A ONE-MANUFACTURER-TWO-RETAILER SUPPLY CHAIN IN A PRICE-SENSITIVE AND STOCK-STIMULATING UNCERTAIN DEMAND MARKET WITH DEMAND LEAKAGE

Kuo-Hsien Wang (Corresponding Author)
Department of Business Administration, Takming University of Sciences and Technology
Taipei, **TAIWAN**

Yuan-Chih Huang
Department of Business Administration, Takming University of Sciences and Technology, Taipei
TAIWAN

Che-Tsung Tung
Department of International Trade, Takming University of Sciences and Technology
Taipei, **TAIWAN**

ABSTRACT

We address a decentralized supply chain in which a manufacturer supplies an item to two retailers who compete with each other in an uncertain demand market, considering the effects of price sensitivity, demand stimulation and demand leakage. The objective is to coordinate the chain and reach Pareto improvement through negotiating the two wholesale prices and setting the two buyback prices, from which we find a range for each negotiated wholesale price but only one buyback price for each retailer to achieve our goal. The conflict of interests between the manufacturer and each retailer and the price and inventory competitions between both retailers are confirmed. We also learn that the demand-stimulating effect favors the chain profit, but the demand-leaking effect prompts the high priced retailer to negotiate a cheaper wholesale price and the manufacturer to negotiate an expensive wholesale price with the low priced retailer. Many managerial insights are obtained by the numerical examples.

Keywords: Newsvendor; Demand leakage; Demand stimulation; Price competition; Inventory competition.

INTRODUCTION

Within a decentralized supply chain, if a manufacturer and a retailer are each seeking to optimize their own profits, then a so-called "double-marginalization" phenomenon will be generated (Spengler[22]). This phenomenon will lead the chain to experience poor channel profit performances as a result of a less optimal order quantity in comparison to a coordinated supply chain. Thus, contractual terms enhancing a chain's profit efficiency in a decentralized supply chain setting have become imperative to achieve two purposes, supply chain coordination and Pareto efficiency. A contract is said to coordinate a chain if it maximizes the chain's profit as a whole; a contract is said to be Pareto-efficient if each member's profit is no worse off when the contract is in place than it would be in the event of other default contracts (Bose and Anand, [5]).

A price-only contract is widely considered to be a basic, simple trade-off in the existing literature. In such an agreement, a manufacturer offers no incentive to retailer(s), and the retailer(s) then takes all of the responsibility for excess inventory at the end of the selling period. However, researchers, including Larivieve and Porteus [16], Cachon [7] and Bernstein and Federgruen [4], demonstrated that a price-only contract fails to coordinate a supply chain. Conversely, a return-policy contract mitigating the risk of over-stocking due to market demand uncertainty is a commitment made by a manufacturer to accept his partner's unsold products (Padmanabhan and Png, [18]). Pasternack [19], who was the first to analyze manufacturer-retailer channel coordination through return policies for seasonal items, contended that return policies could be used as an instrument for supply chain coordination. Since then, a number of related articles have been published. Emmons and Gilbert [17] investigated the role of return policies in pricing and inventory decisions for catalogue goods. Meanwhile, Lau et al. [17] studied the problem of demand uncertainty and return policies for a seasonal product. Tsay [24] researched a quantity flexibility contract in a newsvendor supply chain, whereas Yao et al. [29] addressed demand uncertainty and manufacturer return policies for style-good retailing competition. Bose and Anand [5] contributed to a practical finding on return policies with exogenous pricing. Yao et al. [28] analyzed the impact of price-sensitivity factors on a return policy coordinating a supply chain, and Chen [8] discussed return policies with a wholesale-price-discount contract in the context of a newsvendor setting. Recently, Zheng and Negenborn [31] proposed a negotiation model between a supplier and a buyer under demand uncertainty with fixed and elastic demands. Qi et al. [20] analyzed game theory in a one manufacturer and two retailers supply chain with customer market search, allowing customers to go to another retailer if stock out occurs. In this paper, we develop a (w,b) contract in a one-manufacturer-two-retailer decentralized supply chain setting. In our (w,b) contract, the manufacturer negotiates a wholesale price with each retailer that is cheaper than that in a price-only contract, and provides each retailer with a buyback price to accept all unsold products. Unlike the existing works in the literature, we find that although there is a range for each negotiated wholesale price, there is only one buyback price for each retailer to coordinate the chain.

In 1980, revenue management (RM) first emerged in the airline industry and has since been applied to areas such as media, utilities and retail trade-offs (Weatherford and Bodily, [27]). Smith et al. [21] thus found that RM increases revenue by 2-8% without increasing the supply of products. Typically, one of the underlying principles of RM is to divide a single market into multiple submarkets with different retail prices, such as a virtual store vs. a physical store. Accordingly, Zhang et al. [30] modeled demand leakage as a function of price difference across a market and investigated the impact of demand leakage on a firm's inventory and pricing decisions. Their result indicates that more customer segments do not necessarily outperform a single segment, particularly in a highly uncertain demand market. Wang et al. [26] addressed a supply chain in which two retailers sell two homogeneous items in a stochastic demand market, allowing demand leakage from the high-priced item to the low-priced one. They found that high demand uncertainty decreases the chain's profit, whereas it will enhance the chain's profit efficiency if a return-policy contract is in place. Rather than Wang et al. [26], we discuss a supply chain in which two retailers compete with each other, selling a single item in an uncertain demand market and allowing demand leakage from the high-priced product to the low-priced product. Thus, each retailer faces a stochastic demand incorporating the effect of demand leakage, from which we learn that the leaking effect is not beneficial to the chain profit if our (w,b) contract is implemented. This suggests that the manufacturer favors a market without demand leakage. This result conforms to that of Zhang et al. [30], but is opposite to that if our contract in not in place.

Empirically, the stock-level demand stimulation effect has been recognized in both marketing and operations research on inventory management. For some items, such as books, magazines, fashion apparel or 3c products, displaying a large stockpile of inventory on shelf space can actually increase sales. This phenomenon is called the customers' impulse-purchase and was first introduced by Balakrishnan et al. [2] in terms of the effects of increasing product visibility, kindling latent demand, signaling a popular product and providing an assurance of future availability. Prior to this discovery, Dana and Petruzzi [10] also claimed that higher stock levels can increase sales because the consumer utility increases as the item's fill rate increases. A number of articles, such as those by Corstjens and Doyle [9], Bultez and Naert [6] and

Eliashberg and Steinberg [12], studied inventory demand stimulation and developed mathematical models for shelf space allocation. Gupta and Vrat [15], Baker and Urban [1], Goh [14], Urban [25] and Balakrishnan et al. [3] all stressed the use of an optimal inventory policy with stock-level-dependent demand functions. From a retailer's perspective, Stavrulaki [23] managed inventory decisions in the framework of a single-period, stock-level-dependent demand setting that is solved using the heuristic solution approach. In addition, Devangan et al. [11] considered an individually rational buyback contract with inventory level dependent demand, in which they assumed that shelf space inventory is used a lever to stimulate demand, and used the Shapley value from cooperative game theory to ensure fairness and individual rationality in their buyback contract.

This paper assumes both retailers' stochastic demands considering the effects of price elasticity, inventory stimulation and demand leakage as well as random demand. The manufacturer offers a (w,b) contract to coordinate the chain and reach Pareto improvement, during which the following questions will be investigated. (1) How will all chain members respond to the effect of demand stimulation and/or demand leakage if our contract is in place? (2) What leads the manufacturer to set a buyback price for each retailer? (3) How will the manufacturer negotiate a wholesale price with each retailer? (4) What is our contract's profit efficiency under different parameter circumstances?

The remainder of this article is organized as follows. Assumptions and notations are given in Section 2, along with relevant models and corresponding analyses. Numerical examples are conducted in Section 3, accompanied by managerial insights. Contributions and potential research directions for future studies are presented in Section 4. All of the proofs are given in the Appendix.

The models

The problem investigated in the study is as follows. A manufacturer supplies a newsvendor item to two retailers who then compete with each other in a stochastic demand market. Each retailer i, $i=1,2$, and $j=3-i$, faces a deterministic demand $d_i=\alpha_i-\beta_i p_i+\gamma_i q_i+L_i(p_i,p_j)$ and a random demand $\varepsilon_i\in[0,\infty)$, where α_i>0 is the realized demand, β_i>0 is the sensitivity parameter of the retail price p_i, o< γ_i <1 is the sensitivity parameter of demand stimulation for the order quantity q_i, $L_i(p_i,p_j)=$

$-\lambda_i (p_i - p_j)^+ + \lambda_j (p_j - p_i)^+$ is the amount of demand leakage from high-priced product to low-priced product with demand leakage rate λ_i and λ_j, and $(\cdot)^+ = \max\{\cdot, 0\}$. Define $f_i(\cdot)$ and $F_i(\cdot)$ as the probability density function and cumulative distribution function for ε_i, respectively; thus, retailer i's total demand is assumed to be $x_i = d_i + \varepsilon_i$ $= \alpha_i - \beta_i p_i + \gamma_i q_i + L_i(p_i, p_j) + \varepsilon_i$. Meanwhile, let c be the unit production cost, w be the unit wholesale price for the two retailers, and zero salvage is assumed for unsold products.

Decentralized supply chain with a price-only contract

In this scenario, each retailer maximizes his expected profit by determining his retail price and order quantity; the manufacturer then determines the wholesale prices to optimize his expected profit subject to the two retailers' optimal retailer prices and orders. Thus, for $i = 1, 2$, and $j = 3 - i$, let $q_i = d_i + z_i$ be the retailer i's order quantity with z_i as a safety stock level. Thus, according to $d_i = \alpha_i - \beta_i p_i + \gamma_i q_i + L_i(p_i, p_j)$, we obtain $q_i = d_i + z_i = \alpha_i - \beta_i p_i + \gamma_i q_i + L_i(p_i, p_j) + z_i$; thus, q_i can be regarded as a function of p_i and z_i below.

$$q_i = \frac{1}{1-\gamma_i}(\alpha_i - \beta_i p_i + L_i(p_i, p_j) + z_i) \qquad (1)$$

Accordingly, retailer i's profit is obtained by

$$\begin{cases} p_i x_i - w q_i = (p_i - w) q_i - p_i (z_i - \varepsilon_i) & \varepsilon_i \le z_i \\ (p_i - w) q_i & \varepsilon_i > z_i \end{cases}$$

and his expected profit, denoted by $E[\pi_i(p_i, z_i)]$, is given by

$$E[\pi_i(p_i, z_i)] = (p_i - w) q_i - p_i \Lambda_i(z_i) \qquad (2)$$

where $\Lambda_i(z_i)=\int_0^{z_i}(z_i-\varepsilon_i)f_i(\varepsilon_i)d\varepsilon_i$ represents leftover inventory. Thus, retailer i's objective in this scenario is to determine p_i and z_i to maximize $E[\pi_i(p_i,z_i)]$. Once p_i and z_i are obtained, q_i can be determined according to Eq. (1). Therefore, the following propositions are provided to solve the optimization problem.

Proposition 1 For $i=1,2$, and $j=3-i$, retailer i's expected profit $E[\pi_i(p_i,z_i)]$ is concave in p_i and z_i if $\left(\beta_i+\min\{\lambda_i,\lambda_j\}\right)p_i f(z_i)>\frac{1}{1-\gamma_i}$.

Proposition 1 provides a sufficient condition to optimize $E[\pi_i(p_i,z_i)]$, indicating that the optimal safety stock level z_i would never occur at where $f_i(z_i)$ is considerably small and needs a large price p_i to meet the condition. This condition also suggests that the effects of price sensitivity β_i and demand leakage λ_i help facilitate the optimization of $E[\pi_i(p_i,z_i)]$, whereas the effect of demand stimulation γ_i counteracts the result. This could be because a higher level of demand stimulation γ_i, according to Eq. (1), needs a larger order quantity to reach the concave $E[\pi_i(p_i,z_i)]$; however, this larger order quantity increases his risk of overstock.

Further, if retailer i's optimal p_i, z_i and q_i exist, their optimal necessary conditions would be given as follows.

$$\begin{cases} q_i+(p_i-w)\dfrac{-\beta_i+l_1}{1-\gamma_i}-\Lambda_i(z_i)=0 & (3)\\ \dfrac{p_i-w}{1-\gamma_i}-p_iF(z_i)=0 & (4)\\ (1-\gamma_i)q_i=\alpha_i-\beta_i p_i+L_i(p_i,p_j)+z_i & (5) \end{cases}$$

where $l_1 = \dfrac{\partial L_i(p_i, p_j)}{\partial p_i} = \begin{cases} -\lambda_1 & p_1 > p_2 \\ -\lambda_2 & p_1 < p_2 \end{cases}$ is negative, And Eqs. (3) and (4) reveal how p_i associates with z_i below.

Proposition 2 Each retailer's optimal retail price p_i positively correlates with his optimal safety stock level z_i.

This positive correlation between price and inventory suggests that each retailer should stockpile a larger safety inventory if an only if he sets a higher price, the managerial meaning of which is given below. If each retailer intends to set a higher price, he then should play a larger inventory because it can offset sales losses due to the higher pricing as well as entice more sales due to the stock-dependent demand. By contrast, if each retailer intends to stockpile a larger inventory to entice more demand, his retail price should be raised to reduce the amount of the deterministic demand to avoid overstock. More importantly, this positive correlation inspires us to prove the unique optimal solution of Eqs. (3)-(5) as follows.

Proposition 3 For $i = 1,2$, and $j = 3 - i$, retailer i's optimal retail price, safety stock level and order quantity uniquely exist in the range of $c < p_i < \dfrac{\alpha_i}{\beta_i}$.

Note that the range of $c < p_i < \dfrac{\alpha_i}{\beta_i}$ in Proposition 3 assures non-negative deterministic demand $\alpha_i - \beta_i p_i$. Once retailer i's optimal values are set, we examine how these values will interact with the wholesale price. Thus, if regarding p_i, z_i and q_i as $p_i(w)$, $z_i(w)$ and $q_i(w)$, respectively, in Eqs. (3)-(5), the following result is obtained.

Proposition 4 Each retailer's optimal safety stock level and order quantity decrease, but the optimal retail price increases in the wholesale price.

This result clearly explains the conflict of interests between the manufacturer and each retailer in the following patterns. If the manufacturer sets a higher wholesale price, it can not only stop each retailer from placing larger order but also urge them to set higher retail prices to customers; the consequence is then to reduce the amounts of sales and thereby all members' profits. Conversely, although the manufacturer's cheaper wholesale price can attract both retailers' large orders and allow them to set cheaper retail prices to customers; the cheaper-retail-price setting, however, will impair all members' profit margins despite the large orders.

We next investigate how both retailers would compete with each other in this competitive environment.

Proposition 5 Both retailers' optimal values are positively correlated.
This result reveals price competition and inventory competition between both retailers as follows. The price competition implies that if one of the two retailers would slash his price for demand stimulation, the other should follow; consequently, both retailers' marginal profits decrease. The inventory competition implies that if one of the two retailers would order more products for demand stimulation, the other should follow; consequently, both retailers' risks of overstock increase. The two competitions, meanwhile, lead the two retailers to a competing equilibrium below.

Proposition 6 There exists a unique Nash equilibrium between the two retailers in the competing market.
As for the manufacturer's objective in this scenario, he will determine the wholesale price that maximizes his expected profit $E[\pi(w)]=(w-c)(q_1+q_2)$ subject to both retailers' optimal necessary conditions, that is, $\max_{w} E[\pi(w)]$ s.t Eqs. (3)-(5). To this end, the following results are needed.

Proposition 7 The manufacturer's expected profit $E[\pi(w)]$ is concave in w, and the optimal w uniquely exists in $w>c$.
After completing the decentralized supply chain with a price-only contract, a centralized supply chain is developed as follows.

Centralized supply chain

In this scenario, we assume that the manufacturer can sell the item himself. Thus, similar to Eq. (2), if we let $p=(p_1,p_2)$ and $z=(z_1,z_2)$ for convenience, his expected profit is then given by

$$E[\pi^c(p,z)] = \sum_{i=1}^{2}\left((p_i - c)q_i - p_i\Lambda_i(z)\right) \qquad (6)$$

Thus, his objective is to determine p_i and z_i, $i = 1,2$ to maximize $E[\pi^c(p,z)]$. We first note that in this scenario, q_i in Eq. (1) now depends on p_i, z_i and p_j. In this way, the concave $E[\pi^c(p,z)]$ and its optimal values are obtained by the following result.

Proposition 8 $E[\pi^c(p,z)]$ is concave in p and z, and the optimal p_i, z_i and q_i uniquely exist in the following equations. For $i = 1,2$, $j = 3 - i$

$$\begin{cases} q_i + (p_i - c)\dfrac{-\beta_i + l_1}{1-\gamma_i} - \Lambda_i(z_i) + (p_j - c)\dfrac{l_2}{1-\gamma_j} = 0 \\ \dfrac{p_i - c}{1-\gamma_i} - p_i F(z_i) = 0 \\ (1-\gamma_i)q_i = \alpha_i - \beta_i p_i + L_i(p_i, p_j) + z_i \end{cases}$$

where $l_2 = \dfrac{\partial L_i(p_i, p_j)}{\partial p_j} = \begin{cases} \lambda_1 & p_1 > p_2 \\ \lambda_2 & p_1 < p_2 \end{cases} = -l_1$

Decentralized supply chain with our (w,b) contract

Using the former two scenarios as benchmarks, we develop our (w,b) contract below.
Before the selling period:
(1) The manufacturer will accept retailer i's, $i = 1,2$, all unsold products at the end of the selling period at a buyback price b_i if retailer i plays order up to the level of q_i^c as in the centralized supply chain, where q_i^c is the optimal order in the centralized supply chain.

(2) The manufacturer then negotiates a wholesale price w_i with retailer i that is cheaper than that in the price-only contract.
(3) The game's objective is to coordinate the chain and reach Pareto improvement through negotiating the two buyback prices and the two wholesale prices.
After the selling period:

(1) The manufacturer accepts all of the two retailers' unsold products.

Accordingly, $x_i = d_i + \varepsilon_i = \alpha_i - \beta_i p_i + \gamma_i q_i^c + L_i(p_i, p_j) + \varepsilon_i$ is retailer i's demand if his order is up to q_i^c, and his expected profit is then given by

$$E[\pi_i^b(p_i)] = (p_i - w_i)q_i^c - (p_i - b_i)\Lambda_i(z_i) \qquad (7)$$

where we regard retailer i's profit in Eq. (7) as $E[\pi_i^b(p_i)]$ rather than $E[\pi_i^b(p_i, z_i)]$ because $z_i = z_i(p_i)$ satisfies $(1-\gamma_i)q_i^c = \alpha_i - \beta_i p_i + L_i(p_i, p_j) + z_i$, according to Eq. (1). Further, Eq. (7) implies that even retailer i accepts q_i^c, he still could optimize his $E[\pi_i^b(p_i)]$ by setting p_i that may not coordinate the chain. Thus, the following results are derived.

Proposition 9 For $i = 1, 2$, retailer i's expected profit $E[\pi_i^b(p_i)]$ is concave in p_i, and the optimal p_i and z_i uniquely exist in the following equations.

$$q_i^c - \Lambda_i(z_i) - (p_i - b_i)F(z_i)(\beta_i - l_1) = 0 \qquad (8)$$

$$(1-\gamma_i)q_i^c = \alpha_i - \beta_i p_i + L_i(p_i, p_j) + z_i \qquad (9)$$

But, as analyzed, retailer i's order q_i^c will coordinate the chain only when the optimal values in Eqs. (8) and (9) are exactly the same as in the centralized supply chain, that is, $p_i = p_i^c$ and $z_i = z_i^c$, according to which we obtain each retailer's buyback price below.

Proposition 10 In our (w,b) contract, $b_i = p_i^c - \dfrac{q_i^c - \Lambda_i(z_i^c)}{(\beta_i - l_1)F_i(z_i^c)}$, $i = 1, 2$ is retailer i's buyback price to coordinate the chain.

After the buyback prices, the manufacturer will negotiate a wholesale price with each retailer to reach Pareto efficiency; thus, according to Eq. (7), retailer i's expected profit is now a function of the negotiated wholesale price w_i, which is given by

$E[\pi_i^b(w_i)] = (p_i^c - w_i)q_i^c - (p_i^c - b_i)\Lambda_i(z_i^c)$, and the manufacturer's expected profit is $E[\pi_m^b(w)] = \sum_{i=1}^{2}\left((w_i - c)q_i^c - b_i\Lambda_i(z_i^c)\right)$, where $w = (w_1, w_2)$ and b_i is given in Proposition 10. Thus, $E[\pi_i^b(w_i)] > \pi_i^*$ and $E[\pi_m^b(w)] > \pi_m^*$ are needed to attain a win-win game, where π_i^* and π_m^* are retailer i's and the manufacturer's optimal expected profits in the price-only contract, respectively. Accordingly, a range for each negotiated wholesale price is obtained below.

Proposition 11 In our (w,b) contract, a range for each negotiated wholesale price to reach Pareto efficiency is given by the following constraints. For $i = 1, 2$

$$w_i < \frac{1}{q_i^c}\left(p_i^c q_i^c - (p_i^c - b)\Lambda_i(z_i^c) - \pi_i^*\right) \tag{10}$$

$$\sum_{i=1}^{2} q_i^c w_i > \sum_{i=1}^{2}\left(cq_i^c + b_i\Lambda_i(z_i^c)\right) + \pi_m^* \tag{11}$$

$$w_i < w^* \tag{12}$$

Note that $w_i < w^*$ assures the cheaper negotiated w_i than the optimal w^* in the price-only contract.

Numerical examples

The parameters of our examples are as follows: c =5, α_1 =80, α_2 =180, β_1=3, β_2 =8 and $\varepsilon \sim U[0,50]$. Define $I\,\% = \frac{\pi - \pi^*}{\pi^*}$ as a profit increment from the price-only contract optimal profit π^* to our contract profit π. Table 1 includes four cases in which case 1 vs. case 2 and case 3 vs. case 4 are for demand leakage, and case 1 vs. case 3 and case 2 vs. case 4 are for demand stimulation. Table 2 tabulates all of our negotiated values by comparing them with those in the price-only contract, along with our Pareto-improved illustration in Fig 1. Figs 2-5 display each parameter's impact on the chain's profit.

Regarding the demand leakage, Table 1 obviously confirms that price differential $p_1 - p_2$ with leaking effect is less than that without leaking effect, which is because the effect prevents retailer 1 from setting a higher price to retain his sales. This effect also leads to cheaper prices, except for p_2^c. Taking case 1 vs. case 2 as an example, only p_2^c=16.40 in case 1 is higher than p_2^c=15.76 in case 2. This is consistent with Zhang et al. [30] that if the manufacturer can sell the item himself in a market without demand leakage, he will divide the market into two submarkets, one with a high profit margin and the other with a low profit margin, leading to more profit as a whole. Thus, Table 1 shows better π^c=2020.08 and π^c=1348.28 in cases 2 and 4, respectively. Meanwhile, the leaking effect urges both retailers' large inventories in the price-only contract, which benefits retailer 2 and the manufacturer. It, however, is disadvantageous to retailer 1. As shown in Table 1, π_1^*=191.06 as q_1^*=45.28 in case 1 is worse than π_1^*=263.22 as q_1^*=43.07 in case 2.

As for the demand stimulation, Table 1 shows that it benefits all members in all cases. Case 1 vs. case 3, for example, shows that all members' and chain's profits in case 1 overshadow those in case 3. This is conceivable because as shown by Table 1, this stimulating effect not only prompts both retailers' large inventories but also allows cheaper wholesale prices and higher retail prices. Table 1 further identifies our previous finding of the price and the inventory competitions, showing that p_1^* and q_1^* positively correlate with p_2^* and q_2^*, respectively, in all cases.

According to Propositions 10 and 11, the Pareto-improved conditions are as follows: b_1=8.851, b_2=4.788, $w_1 \le 13.685$, $w_2 \le 12.622$ and $76.909w_1 + 151.531w_2 \ge 2523.73$; thus, the right-hand-side triangle in Fig 1 is for negotiating the wholesale prices. Table 2 thus assumes w_1=13.00 and w_2=12.00; clearly, chain coordination and a win-win situation are verified. Additionally, retailer i's, the manufacturer's and the chain's profit increments are 27.57%, 32.91%, 27.36% and 28.41%, respectively. This is in accordance with our previous analysis that retailer 2 profits more from the leaking effect. Thus, in this case, retailer 1 could ask the manufacturer for a cheaper w_1, and the manufacturer could negotiate with retailer 2 a higher w_2.

Figs 2-3 show the impacts of demand-stimulating parameters γ_1 and γ_2 in a market without demand leakage. Fig 2 indicates that γ_1 increases the chain's profit, from p_t =1002.37 or p_c =1348.28 at γ_1=0 to p_t =1176.96 or p_c =1583.26 at γ_1=0.2. Our contract's generated profit increment, however, remains almost unchanged, from I %=34.51% to I %=34.52%. This is because the non-leaking effect keeps the stimulated demand γ_1 in a high profit margin submarket no matter the type of contract. Compared to γ_1, Fig 3 shows that γ_2 contributes to more profit, from p_t =1002.37 or p_c =1348.28 at γ_2=0 to p_t =1204.14 or p_c =1594.30 at γ_2=0.2. A higher γ_2, however, reduces our contract's efficiency from I %=34.51% to I %=32.40%, which is attributed to its generated demand remaining in a low profit margin submarket.

Figs 4-5 show the impacts of demand-leaking parameters λ_1 and λ_2 in a market without demand stimulation. Fig 4 shows an increasing p_t and a decreasing p_c, from 1002.37 and 1348.28 at λ_1=0 to 1095.21 and 1319.08 at λ_1=4, respectively, with a decreasing I % from 34.51 to 20.44. This is understandable because a large λ_1 allows more sales to leak to the low profit margin submarket. Fig 5 illustrates unchanged p_t, p_c and I % in response to a various λ_2, which is obvious because demand leaks from retailer 1 to retailer 2.

CONCLUSION

This study investigated a decentralized supply chain in which a manufacturer supplies an item to two retailers who compete with each other in a stochastic demand market, considering the effects of price sensitivity, demand stimulation and demand leakage. The manufacturer offers contractual terms, including cheaper wholesale prices and buyback prices, to operate the chain as a centralized supply chain.

During the course of our (w,b) contract, the following contributions and managerial insights are concluded.

In a decentralized supply chain with a price-only contract:

(1) there is a conflict of interests between the manufacturer and each retailer;

(2) there are price and inventory competitions between the two retailers, which impair both retailers' profits and increase the risk of overstock;

(3) a double marginalization thus occurs in the aftermath of the conflicts and the two competitions.

In a decentralized supply chain with our (w,b) contract:

(1) each wholesale price is negotiated by the manufacturer and each retailer according to Proposition 11, but the manufacturer determines each buyback price according to Proposition 10;

(2) the manufacturer prefers a market without demand leakage, as it yields more profit as a whole;

(3) if demand leaks from retailer 1 to retailer 2, retailer 1 should ask the manufacturer for a cheaper wholesale price, and the manufacture could negotiate a higher wholesale price with retailer 2;

(4) all chain members benefit from the effect of demand stimulation;

(5) all of the examples conducted confirm our contract's excellent profit efficiency by at least 20.44%.

In future studies, our Stackelberg game could consider the manufacturer as a follower and the two retailers as two leaders in a supply chain, such as a smartphone maker and two telecommunication providers. In such a case, aside from the negotiations between the manufacturer and each retailer, the two retailers (leaders) might have two options to play the game: competing with each other without coordinating the chain or cooperating in the chain as a centralized supply chain; whichever game is used, it will be interesting and challenging for our further explorations.

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Table 1 Optimal values comparisons between the decentralized and centralized supply chains

	w^*	p_1^* / p_1^c	p_2^* / p_2^c	q_1^* / q_1^c	q_2^* / q_2^c	π_1^*	π_2^*	π_m^*	π^* / π^c	$I\%$
$\gamma_1 = 0.2, \gamma_2 = 0.3$ $\lambda_1 = 3, \lambda_2 = 5$	13.94	19.55/18.95	18.52/16.40	45.28/76.91	75.14/151.53	191.06	286.46	1076.12	1553.63/1995.06	28.41
$\gamma_1 = 0.2, \gamma_2 = 0.3$ $\lambda_1 = 0, \lambda_2 = 0$	13.99	23.73/20.74	19.31/15.76	43.07/81.52	64.57/146.72	263.22	268.43	968.18	1499.83/2020.08	34.68
$\gamma_1 = 0.0, \gamma_2 = 0.0$ $\lambda_1 = 3, \lambda_2 = 5$	14.05	19.24/17.77	18.29/15.87	32.96/56.92	48.08/92.99	136.03	179.58	733.08	1048.69/1322.10	26.07
$\gamma_1 = 0.0, \gamma_2 = 0.0$ $\lambda_1 = 0, \lambda_2 = 0$	14.08	22.98/19.73	18.99/15.14	30.42/58.13	40.96/92.35	184.47	169.43	648.47	1002.37/1348.28	34.51

Table 2 Comparisons between the price-only and our (w,b) contracts at $\gamma_1 = 0.2, \gamma_2 = 0.3, \lambda_1 = 3, \lambda_2 = 5$

$price-only$	$w^* = 13.94$		$\pi_1^* = 191.06$	$\pi_2^* = 286.46$	$\pi_m^* = 1076.12$	$\pi^* = 1553.63$
$our \quad (w,b)$	$b_1 = 8.851$ $b_2 = 4.788$	$take$ $w_1 = 13.00$ $w_2 = 12.00$	$\pi_1 = 243.73$ $I\% = 27.57\%$	$\pi_2 = 380.74$ $I\% = 32.91\%$	$\pi_m = 1370.58$ $I\% = 27.36\%$	$\pi_m = 1995.06$ $I\% = 28.41\%$

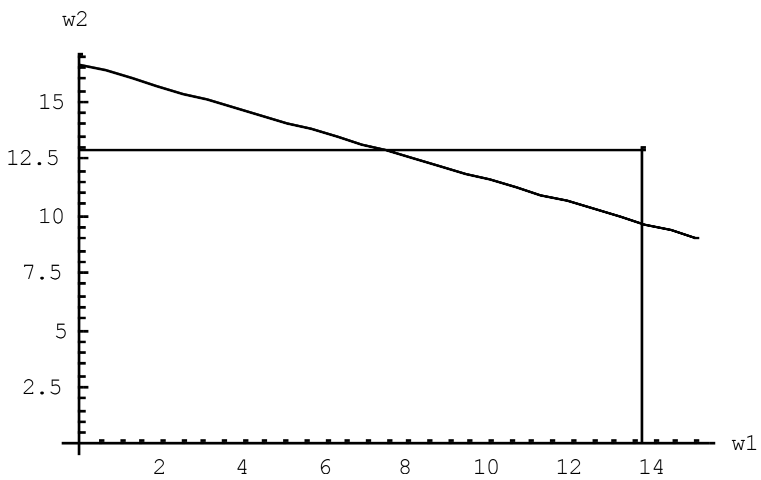

Fig 1 Graphics of Pareto-improved conditions

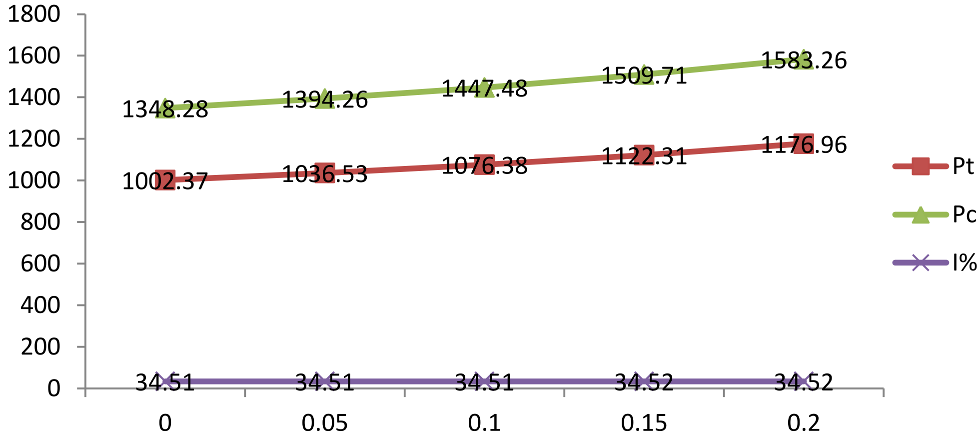

Fig 2 $\gamma_2 = 0, \lambda_1 = 0, \lambda_2 = 0, \gamma_1 = 0.00 - 0.20$ step 0.05

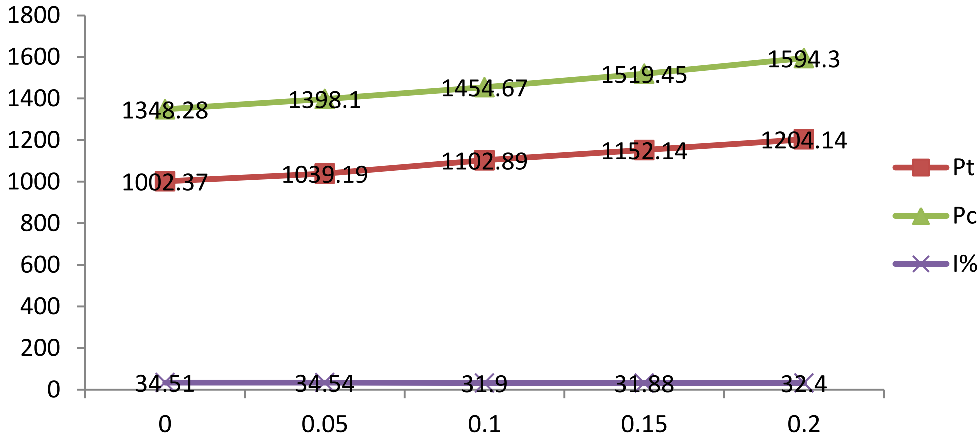

Fig 3 $\gamma_1 = 0, \lambda_1 = 0, \lambda_2 = 0, \gamma_2 = 0.00 - 0.20$ step 0.05

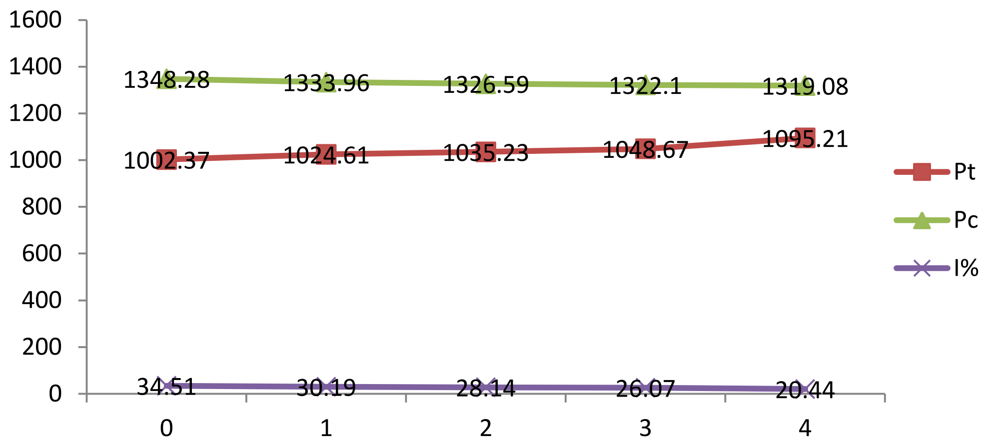

Fig 4 $\gamma_1 = 0, \gamma_2 = 0, \lambda_2 = 0, \lambda_1 = 0-4$ step 1

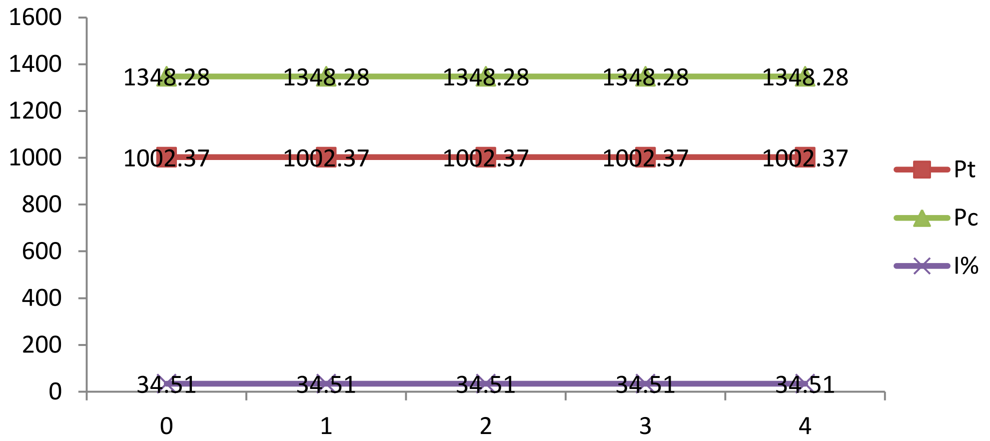

Fig 5 $\gamma_1 = 0, \gamma_2 = 0, \lambda_1 = 0, \lambda_2 = 0-4$ step 1

Appendix

Proof of Proposition 1

According to Eq. (1), we obtain $\frac{\partial q_i}{\partial p_i} = \frac{-\beta_i + l_1}{1-\gamma_i}$ and $\frac{\partial q_i}{\partial z_i} = \frac{1}{1-\gamma_i}$, where $l_1 = \frac{\partial L_i(p_i, p_j)}{\partial p_i} = \begin{cases} -\lambda_1 & p_1 > p_2 \\ -\lambda_2 & p_1 < p_2 \end{cases}$ is negative. Thus, from Eq. (2), we have

$$\frac{\partial E[\pi_i(p_i, z_i)]}{\partial p_i} = q_i - (p_i - w)\frac{-\beta_i + l_1}{1-\gamma_i} - \Lambda_i(z_i)$$

$$\frac{\partial E[\pi_i(p_i, z_i)]}{\partial z_i} = \frac{p_i - w}{1-\gamma_i} - p_i F(z_i)$$

$\frac{\partial^2 E[\pi_i(p_i, z_i)]}{\partial p_i^2} = \frac{2(-\beta_i + l_1)}{1-\gamma_i}$, $\frac{\partial^2 E[\pi_i(p_i, z_i)]}{\partial z_i^2} = -p_i f(z_i)$ and

$\frac{\partial^2 E[\pi_i(p_i, z_i)]}{\partial p_i \partial z_i} = \frac{1}{1-\gamma_i} - F_i(z_i)$. Its corresponding Hessian matrix is

$$H=\begin{bmatrix}\dfrac{2(-\beta_i+l_1)}{1-\gamma_i} & \dfrac{1}{1-\gamma_i}-F_i(z_i)\\ \dfrac{1}{1-\gamma_i}-F_i(z_i) & -p_i f(z_i)\end{bmatrix}$$

The first-order minor is $|H_1|=\dfrac{2(-\beta_i+l_1)}{1-\gamma_i}$ <0

The second-order minor is

$$|H_2|=\begin{vmatrix}\dfrac{2(-\beta_i+l_1)}{1-\gamma_i} & \dfrac{1}{1-\gamma_i}-F_i(z_i)\\ \dfrac{1}{1-\gamma_i}-F_i(z_i) & -p_i f(z_i)\end{vmatrix}>\frac{2(\beta_i-l_1)p_i f_i(z_i)}{1-\gamma_i}-(\frac{1}{1-\gamma_i})^2$$

$$\geq\frac{2(\beta_i+\min\{\lambda_1,\lambda_2\})p_i f_i(z_i)}{1-\gamma_i}-(\frac{1}{1-\gamma_i})^2 >0 \text{ holds}$$

if $(\beta_i+\min\{\lambda_1,\lambda_2\})p_i f_i(z_i)>\dfrac{1}{1-\gamma_i}$, and this proves the negative-definite H, that is, $E[\pi_i(p_i,z_i)]$ is concave in p_i and z_i.

Proof of Proposition 2

If we regard $p_i=p_i(z_i)$ in Eq. (3) and take the derivative w.r.t z_i, it yields $\dfrac{-\beta_i+l_1}{1-\gamma_i}p_i^{'}(z_i)+\dfrac{1}{1-\gamma_i}+p_i^{'}(z_i)\dfrac{-\beta_i+l_1}{1-\gamma_i}-F_i(z_i)=0$. Thus, $p_i^{'}(z_i)=\dfrac{1-(1-\gamma_i)F_i(z_i)}{2(\beta_i-l_1)}$ >0, and this proves the optimal p_i increases in z_i. Next, regard $z_i=z_i(p_i)$ in Eq. (4) and take the derivative w.r.t p_i, yielding $z_i^{'}(p_i)=\dfrac{1-(1-\gamma_i)F_i(z_i)}{(1-\gamma_i)p_i f_i(z_i)}$ >0 ,which proves the optimal z_i increases in p_i, and we complete the proof of positive correlation between the optimal p_i and z_i if they exist.

Proof of Proposition 3

We prove Proposition 3 by iterating $z_i^{(k)}$ and $p_i^{(k)}$, $k=1,2,\cdots$ as follows. First, set $z_i^{(0)}$ =0, the possible minimum of z_i. Then, a sequence of a pair of ($p_i^{(k)}, z_i^{(k)}$) is constructed according to following steps. For $i=1,2,\ j=3-i$, set $k=1$,

Step 1: Find $p_i^{(k)}$ by solving Eq. (3) with $z_i=z_i^{(k-1)}$

Step 2: Find $z_i^{(k)}$ by solving Eq. (4) with $p_i=p_i^{(k)}$

Step 3: Let $k=k+1$ and repeat Steps 1-3

Consequently, according to Proposition 2, the obtained sequence monotonically increases, that is, $z_i^{(k-1)}<z_i^{(k)}$ and $p_i^{(k-1)}<p_i^{(k)}$ for $k=1,2,\cdots$. Meanwhile, the positive value $\alpha_i-\beta_i p_i$ requires that $p_i^{(k)}$ is bounded above by $\dfrac{\alpha_i}{\beta_i}$, which concludes the convergence of $p_i^{(k)}$.

Therefore, the sequence ($p_i^{(k)}, z_i^{(k)}$) simultaneously converges to the solution of Eqs. (3) and (4), which is also unique due to the concave $E[\pi_i(p_i, z_i)]$.

Proof of Proposition 4

We regard $p_i = p_i(w)$ and $z_i = z_i(w)$ in Eqs. (3)-(4) and take the derivative w.r.t w, yielding the following equations.

$$\begin{cases} \frac{2(-\beta_i + l_1)}{1-\gamma_i} p_i^{'}(w) + \left(\frac{1}{1-\gamma_i} - F_i(z_i)\right) z_i^{'}(w) = \frac{-\beta_i + l_1}{1-\gamma_i} \\ \left(\frac{1}{1-\gamma_i} - F_i(z_i)\right) p_i^{'}(w) - p_i f_i(z_i) z_i^{'}(w) = \frac{1}{1-\gamma_i} \end{cases} \quad \text{(A1)}$$

Let $\Delta = \begin{vmatrix} \frac{2(-\beta_i + l_1)}{1-\gamma_i} & \frac{1}{1-\gamma_i} - F_i(z_i) \\ \frac{1}{1-\gamma_i} - F_i(z_i) & -p_i f(z_i) \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} \frac{-\beta_i + l_1}{1-\gamma_i} & \frac{1}{1-\gamma_i} - F_i(z_i) \\ \frac{1}{1-\gamma_i} & -p_i f(z_i) \end{vmatrix}$ and

$\Delta_2 = \begin{vmatrix} \frac{2(-\beta_i + l_1)}{1-\gamma_i} & \frac{-\beta_i + l_1}{1-\gamma_i} \\ \frac{1}{1-\gamma_i} - F_i(z_i) & \frac{1}{1-\gamma_i} \end{vmatrix}$. Then, Δ>0 and Δ_1>0 because of the assumption in Proposition 1; clearly, Δ_2<0. Thus, (A1) implies $p_i^{'}(w) = \frac{\Delta_1}{\Delta} > 0$ and $z_i^{'}(w) = \frac{\Delta_2}{\Delta} < 0$, which implies that the optimal p_i increases but z_i decreases in w. Further, if we regard $q_i = q_i(w)$ in Eq. (5) and then take the derivative w.r.t w, we obtain that $q_i^{'}(w) = \frac{-\beta_i + l_1}{(1-\gamma_i)^2 \Delta}\left((\beta_i - l_1) p_i f_i(z_i) + 2F_i(z_i)\right)$<0, proving the optimal q_i decreases in w.

Proof of Proposition 5

For $i = 1, 2$, $j = 3 - i$, we regard $p_i = p_i(p_j)$ and $z_i = z_i(p_j)$ in Eq. (3) and take the derivative w.r.t p_j, yielding

$$\frac{-\beta_i + l_1}{1-\gamma_i} p_i^{'}(p_j) + \frac{1}{1-\gamma_i} z_i^{'}(p_j) + \frac{l_2}{1-\gamma_i} + p_i^{'}(p_j) \frac{-\beta_i + l_1}{1-\gamma_i} - F_i(z_i) z_i^{'}(p_j) = 0 \quad \text{(A2)}$$

where $l_2 = \frac{\partial L_i(p_i, p_j)}{\partial p_j} = \begin{cases} \lambda_1 & p_1 > p_2 \\ \lambda_2 & p_1 < p_2 \end{cases} = -l_1$. Substitute $z_i^{'}(p_j) = z_i^{'}(p_i) p_i^{'}(p_j) = \frac{1-(1-\gamma_i)F_i(z_i)}{(1-\gamma_i) p_i f_i(z_i)} p_i^{'}(p_j)$ into (A2); we obtain $p_i^{'}(p_j) = \frac{A}{B}$>0, where $A = \frac{l_2 p_i f_i(z_i)}{1-\gamma_i} > 0$ and $B = \frac{2(\beta_i - l_1) p_i f_i(z_i)}{1-\gamma_i} - \left(\frac{1}{1-\gamma_i} - F_i(z_i)\right)^2 > 0$. Once $p_i^{'}(p_j)$>0 is obtained, $z_i^{'}(z_j) = z_i^{'}(p_i) p_i^{'}(p_j) p_j^{'}(z_j)$>0 and $q_i^{'}(q_j) = q_i^{'}(z_i) z_i^{'}(z_j) z_j^{'}(q_j)$>0 are accordingly obtained, and this proves the positive correlation among both retailers' optimal values.

Proof of Proposition 6

We use the scheme in Proposition 3 to help prove Proposition 6 as follows. First, set $p_2 = c$

Step 1: Use Steps 1-3 in Proposition 3 to obtain a convergent (p_1, z_1)

Step 2: Use this convergent (p_1, z_1) and Steps 1-3 in Proposition 3 to obtain a convergent (p_2, z_2)

Step 3: Use this convergent (p_2, z_2) and go to Step 1

Then, according to Propositions 2 and 5, both sequences (p_1, z_1) and (p_2, z_2) are increasing and bounded above, which thus simultaneously converge to solutions that satisfy both retailers' optimal necessary conditions.

Proof of Proposition 7

The manufacturer maximizes his expected profit $E[\pi(w)] = (w-c)(q_1+q_2)$ subject to both retailers' optimal necessary conditions in Eqs. (3)-(5), $i=1,2$, $j=3-i$. First, $E^{'}[\pi(w)] = q_1(w)+q_2(w)+(w-c)\left(q_1^{'}(w)+q_2^{'}(w)\right)$. For any $w>c$, if $w_1>w$, because $q_i^{'}(w)<0$, we have $E^{'}[\pi(w_1)] - E^{'}[\pi(w)] < \left(q_1(w_1)-q_1(w)\right)+\left(q_2(w_1)-q_2(w)\right) + (w-c)\left(\left(q_1^{'}(w_1)-q_1^{'}(w)\right)+\left(q_2^{'}(w_1)-q_2^{'}(w)\right)\right)$. Thus, let $w_1 \to w^+$, $E^{'}[\pi(w_1)] - E^{'}[\pi(w)]<0$ holds and this implies $E^{''}[\pi(w)]<0$, the concave $E[\pi(w)]$ in w.

To prove the unique optimal w, we need $\lim_{w\to c^+} q_i(w) = q_i(c) > 0$ because it represents the amount of order when the manufacturer himself sells the item, and $\lim_{w\to\infty} q_i(w) = 0$ representing zero inventory because $q_i^{'}(w)<0$. Thus, $\lim_{w\to c^+} E^{'}[\pi(w)] = q_1(c)+q_2(c)>0$ and $\lim_{w\to\infty} E^{'}[\pi(w)]<0$ are obtained. Moreover, because $E^{'}[\pi(w)]$ decreases in w, there exists only one $w>c$ such that $E^{'}[\pi(w)]=0$, and this completes the proof.

Proof of Proposition 8

To prove $E[\pi^c(p,z)]$, where $p=(p_1,p_2)$ and $z=(z_1,z_2)$, is concave, we first fix p_2 and prove $E[\pi^c(p,z)]$ is concave in p_1, z_1 and z_2 as follows. Thus, from Eq. (6), we have

$$\frac{\partial E[\pi^c(p,z)]}{\partial p_1} = q_1 + (p_1-c)\frac{-\beta_1+l_1}{1-\gamma_1} - \Lambda_1(z_1) + (p_2-c)\frac{l_2}{1-\gamma_2}$$

$$\frac{\partial E[\pi^c(p,z)]}{\partial z_1} = \frac{p_1-c}{1-\gamma_1} - p_1 F_1(z_1)$$

$$\frac{\partial E[\pi^c(p,z)]}{\partial z_2} = \frac{p_2-c}{1-\gamma_2} - p_2 F_2(z_2)$$

$$\frac{\partial^2 E[\pi^c(p,z)]}{\partial p_1^2} = \frac{2(-\beta_1+l_1)}{1-\gamma_1}, \quad \frac{\partial^2 E[\pi^c(p,z)]}{\partial z_1^2} = -p_1 f_1(z_1)$$

$$\frac{\partial^2 E[\pi^c(p,z)]}{\partial z_2^2} = -p_2 f_2(z_2), \quad \frac{\partial^2 E[\pi^c(p,z)]}{\partial p_1 \partial z_1} = \frac{1}{1-\gamma_1} - F_1(z_1)$$

$$\frac{\partial^2 E[\pi^c(p,z)]}{\partial p_1 \partial z_2} = 0, \qquad \frac{\partial^2 E[\pi^c(p,z)]}{\partial z_1 \partial z_2} = 0$$

The corresponding Hessian matrix is

$$H = \begin{bmatrix} \frac{2(-\beta_1 + l_1)}{1-\gamma_1} & \frac{1}{1-\gamma_1} - F_1(z_1) & 0 \\ \frac{1}{1-\gamma_1} - F_1(z_1) & -p_1 f_1(z_1) & 0 \\ 0 & 0 & -p_2 f_2(z_2) \end{bmatrix}$$

The first-order minor $|H_1| = \frac{2(-\beta_1 + l_1)}{1-\gamma_1} < 0$

The second-order minor $|H_2| = \begin{vmatrix} \frac{2(-\beta_1 + l_1)}{1-\gamma_1} & \frac{1}{1-\gamma_1} - F_1(z_1) \\ \frac{1}{1-\gamma_1} - F_1(z_1) & -p_1 f_1(z_1) \end{vmatrix} > 0$

The third-order minor $|H_3| = \begin{vmatrix} \frac{2(-\beta_1 + l_1)}{1-\gamma_1} & \frac{1}{1-\gamma_1} - F_1(z_1) & 0 \\ \frac{1}{1-\gamma_1} - F_1(z_1) & -p_1 f_1(z_1) & 0 \\ 0 & 0 & -p_2 f_2(z_2) \end{vmatrix}$

$$= -p_2 f_2(z_2) \begin{vmatrix} \frac{2(-\beta_1 + l_1)}{1-\gamma_1} & \frac{1}{1-\gamma_1} - F_1(z_1) \\ \frac{1}{1-\gamma_1} - F_1(z_1) & -p_1 f_1(z_1) \end{vmatrix} < 0$$

Thus, H is negative-definite, proving the concave $E[\pi^c(p,z)]$ in p_1, z_1 and z_2. Likewise, the concave $E[\pi^c(p,z)]$ in p_2, z_2 and z_1 for a fixed p_1 can be drawn. To conclude the concave $E[\pi^c(p,z)]$ in p_1, z_1, p_2 and z_2, we next consider the optimal necessary condition of p_i, z_i and z_j for a fixed p_j, $i = 1, 2$, $j = 3 - i$ as follows.

$$q_i + (p_i - c)\frac{-\beta_i + l_1}{1-\gamma_i} - \Lambda_i(z_i) + (p_j - c)\frac{l_2}{1-\gamma_j} = 0 \qquad \text{(A3)}$$

$$\frac{p_i - c}{1-\gamma_i} - p_i F_i(z_i) = 0 \qquad \text{(A4)}$$

$$\frac{p_j - c}{1-\gamma_j} - p_j F_j(z_j) = 0 \qquad \text{(A5)}$$

According to (A3)-(A5), we will prove that p_i, z_i and z_j all increase in p_j. First, take the derivative w.r.t p_j in (A5), yielding $z_j^{'}(p_j)=\frac{1}{p_j f_j(z_j)}\left(\frac{1}{1-\gamma_j}-F_j(z_j)\right)$>0. Second, taking the derivative w.r.t p_j in (A3) and (A4), respectively, it yields

$$\begin{cases}\frac{2(-\beta_i+l_1)}{1-\gamma_i}p_i^{'}(p_j)+\left(\frac{1}{1-\gamma_i}-F_i(z_i)\right)z_i^{'}(p_j)=\frac{-l_2}{1-\gamma_i}+\frac{-l_2}{1-\gamma_j}\\ \left(\frac{1}{1-\gamma_i}-F_i(z_i)\right)p_i^{'}(p_j)-p_i f_i(z_i)z_i^{'}(p_j)=0\end{cases}$$

Let $\Delta=\begin{vmatrix}\frac{2(-\beta_i+l_1)}{1-\gamma_i} & \frac{1}{1-\gamma_i}-F_i(z_i)\\ \frac{1}{1-\gamma_i}-F_i(z_i) & -p_i f(z_i)\end{vmatrix}$, $\Delta_1=\begin{vmatrix}\frac{-l_2}{1-\gamma_i}+\frac{-l_2}{1-\gamma_j} & \frac{1}{1-\gamma_i}-F_i(z_i)\\ 0 & -p_i f(z_i)\end{vmatrix}$ and

$\Delta_2=\begin{vmatrix}\frac{2(-\beta_i+l_1)}{1-\gamma_i} & \frac{-l_2}{1-\gamma_i}+\frac{-l_2}{1-\gamma_j}\\ \frac{1}{1-\gamma_i}-F_i(z_i) & 0\end{vmatrix}$. Obviously, Δ>0, Δ_1>0 and Δ_2>0; thus, $p_i^{'}(p_j)=\frac{\Delta_1}{\Delta}>0$ and $z_i^{'}(p_j)=\frac{\Delta_2}{\Delta}>0$, and we prove that p_i, z_i and z_j all increase in p_j. Knowing the increases of p_i, z_i and z_j in p_j, $i=1,2$, $j=3-i$, similar to Proposition 6, we construct two sequences (p_1, z_1, z_2) and (p_2, z_2, z_1), respectively, as follows. Set $p_2=c$

Step 1: Solve (A3)-(A5) to obtain (p_1, z_1, z_2)

Step 2: Use this obtained p_1 to solve (A3)-(A5) to obtain (p_2, z_2, z_1)

Step 3: Use this obtained p_2, and repeat Steps 1-3

Therefore, both (p_1, z_1, z_2) and (p_2, z_2, z_1) are increasing and will simultaneously converge to the solution of (A3)-(A5) for $i=1,2$, $j=3-i$. Thus, the uniqueness of optimal p_1, z_1, p_2 and z_2 in the centralized supply chain within the range of $p_i<\frac{\alpha_i}{\beta_i}$ is proven, as is the concave $E[\pi^c(p,z)]$ in p_1, z_1, p_2 and z_2.

Proof of Proposition 9

According to Eq. (7), $E[\pi_i^b(p_i)]=(p_i-w_i)q_i^c-(p_i-b_i)\Lambda_i(z_i)$ where z_i satisfies $(1-\gamma_i)q_i^c=\alpha_i-\beta_i p_i+L_i(p_i,p_j)+z_i$. Thus,

$E^{'}[\pi_i^b(p_i)]=q_i^c-\Lambda_i(z_i)-(p_i-b_i)F_i(z_i)(\beta_i-l_1)$ and

$E^{''}[\pi_i^b(p_i)]=-2F_i(z_i)(\beta_i-l_1)-(p_i-b_i)f_i(z_i)(\beta_i-l_1)^2$<0, which proves the concave $E[\pi_i^b(p_i)]$ in p_i. To prove the unique solution $E^{'}[\pi_i^b(p_i)]$=0, note that

$(1-\gamma_i)q_i^c = \alpha_i - \beta_i p_i + L_i(p_i, p_j) + z_i$ implies $z_i \to (1-\gamma_i)q_i^c - \alpha_i - \lambda_j p_j < q_i^c$ as $p_i \to 0$, and $\Lambda_i(z_i) = \int_0^{z_i} (z_i - \varepsilon_i) f_i(\varepsilon_i) d\varepsilon_i < \int_0^{z_i} z_i f_i(\varepsilon_i) d\varepsilon_i < z_i < q_i^c$ as $p_i \to 0$; thus, $\lim_{p_i \to 0^+} E'[\pi_i^b(p_i)] = \lim_{p_i \to 0^+} \left(q_i^c - \Lambda_i(z_i) - (p_i - b_i) F_i(z_i)(\beta_i - l_1) \right) > 0$. Next, for a fixed q_i^c, $p_i \to \infty$ implies $z_i \to \infty$; thus, $\Lambda_i(z_i) \to \infty$, $F_i(z_i) \to 1$ and $\lim_{p_i \to \infty} E'[\pi_i^b(p_i)] = -\infty$. The unique solution of $E'[\pi_i^b(p_i)] = 0$ is then proven as the consequence of $E''[\pi_i^b(p_i)] < 0$.

Proof of Proposition 10

Each retailer's optimal p_i and z_i, $i = 1, 2$, in our contract are obtained according to Eqs. (8) and (9); thus, if our contract coordinates the chain, the manufacturer must set a buyback price b_i such that each retailer's optimal p_i and z_i are exactly the same as p_i^c and z_i^c in the centralized supply chain, respectively. Therefore, substituting $p_i = p_i^c$ and $z_i = z_i^c$ in Eq. (8) yields $b_i = p_i^c - \dfrac{q_i^c - \Lambda_i(z_i^c)}{(\beta_i - l_1) F_i(z_i^c)}$.

Proof of Proposition 11

It is easily obtained by our Pareto-improved conditions $E[\pi_i^b(w_i)] > \pi_i^*$ and $E[\pi_m^b(w)] > \pi_m^*$, $i = 1, 2$.