ESTIMATION POWER OF PRINCIPAL COMPONENT, MAXIMUM LIKELIHOOD AND GENERALIZED LEAST SQUARES OF STOCK RETURNS

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ABSTRACT

This paper seeks to study comparison of three estimation methods applied in Factor Analysis which includes Principal Component Analysis, Maximum Likelihood and the Generalized or Weighted Least Squares estimation methods using data collected from stock exchange. Our result shows that three estimation methods gave the same number of underlying factors i.e. each method extracted six (6) factors. However, the Maximum likelihood estimate appears to be the best method judging by their percentage of variance explained by the six extracted factors and the residual matrices.

INTRODUCTION

Factor analysis is a statistical data reduction that is used to explain variability among observed random variables in terms of fewer unobserved random variables called factors. The multiple factor analysis model has the form

$$X = A\eta + \varsigma \qquad (1)$$

where X is px1 vector with population covariance matrix Σ, η is the factor vector with unknown dimensions. ς is the error vector with elements $e_1, e_2, ..., e_p$, A is the vector loading matrix , moreover

$$\eta\varsigma = 0 \qquad (2)$$

By computing the covariance of X in (1), we obtain

$$\Sigma = ALA^1 + \mu^2 \qquad (3)$$

where $L = \eta\eta^1$ and $\mu^2 = CC^1$ are diagonal matrix. X is the observed vector and factor analysis assumes that X is the observable realization of unobservable or latent factors indicated by η.

The existence of internal attributes, or factors, is the central assertion in factor analytic theory (Tucker & MacCallum, 1997). While these factors are not directly observable, much of the variation in the phenomena that researchers witness and measure is attributable to these underlying traits (Bartholomew, 1984; Cureton & D'Agostino, 1983; Stevens, 2002). Moreover, factor analytic theory asserts that these hypothetical, internal attributes are more "fundamental" than the surface attributes which we observe (Tucker & MacCallum, 1997; Coughlin, 2013).

Factor analysis involves three steps. The first is to choose a factor extraction method. Such methods includes principal component analysis, unweighted least squares, generalized least squares, maximum likelihood, etc. Information on relative strengths and weakness of these

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techniques is scarce. To examine the relative strengths of these competing perspectives concerning formative measurement models and their associated constructs, this research was carried. The rest of the paper is organized as follows. Section 2 presents related literature. Section 3 contains the data. The methodology and empirical results are discussed in section 4. Result is provided in Section 5 while section 6 contains the conclusion.

REVIEW OF LITERATURE

Diana (2010) identified the effect of using the Maximum Likelihood (ML) and the Diagonally Weighted Least Squares (DWLS) procedures applied to stimulated sets of data, which have different distributions and include variables that can take different numbers of possible values. The five data- sets used in the study were artificially generated and have a sample size of 500 cases, which meets the requirement of 5-20 cases per parameter estimate. One data -set represents the ideal situation of having a perfectly normal distribution and continuous variables. The other four data sets consist of ordinal variable. Results were also compared to the ideal situation of a data set consisting of continuous, normally distributed variables. The outcomes of the study indicated that Maximum Likelihood (ML) provides accurate results when data are continuous and uniformly distributed, but is not as precise with ordinal data that is not treated as continuous, especially when variables have a smaller number of categories and data do not meet the assumption of multivariate normality. In contrast, the Diagonally Weighted Least Squares (DWLS) provides more accurate parameter estimates, and a model fit that is more robust to variable type and non-normality. Although, the results presented support the effectiveness of the Diagonally Weighted Least Squares estimation method with ordinal multivariate non-normal data, they are based on testing only one model.

Olisah (2006) compared principal components analysis and maximum likelihood methods using weekly stock prices of twelve (12) selected stocks four (4) sectors of the Nigerian Stock Exchange from year 2012- 2014 comprising of 150 weeks. Results show that for the two solution methods, three factors were estimated. The cumulative proportion of total sample variance explained by the factors is larger for principal component factoring than for maximum likelihood factoring. The elements of the residual matrix of the maximum likelihood method were smaller than those of the residual matrix corresponding to the principal component method.

Data

The data for this research work is a secondary data collected from recorded daily official list of the Nigerian Stock Exchange in Port Harcourt, Rivers state. These data were daily opening and closing prices of quoted companies or stocks in the Stock Exchange. Thus, the monthly rate of stock returns were calculated which is given below as:

$$Monthly\ rate\ of\ stock\ returns = \frac{current\ month\ closing\ price - previous month\ closing\ price}{previous month\ closing\ price}$$

Estimation method

This research work was analyzed with the help of statistical packages; IBM SPSS (version 20) and Micro-Excel (version 2007). According to Morrison (1988), factor analysis aims to describe the covariance relationship among many variables in terms of a few underlying, but unobservable random quantities called factors. A successful factor analysis is usually one in which different factors influence or load onto disjoint subsets of variables. The three estimation methods employed for this research work is compared with the following: The number of underlying factors, the cumulative proportion of total (standardized) sample variance explained by the factors and the residual matrix.

Principal Component Method (PC)

According to Johnson and Wichern (1992) and Morrison (1988), factor analysis try to describe the covariance relationship among many variables in terms of a few underlying common factors. Because the sample covariance matrix S is an unbiased estimator of the population covariance matrix $\sum$, we usually do factor analysis on the sample covariance matrix S or the correlation matrix R. Johnson and Wichern (1992) says the principal component analysis of the sample covariance matrix S is specified in terms of its eigenvalue-eigenvector pairs ($\lambda_1 e_1$), ($\lambda_2 e_2$),…, ($\lambda_p e_p$)

where $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_p \geq o$ for i = 1, 2, …, p . Let m<p be the number of common factors, the matrix of the estimated factor loadings { l_{ij} } is given by

$$L = \left[\sqrt{\lambda_1}\, \ell_1, \sqrt{\lambda_2}\, \ell_2, \ldots \sqrt{\lambda_m}\, \ell_m \right]$$

The estimated specific variances are provided by the diagonal elements of the matrix S – LL´ , so,

$$\Psi = \begin{pmatrix} \Psi_1 & o\ldots & o \\ o & \Psi_2 \ldots & o \\ \vdots & \vdots & \vdots \\ o & o\ldots & \Psi_p \end{pmatrix}$$

with $\Psi_i = S_{ii} - \sum_{j=i}^{n} l^2 ij$ for i = 1, 2, …, p

Communalities are estimated as $h^2_i = l^2_{i1} + l^2_{i2} + \ldots + l^2_{im}$

The principal component factor analysis of the sample correlation matrix is obtained by starting with sample correlation matrix R in place of sample covariance matrix S. Ideally; the contributions of the first few factors to the sample variances of the variable should be large. The contribution to the sample variance S_{ii} from the first common factor is l^2_{ii}. The contribution to the total sample variance,

$S_{11} + S_{22} + \ldots + S_{pp}$ = tr (s) from the first common factor is then

$$l^2_{11} + l^2_{21} + \ldots + l^2_{p1} = \left\{ (\sqrt{\lambda_1}\, \ell_1)' (\sqrt{\lambda_1}\, \ell_1) \right\} = \lambda_1.$$

Since the eigenvector ℓ_1 has unit length.

In general,

$$\begin{pmatrix} \text{for a factor analysis of S} \\ \text{Proportion of total} \\ \text{sample variance} \\ \text{due to } j_{th} \text{ factor.} \end{pmatrix} = \begin{cases} \dfrac{\lambda j}{S_{11} + S_{22} + \ldots + S_{pp}} \\ \dfrac{\lambda j}{P} & \text{for a factor analysis of R} \end{cases}$$

where

$\lambda_j$is the eigenvalue for the jth factor

P is the number of variables

Maximum Likelihood Method (MLE)

A maximum likelihood estimator is a value of the parameter such that the likelihood function is maximum. According to Johnson and Wichern (1992), if the common factors F and the specific factors ε can be assumed that to be normally distributed, then the maximum likelihood estimates of the factor loadings and specific variances may be obtained. When F_j and ε_j are jointly normal, the observations

$X_j - \mu = L F_j + \varepsilon_j$ are then normal, and the likelihood is

$$L(\mu, \Sigma) = (2\pi)^{-np/2} |\Sigma|^{-n/2} e^{-(1/2)\mathrm{tr}\left\{\Sigma^{-1}\left[\sum_{j=i}^{n}(x_j - \bar{x})(x_j - \bar{x})' + n(\bar{x} - \mu)(\bar{x} - \mu)'\right]\right\}}$$

$$= (2\pi)^{-(n-1)p/2}|\Sigma|^{-(n-1)/2} e^{-(1/2)\,\mathrm{tr}\left(\Sigma^{-1}\sum_{j=i}^{n}(x_j - \bar{x})(x_j - \bar{x})'\right)} (2\pi)^{-p/2}|\Sigma|^{-1/2} e^{-(n/2)(\bar{x}-\mu)^1\Sigma^{-1}(\bar{x}-\mu)'}$$

Which depends on L and Ψ through $\Sigma = LL' + \Psi$

It is desirable to make L well defined by imposing the computationally convenient uniqueness condition.

$L'\Psi^{-1}L = \Delta$ which is a diagonal matrix.

The maximum likelihood estimates of $\hat{L}$ and $\hat{\Psi}$ must be obtained by numerical maximization of L (μ, Σ).

The maximum likelihood estimates of the communalities are

$\hat{h}^2_i = \hat{l}^2_{i1} + \hat{l}^2_{i2} + \ldots + \hat{l}^2_{im}$ for $i = 1, 2, \ldots, p$

so

$$\left(\begin{array}{l}\text{Proportion of total sample}\\ \text{variance due to jth factor}\end{array}\right) = \frac{\hat{l}^2_{1j} + \hat{l}^2_{2j} + \ldots + \hat{l}^2_{pj}}{S_{11} + S_{22} + \ldots + S_{pp}}$$

whenever the maximum likelihood analysis pertains the correlation matrix, we call

$\hat{h}^2 i = \hat{l}^2_{i1} + \hat{l}^2_{i2} + \ldots + \hat{l}^2_{im}$ for $i = 1, 2, \ldots, p$, the maximum likelihood estimates of the communalities and evaluate the importance of factors on the basis of

$$\left(\begin{array}{l}\text{Proportion of total sample}\\ \text{variance due to jth factor}\end{array}\right) = \frac{\hat{l}^2_{1j} + \hat{l}_{2j} + \ldots + \hat{l}^2_{pj}}{p}$$

Weighted or Generalized Least Square (GLS)

Bartlett (1937) has suggested that weighted least squares be used to estimate the common factor values using the factor model.

$$\underset{(px1)}{X} - \underset{(px1)}{\mu} = \underset{(pxm)}{L}\ \underset{(mx1)}{F} + \underset{(px1)}{\varepsilon}$$

Where μ, L and Ψ are known and $\varepsilon' = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_p)$ are regarded as errors and $\mathrm{var}(\varepsilon_i) = \Psi_i$, $i = 1, 2, \ldots, p$ need not be equal.

Given that the sum of squared error weighted by the reciprocal of their variances is

$$\sum_{j=1}^{p} \frac{\varepsilon_i^2}{\Psi} = \varepsilon' \Psi_i^{-1} \varepsilon$$
$$= (x - \mu - LF)' \Psi^{-1} (x - \mu - LF)$$

Bartlett proposed choosing the estimates $\hat{f}$ of f to minimize

$$\hat{f} = (L' \Psi^{-1} L)^{-1} L' \Psi^{-1} (x - \mu)$$

From above f, we can take estimates $\hat{L}$, $\hat{\Psi}$ and $\hat{\mu} = \bar{x}$ as the true values and obtain the factor scores for the jth case as

$$\hat{f}_j = (\hat{L}' \hat{\Psi}^{-1} \hat{L})^{-1} \hat{L}' \hat{\Psi}^{-1} (x_j - \bar{x})$$

Since $\hat{L}$ and $\hat{\Psi}$ are maximum likelihood estimates, they must satisfy the uniqueness condition

$\hat{L}' \hat{\Psi}^{-1} \hat{L} = \hat{\Delta}$, a diagonal matrix.

The factor scores obtained by weighted least squares from the maximum likelihood estimates are

$$\hat{f}_j = (\hat{L}' \hat{\Psi} \hat{L})^{-1} \hat{L}' \hat{\Psi}^{-1} (x_j - \hat{\mu}) = \hat{\Delta}^{-1} \hat{L}^{1'} \hat{\Psi}^{-1} (x_j - \bar{x}) \qquad j = 1, 2, \ldots, n$$

If the correlation matrix is factored

$$\hat{f}_j = (\hat{L}'_z \hat{\Psi}_z^{-1} \hat{L}_z)^{-1} \hat{L}'_z \hat{\Psi}_z^{-1} z_j = \hat{\Delta}_z^{-1} \hat{L}'_z \hat{\Psi}_z^{-1} z_j \qquad j = 1, 2, \ldots, n$$

where

$z_j = D^{-1/2} (x_j - \bar{x})$ and

$$\hat{\rho} = \hat{L}_z \hat{L}'_z + \hat{\Psi}_z$$

For standardized data since $L = \left[\sqrt{\lambda_1}\, \ell_1 \mid \sqrt{\lambda_2}\, \ell_2 \mid \ldots \mid \sqrt{\lambda_m}\, \ell_m \right]$

We have

$$F_j = \begin{bmatrix} \frac{1}{\sqrt{\lambda_1}} \hat{\ell}'_1 (x_j - \bar{x}) \\ \frac{1}{\sqrt{\lambda_2}} \hat{\ell}'_2 (x_j - \bar{x}) \\ \vdots \qquad \vdots \\ \frac{1}{\sqrt{\lambda_m}} \hat{\ell}'_m (x_j - \bar{x}) \end{bmatrix}$$

For these factor scores

$$\frac{1}{n} \sum_{j=1}^{n} \hat{f}_j = o \qquad \text{(Sample mean)}$$

and

$$\frac{1}{n-1} \sum_{j=1}^{n} \hat{f}_j \hat{f}_j = 1 \qquad \text{(Sample covariance)}$$

RESULT
The underlying factors

Factor loadings matrix for the Principal Component Method

$L_P =$

	F_1	F_2	F_3	F_4	F_5	F_6
X_1	0.686	0.397	0.456	-0.090	-0.075	-0.067
X_2	0.302	0.503	-0.632	0.082	-0.008	-0.003
X_3	-0.732	0.398	0.320	-0.036	0.137	0.034
X_4	-0.079	0.093	0.142	0.077	0.826	0.040
X_5	0.713	0.080	0.592	-0.035	-0.092	-0.020
X_6	0.191	0.621	-0.430	0.070	-0.092	0.038
X_7	0.026	0.011	-0.005	0.124	0.069	0.849
X_8	0.049	0.528	-0.341	-0.164	0.116	-0.101
X_9	-0.129	0.014	-0.007	-0.735	-0.158	-0.017
X_{10}	0.437	0.108	0.052	-0.068	0.109	0.173
X_{11}	-0.093	-0.349	0.062	0.417	-0.384	0.019
X_{12}	0.002	0.014	0.025	0.369	0.176	-0.483
X_{13}	-0.543	0.298	0.456	0.001	-0.182	0.027
X_{14}	-0.520	0.350	0.309	0.064	-0.166	0.015
X_{15}	0.745	0.369	0.349	-0.125	0.009	-0.034
X_{16}	-0.568	0.365	0.465	0.095	0.063	-0.002
X_{17}	0.603	0.221	0.168	0.405	-0.070	0.072
X_{18}	-0.379	0.690	-0.244	0.151	-0.203	0.018

Factor loadings matrix for the Maximum Likelihood Method

$L_M =$

	F_1	F_2	F_3	F_4	F_5	F_6
X_1	0.458	-0.082	-0.108	0.754	0.157	0.139
X_2	0.239	-0.048	0.356	0.094	-0.540	0.373
X_3	-0.267	0.248	0.502	-0.159	0.680	0.152
X_4	0.145	0.989	0.000	0.000	-0.002	0.000
X_5	0.484	-0.075	-0.319	0.684	0.169	-0.219
X_6	0.156	-0.019	0.441	0.202	-0.362	0.268
X_7	0.042	-0.011	0.003	-0.021	0.002	0.013
X_8	0.021	-0.007	0.274	0.118	-0.135	0.483
X_9	-0.229	0.020	0.001	0.102	0.022	0.020
X_{10}	0.220	-0.023	-0.085	0.237	-0.102	0.004
X_{11}	0.005	-0.013	-0.039	-0.139	-0.049	-0.405
X_{12}	0.017	0.006	0.010	0.001	0.000	-0.026
X_{13}	-0.148	0.000	0.305	-0.072	0.617	-0.001
X_{14}	-0.134	0.044	0.336	-0.080	0.436	-0.018
X_{15}	0.444	-0.037	-0.138	0.750	0.020	0.178
X_{16}	-0.121	0.193	0.421	-0.022	0.537	-0.145
X_{17}	0.989	-0.143	0.007	-0.007	0.001	0.000
X_{18}	-0.109	-0.018	0.970	0.103	-0.083	-0.050

Factor loadings matrix for the Generalized Least square Factor Analysis

$L_G =$

	F_1	F_2	F_3	F_4	F_5	F_6
X_1	-0.364	-0.152	0.253	0.746	0.216	0.171
X_2	0.081	-0.088	0.437	0.076	-0.485	0.419
X_3	0.444	0.283	0.163	-0.271	0.715	0.145
X_4	-0.215	0.954	0.204	0.001	0.002	0.000
X_5	-0.514	-0.147	0.123	0.717	0.194	-0.220
X_6	0.196	-0.048	0.451	0.167	-0.305	0.354
X_7	-0.030	-0.017	0.026	-0.022	0.001	0.011
X_8	0.162	-0.013	0.213	0.076	-0.086	0.508
X_9	0.172	0.055	-0.142	0.099	0.032	0.027
X_{10}	-0.195	-0.056	0.103	0.252	-0.087	0.016
X_{11}	-0.220	-0.013	-0.019	-0.113	-0.075	-0.404
X_{12}	-0.006	0.003	0.020	0.002	0.000	-0.021
X_{13}	0.269	0.021	0.083	-0.149	0.629	-0.014
X_{14}	0.282	0.062	0.129	-0.148	0.454	-0.012
X_{15}	-0.372	-0.105	0.234	0.745	0.075	0.198
X_{16}	0.316	0.205	0.223	-0.102	0.559	-0.145
X_{17}	-0.714	-0.295	0.634	-0.009	0.001	0.000
X_{18}	0.740	-0.011	0.671	0.004	-0.003	-0.002

The cumulative total (standardized) variance explained

Table 1.0: Six-factor solution for Principal Component Method

VARIABLE	ESTIMATED FACTOR LOADINGS						COMMUNALITIES h^2	SPECIFIC VARIANCES $\Psi=1-h^2$
	F_1	F_2	F_3	F_4	F_5	F_6		
X_1	0.686	0.397	0.456	-0.090	-0.075	-0.067	0.854355	0.145645
X	0.302	0.503	-0.632	0.082	-0.008	-0.003	0.750434	0.249566
X_3	-0.732	0.398	0.320	-0.036	0.137	0.034	0.817849	0.182151
X_4	-0.079	0.093	0.142	0.077	0.826	0.040	0.724859	0.275141
X_5	0.713	0.080	0.592	-0.035	-0.092	-0.020	0.875322	0.124678
X_6	0.191	0.621	-0.430	0.070	-0.092	0.038	0.62183	0.37817
X_7	0.026	0.011	-0.005	0.124	0.069	0.849	0.74176	0.25824
X_8	0.049	0.528	-0.341	-0.164	0.116	-0.101	0.448019	0.551981
X_9	-0.129	0.014	-0.007	-0.735	-0.158	-0.017	0.582364	0.417636
X_{10}	0.437	0.108	0.052	-0.068	0.109	0.173	0.251771	0.748229
X_{11}	-0.093	-0.349	0.062	0.417	-0.384	0.019	0.456	0.544
X_{12}	0.002	0.014	0.025	0.369	0.176	-0.483	0.401252	0.598749
X_{13}	-0.543	0.298	0.456	0.001	-0.182	0.027	0.625443	0.374557
X_{14}	-0.520	0.350	0.309	0.064	-0.166	0.015	0.520258	0.479742
X_{15}	0.745	0.369	0.349	-0.125	0.009	-0.034	0.829849	0.170151
X_{16}	-0.568	0.365	0.465	0.095	0.063	-0.002	0.685072	0.314928
X_{17}	0.603	0.221	0.168	0.405	-0.070	0.072	0.614783	0.385217
X_{18}	-0.379	0.690	-0.244	0.151	-0.203	0.018	0.743611	0.256389
Cumulative proportion of total (standardized) sample variance explained	0.21212	0.34479	0.46246	0.52585	0.58518	0.64138		

Table 2.0 : Six-factor solution for Maximum Likelihood Method

VARIABLE	ESTIMATED FACTOR LOADINGS F_1	F_2	F_3	F_4	F_5	F_6	COMMUNALITIES h^2	SPECIFIC VARIANCES $\Psi=1-h^2$
X_1	0.458	-0.082	-0.108	0.754	0.157	0.139	0.840638	0.159362
X_2	0.239	-0.048	0.356	0.094	-0.540	0.373	0.625726	0.374274
X_3	-0.267	0.248	0.502	-0.159	0.680	0.152	0.895582	0.104418
X_4	0.145	0.989	0.000	0.000	-0.002	0.000	0.99915	0.00085
X_5	0.484	-0.075	-0.319	0.684	0.169	-0.219	0.88602	0.11398
X_6	0.156	-0.019	0.441	0.202	-0.362	0.268	0.46285	0.53715
X_7	0.042	-0.011	0.003	-0.021	0.002	0.013	0.002508	0.997492
X_8	0.021	-0.007	0.274	0.118	-0.135	0.483	0.341004	0.658996
X_9	-0.229	0.020	0.001	0.102	0.022	0.020	0.06413	0.93587
X_{10}	0.220	-0.023	-0.085	0.237	-0.102	0.004	0.122743	0.877257
X_{11}	0.005	-0.013	-0.039	-0.139	-0.049	-0.405	0.187462	0.812538
X_{12}	0.017	0.006	0.010	0.001	0.000	-0.026	0.001102	0.998898
X_{13}	-0.148	0.000	0.305	-0.072	0.617	-0.001	0.500803	0.499197
X_{14}	-0.134	0.044	0.336	-0.080	0.436	-0.018	0.329608	0.670392
X_{15}	0.444	-0.037	-0.138	0.750	0.020	0.178	0.812133	0.187867
X_{16}	-0.121	0.193	0.421	-0.022	0.537	-0.145	0.539009	0.460991
X_{17}	0.989	-0.143	0.007	-0.007	0.001	0.000	0.998669	0.001331
X_{18}	-0.109	-0.018	0.970	0.103	-0.083	-0.050	0.973103	0.026897
Cumulative proportion of total (standardized) sample variance explained	0.10904	0.17111	0.28854	0.38837	0.49038	0.53235		

Table 3.0 : Six-factor solution for Generalized Least Squares Method

VARIABLE	ESTIMATED FACTOR LOADINGS F_1	F_2	F_3	F_4	F_5	F_6	COMMUNALITIES h^2	SPECIFIC VARIANCES $\Psi=1-h^2$
X_1	-0.364	-0.152	0.253	0.746	0.216	0.171	0.852022	0.147978
X_2	0.081	-0.088	0.437	0.076	-0.485	0.419	0.621836	0.378164
X_3	0.444	0.283	0.163	-0.271	0.715	0.145	0.909485	0.090515
X_4	-0.215	0.954	0.204	0.001	-0.002	0.000	0.997962	0.002038
X_5	-0.514	-0.147	0.123	0.717	0.194	-0.220	0.901059	0.098941
X_6	0.196	-0.048	0.451	0.167	-0.305	0.354	0.490351	0.509649
X_7	-0.030	-0.017	0.026	-0.022	0.001	0.011	0.002471	0.997529
X_8	0.162	-0.013	0.213	0.076	-0.086	0.508	0.343018	0.656982
X_9	0.172	0.055	-0.142	0.099	0.032	0.027	0.064327	0.935673
X_{10}	-0.195	-0.056	0.103	0.252	-0.087	0.016	0.123099	0.876901
X_{11}	-0.020	-0.013	-0.019	-0.113	-0.075	-0.404	0.18254	0.81746
X_{12}	-0.006	0.003	0.020	0.002	0.000	-0.021	0.00089	0.99911
X_{13}	0.269	0.021	0.083	-0.149	0.629	-0.014	0.497729	0.502271
X_{14}	0.282	0.062	0.129	-0.148	0.454	-0.012	0.328173	0.671827
X_{15}	-0.372	-0.105	0.234	0.745	0.075	0.198	0.804019	0.195981
X_{16}	0.316	0.205	0.223	-0.102	0.559	-0.145	0.53552	0.46448
X_{17}	-0.714	-0.295	0.634	-0.009	0.001	0.000	0.998859	0.001141
X_{18}	0.740	-0.011	0.671	0.004	-0.003	-0.002	0.997991	0.002009
Cumulative proportion of total (standardized) sample variance explained	0.12376	0.19022	0.27908	0.38351	0.48713	0.53619		

Table 4.0 Percentage of Variance Explained

Factors	1	2	3	4	5	6
PC	21.208	13.262	11.767	6.332	5.928	5.622
MLE	10.908	6.208	11.742	9.985	10.204	4.197
GLS	12.374	6.651	8.892	10.444	10.364	4.904

CONCLUSION

We compared three estimation methods in Factor Analysis on stock returns; these stocks were eighteen quoted companies/industries selected from the present eighteen sectors of the Nigerian Stock Exchange (NSE). The monthly rates of return for these stocks were calculated for a period of thirteen (13) years ranging from January 1, 2000 to December 31, 2012. Three estimation methods employed in Factor Analysis namely; Principal Component, Maximum-Likelihood and Generalized Least Squares were applied to the standardized or normalized data and six underlying or hidden factors were recovered in each method.

The cumulative proportion of the total sample variance explained by one, two, three, four, five and six factor solutions and the percentage of variance explained for the three estimation methods as shown in tables 1.0, 2.0, and 3.0 and 4.0. Clearly, the percentage of variance explained is smallest for the maximum-likelihood method compared to the Principal Component and Generalized Least Squares methods.

Comparing the values of the residual matrices for the three estimation methods, the residual matrix that contains smallest positive value is considered to be the best method of estimation. Clearly, the Maximum-Likelihood method contains smallest positive value than those of the residual matrices corresponding to the Principal Component and Generalized Least Squares methods. On this basis, we prefer the Maximum-Likelihood approach of the Factor Analysis which implies that the Maximum-Likelihood approach is more efficient, consistent and reliable.

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APPENDIX: Residual matrix for principal component method

	X_{1}	X_{2}	X_{3}	X_{4}	X_{5}	X_{6}	X_{7}	X_{8}	X_{9}	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}
X_{1}	0	-0.00609	0.020539	0.009081	-0.02722	0.007463	0.051295	0.012439	-0.01501	-0.11072	0.040082	0.019719	-0.03249	-0.03082	-0.01356	-0.01516	-0.08298	0.000899
X_{2}	-0.00609	0	0.00226	0.055237	0.028652	-0.12517	-0.01661	-0.06382	0.065447	-0.03647	0.059608	-0.03015	0.062827	0.002747	-0.00881	0.036529	0.023353	-0.04577
X_{3}	0.020539	0.00226	0	-0.04603	-0.02434	-0.02791	-0.0216	0.005482	-0.031	0.040997	0.01296	-0.01551	-0.00995	-0.07428	0.005221	-0.08699	0.0274	-0.01033
X_{4}	0.009081	0.055237	-0.04603	0	0.04031	0.092478	-0.10276	-0.07996	0.164284	-0.06462	0.288621	-0.15116	-0.00919	0.03808	0.006531	-0.03112	0.027983	0.062868
X_{5}	-0.02722	0.028652	-0.02434	0.04031	0	0.000443	0.01321	-0.00039	0.0214	-0.0349	0.034172	0.006101	-0.0258	0.0031	-0.03354	0.006585	-0.0559	0.053444
X_{6}	0.007463	-0.12517	-0.02791	0.092478	0.000443	0	-0.03654	-0.23289	0.062495	0.019039	0.024912	0.01739	-0.0061	0.038518	-0.0195	-0.02001	-0.0597	-0.04195
X_{7}	0.051295	-0.01661	-0.0216	-0.10276	0.01321	-0.03654	0	0.084294	0.10464	-0.13326	-0.04778	0.346086	-0.00337	-0.014	0.036061	-0.00635	-0.08079	-0.02296
X_{8}	0.012439	-0.06382	0.005482	-0.07996	-0.00039	-0.23289	0.084294	0	-0.10139	-0.08403	0.119822	-0.01665	0.036762	-0.00568	0.022694	-0.01025	0.014865	-0.11582
X_{9}	-0.01501	0.065447	-0.031	0.164284	0.0214	0.062495	0.10464	-0.10139	0	0.000408	0.228469	0.237049	-0.04859	-0.00175	-0.03865	0.051618	0.133708	0.049958
X_{10}	-0.11072	-0.03647	0.040997	-0.06462	-0.0349	0.019039	-0.13326	-0.08403	0.000408	0	0.047034	0.072781	0.03763	0.031223	-0.10116	0.025555	-0.0544	0.059072
X_{11}	0.040082	0.059608	0.01296	0.288621	0.034172	0.024912	-0.04778	0.119822	0.228469	0.047034	0	-0.07659	-0.04959	-0.03809	0.053655	0.026346	-0.06734	0.05243
X_{12}	0.019719	-0.03015	-0.01551	-0.15116	0.006101	0.01739	0.346086	-0.01665	0.237049	0.072781	-0.07659	0	0.021218	-0.00374	0.001738	-0.01771	-0.09485	-0.0051
X_{13}	-0.03249	0.062827	-0.00995	-0.00919	-0.0258	-0.0061	-0.00337	0.036762	-0.04859	0.03763	-0.04959	0.021218	0	-0.15825	-0.01589	-0.12381	0.026874	-0.08574
X_{14}	-0.03082	0.002747	-0.07428	0.03808	0.0031	0.038518	-0.014	-0.00568	-0.00175	0.031223	-0.03809	-0.00374	-0.15825	0	-0.01059	-0.11539	0.009678	-0.11382
X_{15}	-0.01356	-0.00881	0.005221	0.006531	-0.03354	-0.0195	0.036061	0.022694	-0.03865	-0.10116	0.053655	0.001738	-0.01589	-0.01059	0	-0.01657	-0.09671	0.019215
X_{16}	-0.01516	0.036529	-0.08699	-0.03112	0.006585	-0.02001	-0.00635	-0.01025	0.051618	0.025555	0.026346	-0.01771	-0.12381	-0.11539	-0.01657	0	0.006798	0.023818
X_{17}	-0.08298	0.023353	0.0274	0.027983	-0.0559	-0.0597	-0.08079	0.014865	0.133708	-0.0544	-0.06734	-0.09485	0.026874	0.009678	-0.09671	0.006798	0	-0.05862
X_{18}	0.000899	-0.04577	-0.01033	0.062868	0.053444	-0.04195	-0.02296	-0.11582	0.049958	0.059072	0.05243	-0.0051	-0.08574	-0.11382	0.019215	0.023818	-0.05862	0

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Residual matrix for maximum likelihood method

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}
X_1	0	-0.00689	0.001836	0.000002	0.000898	0.028896	-0.0081	-0.00251	0.001488	-0.03307	0.017226	0.020646	-0.00872	-0.00636	0.001328	0.008146	0.000189	-0.00047
X_2	-0.00689	0	-0.00254	-0.00026	0.010939	0.013376	-0.00643	0.05095	0.029167	0.007726	0.003736	-0.00573	0.029113	-0.0028	-0.01386	-0.00356	-0.00053	-0.00099
X_3	0.001836	-0.00254	0	0.000803	-0.00191	-0.00948	0.001761	-0.00406	0.002613	0.017549	-0.00808	-0.00686	0.004518	-0.00183	-0.00141	-0.01013	0.00022	-0.00016
X_4	0.000002	-0.00026	0.000803	0	0.000333	0.000447	0.000793	-0.00039	0.000469	0.000643	0.000034	-0.0004	-0.00031	0.000786	0.000253	0.000742	-0.00098	0.000441
X_5	0.000898	0.010939	-0.00191	0.000333	0	-0.01755	-0.00132	-0.0034	0.010549	0.025686	-0.00417	-0.00697	0.010683	0.008434	-0.00109	-0.00312	-0.00055	0.000461
X_6	0.028896	0.013376	-0.00948	0.000447	-0.01755	0	0.005398	-0.10139	0.029663	0.055858	-0.03595	0.006818	0.023749	0.09638	-0.01007	-0.02642	-0.00031	0.00044
X_7	-0.0081	-0.00643	0.001761	0.000793	-0.00132	0.005398	0	-0.01031	-0.00333	0.020891	-0.01079	-0.00632	-0.00343	-0.01321	0.008755	0.001291	-0.00028	0.000449
X_8	-0.00251	0.05095	-0.00406	-0.00039	-0.0034	-0.10139	-0.01031	0	-0.00805	-0.04216	0.010892	0.000385	-0.00019	0.005052	0.016455	0.021664	-0.00073	0.000174
X_9	0.001488	0.029167	0.002613	0.000469	0.010549	0.029663	-0.00333	-0.00805	0	0.003915	0.0138	-0.04982	0.009593	0.014026	-0.01895	0.00834	0.000026	-0.00225
X_{10}	-0.03307	0.007726	0.017549	0.000643	0.025686	0.055858	0.020891	-0.04216	0.003915	0	-0.07015	-0.01589	-0.02051	-0.03944	-0.00068	-0.03159	0.000487	-0.00066
X_{11}	0.017226	0.003736	-0.00808	0.000034	-0.00417	-0.03595	-0.01079	0.010892	0.0138	-0.07015	0	-0.01301	0.028455	0.0153	-0.00976	-0.01994	-0.00045	0.001141
X_{12}	0.020646	-0.00573	-0.00686	-0.0004	-0.00697	0.006818	-0.00632	0.000385	-0.04982	-0.01589	-0.01301	0	-0.00949	-0.00673	-0.01307	0.037941	-0.000018	-0.00014
X_{13}	-0.00872	0.029113	0.004518	-0.00031	0.010683	0.023749	-0.00343	-0.00019	0.009593	-0.02051	0.028455	-0.00949	0	0.002898	-0.00436	0.014629	0.000116	-0.0014
X_{14}	-0.00636	-0.0028	-0.00183	0.000786	0.008434	0.09638	-0.01321	0.005052	0.014026	-0.03944	0.0153	-0.00673	0.002898	0	-0.00902	0.042336	-0.00053	-0.00321
X_{15}	0.001328	-0.01386	-0.00141	0.000253	-0.00109	-0.01007	0.008755	0.016455	-0.01895	-0.00068	-0.00976	-0.01307	-0.00436	-0.00902	0	-0.00347	0.000789	-0.0001
X_{16}	0.008146	-0.00356	-0.01013	0.000742	-0.00312	-0.02642	0.001291	0.021664	0.00834	-0.03159	-0.01994	0.037941	0.014629	0.042336	-0.00347	0	0.00063	0.000502
X_{17}	0.000189	-0.00053	0.00022	-0.00098	-0.00055	-0.00031	-0.00028	-0.00073	0.000026	0.000487	-0.00045	-0.000018	0.000116	-0.00053	0.000789	0.00063	0	0.000241
X_{18}	-0.00047	-0.00099	-0.00016	0.000441	0.000461	0.00044	0.000449	0.000174	-0.00225	-0.00066	0.001141	-0.00014	-0.0014	-0.00321	-0.0001	0.000502	0.000241	0

Residual matrix for generalized least squares method

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}
X_1	0	-0.01204	-0.00068	-0.00104	-0.00473	0.017709	-0.00777	-0.00889	-0.00049	-0.03549	-0.05167	0.019311	-0.00921	-0.00817	-0.0004	0.008908	0.00036	-0.000069
X_2	-0.01204	0	-0.00068	0.001113	0.014725	-0.00313	-0.00588	0.050315	0.028645	0.007805	0.020468	-0.00734	0.028043	-0.00429	-0.01357	-0.00339	-0.000015	-0.000056
X_3	-0.00068	-0.00068	0	-0.00293	0.000265	-0.00695	0.001621	-0.00554	0.002247	0.017816	0.081038	-0.00686	0.001008	-0.00676	-0.0007	-0.01297	0.000005	-0.0003
X_4	-0.00104	0.001113	-0.00293	0	-0.00047	0.001371	0.000484	-0.00012	0.000315	0.000409	-0.04276	-0.00023	-0.00324	-0.00159	-0.00044	-0.00114	-0.00041	-0.00029
X_5	-0.00473	0.014725	0.000265	-0.00047	0	-0.00747	-0.00112	0.00111	0.009708	0.023583	-0.10896	-0.00716	0.009871	0.009595	-0.00258	-0.00508	-0.000084	0.000484
X_6	0.017709	-0.00313	-0.00695	0.001371	-0.00747	0	0.005423	-0.12219	0.028639	0.053796	0.019077	0.0064	0.029535	0.100959	-0.01829	-0.01481	-0.00034	-0.000064
X_7	-0.00777	-0.00588	0.001621	0.000484	-0.00112	0.005423	0	-0.00973	-0.00336	0.020975	-0.01729	-0.00637	-0.00348	-0.01342	0.009108	0.000959	-0.00012	0.00068
X_8	-0.00889	0.050315	-0.00554	-0.00012	0.00111	-0.12219	-0.00973	0	-0.00939	-0.04284	0.041888	-0.00173	-0.00045	0.001033	0.018303	0.02146	-0.00044	0.000508
X_9	-0.00049	0.028645	0.002247	0.000315	0.009708	0.028639	-0.00336	-0.00939	0	0.00365	0.049352	-0.04992	0.009364	0.013852	-0.01951	0.009164	-0.00008	-0.00064
X_{10}	-0.03549	0.007805	0.017816	0.000409	0.023583	0.053796	0.020975	-0.04284	0.00365	0	-0.10826	-0.01623	-0.02142	-0.03984	-0.00091	-0.03221	0.000303	-0.00067
X_{11}	-0.05167	0.020468	0.081038	-0.04276	-0.10896	0.019077	-0.01729	0.041888	0.049352	-0.10826	0	-0.01216	0.081712	0.075775	-0.08796	0.044241	-0.14281	0.147825
X_{12}	0.019311	-0.00734	-0.00686	-0.00023	-0.00716	0.0064	-0.00637	-0.00173	-0.04992	-0.01623	-0.01216	0	-0.00911	-0.00603	-0.01493	0.03898	-0.000061	0.000003
X_{13}	-0.00921	0.028043	0.001008	-0.00324	0.009871	0.029535	-0.00348	-0.00045	0.009364	-0.02142	0.081712	-0.00911	0	0.004347	-0.00455	0.017343	0.000669	-0.000067
X_{14}	-0.00817	-0.00429	-0.00676	-0.00159	0.009595	0.100959	-0.01342	0.001033	0.013852	-0.03984	0.075775	-0.00603	0.004347	0	-0.01119	0.045789	0.000066	0.000373
X_{15}	-0.0004	-0.01357	-0.0007	-0.00044	-0.00258	-0.01829	0.009108	0.018303	-0.01951	-0.00091	-0.08796	-0.01493	-0.00455	-0.01119	0	-0.00433	0.000691	-0.00025
X_{16}	0.008908	-0.00339	-0.01297	-0.00114	-0.00508	-0.01481	0.000959	0.02146	0.009164	-0.03221	0.044241	0.03898	0.017343	0.045789	-0.00433	0	0.00024	-0.00042
X_{17}	0.00036	-0.000015	0.000005	-0.00041	-0.000084	-0.00034	-0.00012	-0.00044	-0.00008	0.000303	-0.14281	-0.000061	0.000669	0.000066	0.000691	0.00024	0	0.00074
X_{18}	-0.000069	-0.000056	-0.0003	-0.00029	0.000484	-0.000064	0.00068	0.000508	-0.00064	-0.00067	0.147825	0.000003	-0.000067	0.000373	-0.00025	-0.00042	0.00074	0