INVESTIGATION ON CONTROLLABILITY, OBSERVABILITY AND STABILITY FOR PLANT OPTIMAL CONTROL PERFORMANCE

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ABSTRACT

This study examines the controllability, observability and stability of control systems as vital factors during the design stage. In order to achieve optimal control performance of controlled systems, the plant should be examined for controllability, observability and stability before applying control techniques. However, in some cases, these factors are neglected during design stages which eventually cause poor performance of the system in the long run. The principles behind controllability, observability and stability of control systems were reviewed in this work. Position control plant model was used for demonstration of control system design and dynamics tests. From the test results, the plant model was controllable and observable since their respective matrix determinants exist. From the Bode plot result, the system is stable. Therefore, the position control plant design can be controlled using any control technique such as open loop or closed loop (i.e. feedback) control technique.

Keywords: Plant, Observability, Controllability, Stability, Control System.

INTRODUCTION

In the world today, every practical and real live process or situation in every field of life including education, politics, agriculture, health, economics, management, engineering, etc can be controlled using control techniques to achieve optimum output. In fact, every life activity or situation can be referred to as a plant which can be controlled to produce better results. In control theory, a plant can be referred to as a combination of process and actuator, often expressed with a transfer function (commonly in the s-domain) which indicates the relationship between an input signal and output signal of a system without feedback, usually determined by physical properties of the system. Advancement in the field of control gave rise to automation and robotics which has helped to achieve improved output and performance of most industrial and home processes. According to Al-Tabey (2012), Micro-Robots are used in medical field for surgical applications. Robots are widely used in medical field for getting minimally invasive surgery efficiently and accurately. The increased need and application of control systems especially robots as stated in Agbaraji and Inyama (2015) is resulting to more research towards robotics and automation. Robots are generally used to perform unsafe, hazardous, highly repetitive, and unpleasant tasks. They have many different functions such as material handling, assembly, arc welding, resistance welding, machine tool load and unload functions, painting, spraying, etc. (Elfasakhany et al, 2011). The four main types of robot arms are revolute, polar, cylindrical and Cartesian coordinate. The degrees of freedom or the number of axis classify the type of robot (Rasouliha, 2004).

Kendrick (1976) presented a survey of applications of control theory to macroeconomics. According to him, control theory has been applied to about fifty different macroeconomic models containing anywhere from one to more than three hundred equations, and including models of the economies of the United States, Canada, United Kingdom, West Germany, France, Belgium, Australia and the Netherland. A wide range of control theory methods has been applied to these models. Deterministic methods for quadratic - linear and general nonlinear models have been used. Uncertainty has been introduced in the form of an additive noise to the systems equations and in the form of uncertainty about parameter values and the models have been solved either with closed loop policies and/or open loop optimal feedback policies.

Most plant designs do not examine the dynamics of the system response to determine the behavior of such systems before applying control techniques. This is confirmed by numerous inaccurate stability analyses, erroneous statements about the existence of stable control, and overly severe constraints on compensator characteristics. The basic difficulty has been a failure to account properly for all dynamic modes of system response. This failure is attributable to a limitation of the transfer function matrix; it fully describes a linear system if and only if the system is controllable and observable. The concepts of controllability and observability were introduced by Kalman (1960) and have been employed primarily in the study of optimal control.

The test for controllability and obervability of a control system is very important because it is used to determine if the plant would be controllable from the input vector and observable from the output vector. Controllability involves determining the state vector from the input vector and also involves the system and input matrices. For the system to be controllable, the determinant of the controllability matrix must exist (Okoro, 2008). On the other hand, observability is concerned with finding the state vector from the output vector and, thus, involves the system and output matrices. For a system to be observable, the determinant of the observability matrix must exist (Okoro, 2008). A system which is not controllable has dynamic modes of behavior which depend solely on initial conditions or disturbance inputs. Also, a system which is not observable has dynamic modes of behavior which cannot be ascertained from measurement of the available outputs (Gilbert, 1963). A plant must be completely controllable and observable and stable in order to perform optimally. It must be stable and robust in order to achieve and maintain optimum performance even in presence of significant uncertainties.

LITERATURE REVIEW

Controllability

Given a Linear Time Invariant (LTI) system that is described by:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

Where

u=u(t), p-dimensional input vector
y=y(t), q-dimensional output vector
x=x(t), n-dimensional state vector
$\dot{x} = \dot{x}(t)$, time derivative of state
A, constant nth order differential transition matrix
B, constant, n row, p column, input matrix
C, constant, q row, n column, output matrix
D, constant, q row, p column, transmission matrix
If n=0 the system is said to be static

According to Padhi, a system is said to be *controllable* at time t_0 if it is possible by means of an unconstrained control vector to transfer the system from any initial state to any other state

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in a finite interval of time. Controllability depends upon the system matrix *A* and the control influence matrix *B*. In more precise manner, Oliver stated that a control system is said to be (completely) controllable if, for all initial times t_0 and all initial states $x(t_0)$, there exists some input function *u(t)* that drives the state vector *x(t)* to any final state at some finite time $t_0 \leq t \leq T$.

Controllability Test

Given a system defined by the linear state equation:

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad x(t) = x_0$$

The controllability matrix is defined as:

$$P = [B\ AB\ A^2B \dots A^{n-1}B]$$

It can be proved that a system is controllable if and only if:

$$rank(P) = n$$

For the general multiple-input (*m*) case, **A** is an *n* x *n* matrix and **B** is *n* x *m*. Then, **P** consists of *n* matrix blocks **B**, **AB**, $\mathbf{A}^2\mathbf{B} \dots \mathbf{A}^{n-1}\mathbf{B}$, each with dimension *n* x *m*, stacked side by side. Thus, **P** has dimension *n* x *n·m*, having more columns than rows.

For the single-input case, **B** consists of a single column, yielding a square *n* x *n* controllability matrix **P**. Therefore, a single-input linear system is controllable if and only if the associated controllability matrix **P** is nonsingular. ($|\mathbf{P}| \neq 0$) (Oliver)

Controllability matrix:

The rank of an m x n matrix **A** is one of the following:

The maximal number of linearly independent columns of **A**

The maximal number of linearly independent rows of **A**

The size of the largest nonsingular submatrix that can be extracted from **A**

Matlab functions:

ctrb() calculates the controllability matrix for state-space systems.

Syntax

```
P=ctrb(sys)
P=ctrb(sys.A,sys.B) or P=ctrb(A,B)
```

rank() provides an estimate of the number of linearly independent rows or columns of a full matrix.

Syntax

```
k = rank(A)
```

Observability

A system is said to be *observable* at time t_0 if, with the system in state $x(t_0)$, it is possible to determine this state from the observation of the output over a finite interval of time (Padhi). Observability depends upon the system matrix *A* and the output matrix *C*. The state $\mathbf{x}(t_0)$ is said to be observable if given any input ***u(t)***, there exists a finite time $T \geq t_0$ such that the knowledge of signal input ***u(t)***, signal output ***y***(t) for $T > t \geq t_0$, and matrices **A**, **B**, **C** and **D,** are sufficient to determine $\boldsymbol{x}(t_0)$. If every state of the system is observable, the system is said to be (completely) observable (Oliver).

Observability Test

Given a system defined by its linear state equation, the observability matrix is defined as:

$$Q = \begin{bmatrix} C \\ CA \\ CA^2 \\ . \\ . \\ . \\ CA^{n-1} \end{bmatrix}$$

It can be proved that a system is observable if and only if:

$$rank(Q) = n$$

For the general multiple-output (p) case, **A** is an n x n matrix and **C** is p x n. Then, **Q** consists of n matrix blocks **C**, **CA**, $\mathbf{CA}^2$ …$\mathbf{CA}^{n-1}$, each with dimension p x n, stacked one on top of another. Thus, **Q** has dimension np xn, having more rows than columns.
For the single-output case, **C** consists of a single row, yielding a square n x n observability matrix **Q**. Therefore, a single-output linear system is observable if and only if the associated observability matrix **Q** is nonsingular. ($|\mathbf{Q}| \neq 0$).
As it happens for the Controllability, the Observability is invariant with respect to coordinate transformation (Oliver).

Matlab function

obsv() calculates the obsevability matrix for state-space systems.
Syntax

```
Q=obsv(sys)
Q=obsv(A, C) or Q=obsv(sys.A, sys.C)
```

PLANT MODEL

The dynamic behaviors of a motion control system used for the demostration are given by the following equations (Phillips et al, 1996):

$$e_a = R_m i_a(t) + L_m \frac{di_a(t)}{dt} + e_m(t)$$

$$e_m = K_m \frac{d\theta_m(t)}{dt}$$

$$T_m = K_t i_a(t)$$

$$T_m = J \frac{d^2\theta_m(t)}{dt^2} + B \frac{d\theta_m(t)}{dt}$$

$$J = J_m + n^2 J_{load}$$

$$\theta_L = n\theta_m$$

After simplification and taking the ratio of $\theta_L(s)/E_a(s)$ the transfer function becomes:

$$\frac{\theta_L}{E_a} = \frac{K_t n}{JL_m s^3 + (R_m J + bL_m)s^2 + (K_t K_m + R_m b)s}$$

Where;
R_m = armature- winding resistance in ohm
L_m = armature - winding inductance in Henry
i_a = armature - winding current in ampere
e_a = armature voltage in volt
e_m = back emf voltage in volt
K_m = back emf constant in volt / (rad/sec)
T_m = torque developed by the motor in N.m
K_t = motor torque constant in N.m/A
J = moment of inertia of motor and robot arm in $kg^2\ m$ /rad

b = viscous - friction coefficient of motor and robot arm in N.m/rad /sec
θ_m = angular displacement of the motor shaft in rad
θ_L = angular displacement of the robot arm in rad
n = gear ratio N_1/N_2

Plant Simulation

Figure 1 shows the position control plant angular position (θ or thetha) of the shaft in matlab/simulink. In this work, the plant has the following specifications: J = 0.02kg.m^2, b = 0.03N.m/rad.s, Kt = 0.023N.m/A, Ke = 0.023V.s/rad, R = 1Ohm, L = 0.23H.

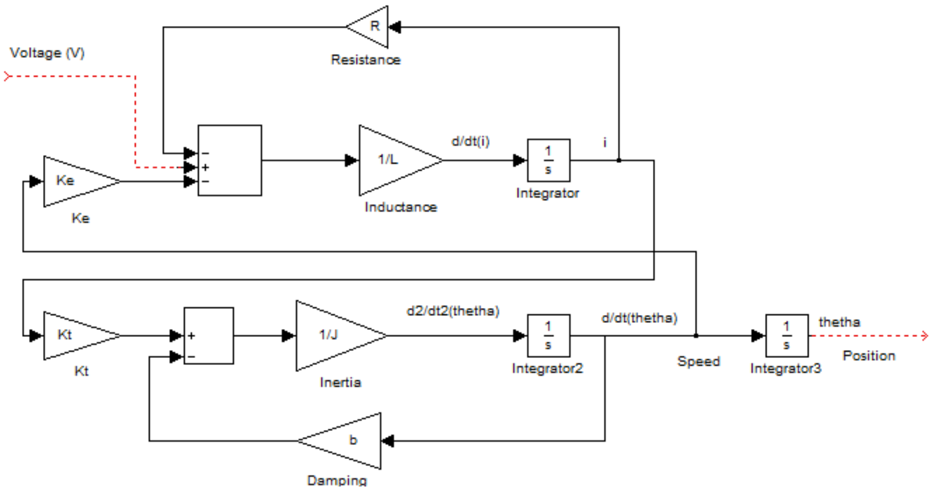

Figure 1: The position control plant model

Substituting the physical parameter values to test for stability and workability of the position control plant as illustrated in figure 2.

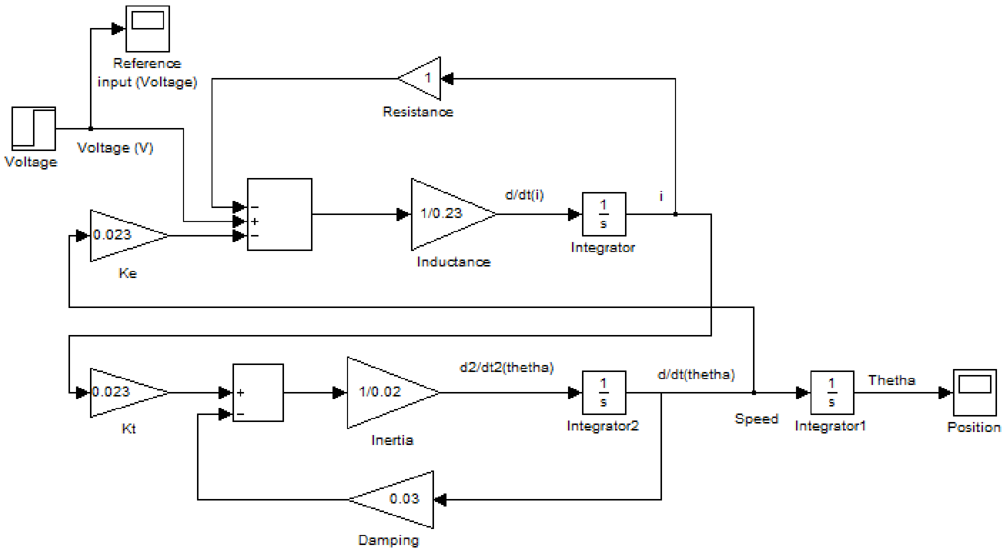

Figure 2: The position control plant model with parameter values

TESTS AND RESULTS

Figures 3a and 3b show the output scope graph of the reference input and the angular position of the robot arm plant.

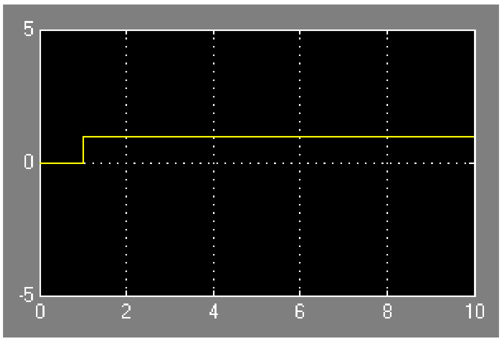

Figure 3a: Plant reference input

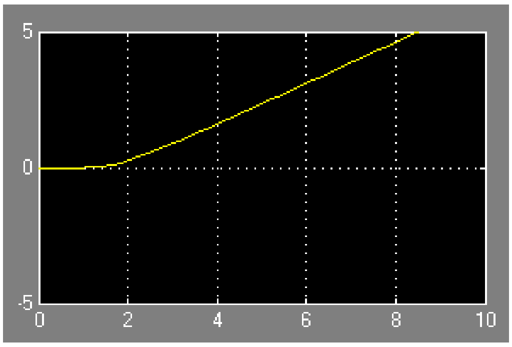

Figure 3b: Angular Position

To generate the state space and transfer function of the position control plant model, we have:

```
>> [A, B, C, D]=linmod('Position_Control_Plant')
A =
        0    1.0000         0
        0   -1.5000    1.1500
        0   -0.1000   -4.3478
B =
        0
        0
   4.3478
C =
    1    0    0
D =
    0
>> [num, den]=ss2tf(A, B, C, D)
num =
        0   -0.0000   -0.0000    5.0000
den =
   1.0000    5.8478    6.6367         0
```

To verify the model extraction of the position control plant model, an open-loop step response of the transfer function was used. Figure 4 shows the open-loop step response of extracted transfer function of the position control plant model.

```
>> step(num, den)
```

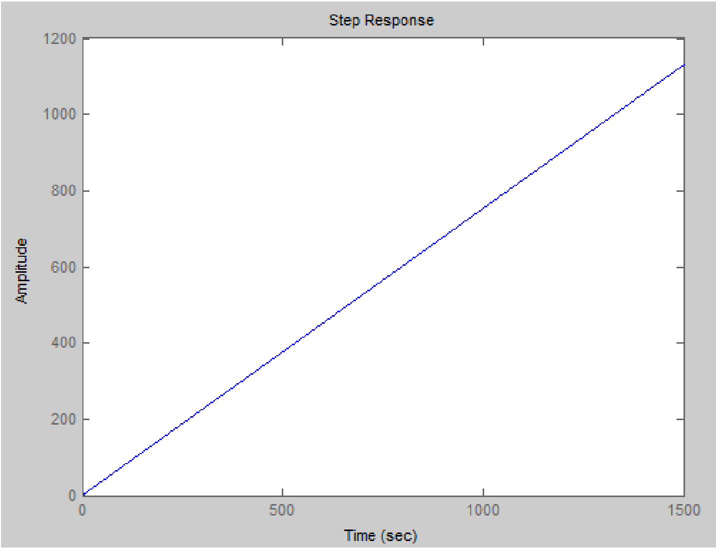

Figure 4: Open-loop step response of extracted transfer function of the plant model

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Controllability and Observability Tests for the Plant

To determine the controllability, the controllability matrix was first determined as follows:

```
>> CO=ctrb(A, B)  %This gives the controllability matrix
CO =
        0         0    5.0000
        0    5.0000  -29.2391
   4.3478  -18.9036   81.6895
>> Control_CO=det(CO) %This gives the controllability matrix determinant
Control_CO =
 -108.6957
```

Working for the observability matrix and test, we have:

```
>> OB=obsv(A, C) %This gives the observability matrix


OB =
   1.0000         0         0
        0    1.0000         0
        0   -1.5000    1.1500
>> Control_OB=det(OB) %This gives the observability matrix determinant
Control_OB =
   1.1500
```

Stability Test for the Position Control Plant

Bode stability plot was used to investigate the stability of the modeled plant in order to determine the optimal control state of the robot arm. Figure 6 illustrates the Bode plot of the transfer function of the plant showing all Stability margins.

```
>> bode(num, den);grid on
```

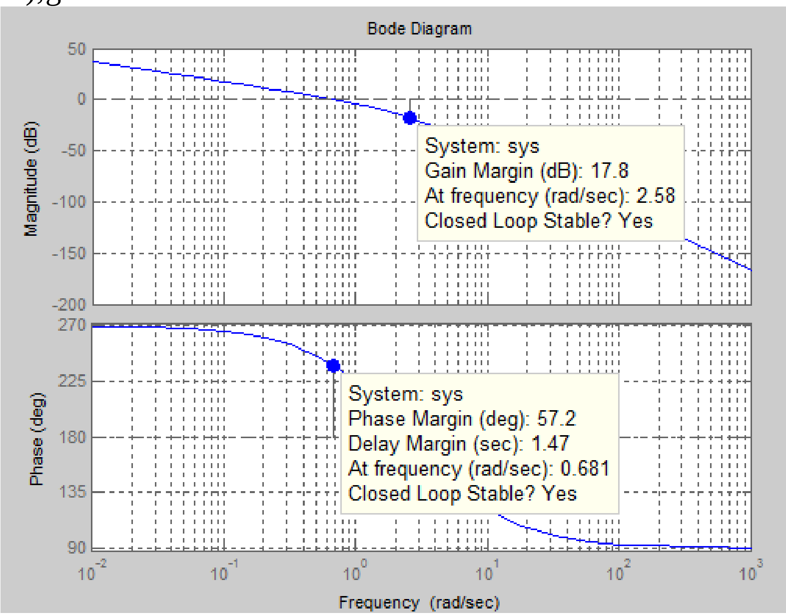

Figure 6: Bode plot of the transfer function of the plant showing all Stability margins.

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CONCLUSION

The examination of a plant for optimal performance involves the study of the controllability, observability and stability of the plant. The plant model was achieved using matlab/simulink. From the controllability and observability test results, thel plant model is both controllable and observable since their respective matrix determinants exist. It can be observed from the Bode plot result that the system is stable. Therefore, since the system met the conditions of controllability, observability and stability, the plant design was successful. Thus, controller design techniques such as the open or closed loop techniques can be employed to achieve desired results of the controlled system.

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