The graph indicates the types of facilities accessed by other facilities. The node x_1 access the nodes x_2 and x_4. Similarly, the node x_5 with facility type t_1 access the facility types t_3, t_1 and t_2 of nodes x_6, x_2 and x_4 respectively.

GSM Mobile Phone Networks

The Groupe Spécial Mobile (GSM) was created in 1982 to provide a standard for a mobile telephone system. The first GSM network was launched in 1991 by Radiolinja in Finland with joint technical infrastructure maintenance from Ericsson. Today, GSM is the most popular standard for mobile phones in the world, used by over 2 billion people across more than 212 countries. GSM is a cellular network with its entire geographical range divided into hexagonal cells. Each cell has a communication tower which connects with mobile phones within the cell. All mobile phones connect to the GSM network by searching for cells in the immediate vicinity. GSM networks operate in only four different frequency ranges. The reason why only four different frequencies suffice is clear: the map of the cellular regions can be properly colored by using only four different colors! So, the vertex coloring algorithm may be used for assigning at most four different frequencies for any GSM mobile phone network, see figure 7 below.

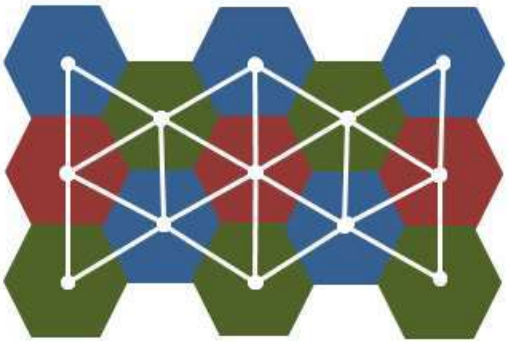

Figure 7 *The cells of a GSM mobile phone network*

Closing Comment

Graph theory has found widespread application in numerous fields. In turn, these fields have stimulated the development of many new graph-theoretical concepts and led to many challenging graph theory problems. We can anticipate that the continued interplay between graph theory and many areas of application will lead to important new developments.

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