PLURIHARMONICITY IN THE SHEAF OF COMPLEX LINES

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ABSTRACT

In this article, we prove an analog of Forelli's Theorem for pluriharmonic functions, Theorem 1. Let D be a complete circular domain in C^n and the function $U, D \to R$,defined in D, satisfies the following conditions:

1) U M- subharmonic somewhere, $U \in Msh\{0\}$
2) For each fixed $z \in D$ the function $U(\lambda z)$ is harmonic in the circle $\{\lambda \in C^n : \lambda z \in D\}$.

Then the function U is pluriharmonic in D.

Keywords: M – sub harmonic functions, plurisubharmonic functions, holomorphic functions, harmonic functions, entire circular domain.

INTRODUCTION

Let's begin with known definitions of circular domains.

Definition 1. Domain $D \in C^n$ is called entire circular if $\lambda z \in D$, for each $z \in D$ and $|\lambda| \leq 1$.

The results of this paragraph are directly related to the following Forelli'stheorem.

The theorem (Forelli). Let D be an entire circular domain in C^n. Let's suppose that the function $U, D \to \mathbf{R}$ has the following features:

1) U is infinitely continuously differentiable in the neighborhood of zero: $U \in C^\infty\{0\}$

2) For each fixed $z \in D$ the function $U(\lambda z)$ is harmonic in the circle $\{\lambda \in C : \lambda z \in D\}$.

Then, the function U is pluriharmonic in D.

The example K continuously differentiable function $U(z) = \frac{z_1^{k+1} \times \overline{z_z}}{|z|^2}$ $U(0) = 0$

show that the gained result is given incorrectly if the condition I - infinite smoothness is substitutedwith the existence of only a finite number of derivatives in the neighborhood $0 \in C^n$. We prove that the condition $U \in C^{\infty}\{0\}$ may be replaced by the more convenient condition M – subharmonic in some neighborhood of zero.

Theorem 1. Let D be an entire circular domain in C^n and the function $U, D \to R$ given in D satisfies the following conditions:

3) U M-subharmonic somewhere, $U \in Msh\{0\}$

4) For each fixed $z \in D$ the function $U(\lambda z)$ is harmonic in the circle $\{\lambda \in C^n : \lambda z \in D\}$

Then, the function U is pluriharmonic in D.

The proof of the Theorem I. Fromthe conditions of Theorem and the formula of Puasson, for each fixed number $|\lambda| \le 1$ and the point $z \in D$ we know $U(\lambda z) = \frac{1}{2\pi}\int_0^{2\pi} U(\tau z)\operatorname{Re}\frac{\xi+\lambda}{\xi-\lambda}dt$ where $\tau = e^{it}$. As in the theorem I, firstly, we look at the case when $U \in C^2(G)$. From the conditions of the theorem for each fixed λ $|\lambda| \le 1$, in the neighborhood G, having derived both sides of the equation (I.2) on Z, we have $\Delta_z U(\lambda z) = \frac{1}{2\pi}\int_0^{2\pi} \Delta_z U(\xi, z)\operatorname{Re}\frac{\xi+\lambda}{\xi-\lambda}dt$, and when $\frac{1}{2\pi}\int_0^{2\pi} \Delta_z U(\xi, z)\operatorname{Re}\frac{\xi+\lambda}{\xi-\lambda}dt = \Delta_z U(0) = 0$.

Let's suppose $\psi(\lambda z) = \frac{1}{2\pi}\int_0^{2\pi} \Delta_z U(\xi z)\operatorname{Re}\frac{\xi+\lambda}{\xi-\lambda}dt$ the function $\psi(\lambda z)$ is holomorphic according to λ in the circle $|\lambda| \le 1$ and $\psi(0, z) = 0$ according to (I.3). Therefore, $\psi(\lambda z)$ equals 0 in fixed $z \in G$ or identically, or takes a neighborhood of zero in some neighborhood of zero. The latter cannot be done because $\operatorname{Re}\psi(\lambda z) \ge 0$.

This implies that $\psi(\lambda z) = 0$ for any fixed λ, $|\lambda| \le 1$ and $z \in G$. So $\square_z U(\lambda z) = 0$ and, that is to say U began coordinates harmonically in the neighborhood. Consequently $U \in C^{\infty}\{0\}$ as a function and U will be pluraharmonic in D accoding to the formed theorem of Forelli.

According to the formula of Poisson and theorems of Fubini, we have
$$(U(\lambda z,\Delta\varphi))=\int_G\left\{\frac{1}{2\pi}\int_0^{2\pi}\Delta_z U(\tau z)\operatorname{Re}\frac{\xi+\lambda}{\xi-\lambda}dt\right\}\Delta_z\varphi dV=\frac{1}{2\pi}\int_0^{2\pi}\left\{\int_G U(\xi,z)\Delta_z\varphi dV\right\}\operatorname{Re}\frac{\xi+\lambda}{\xi-\lambda}dt\geq 0$$
$\tau=e^{it}$.
, where

It is clear that the function $\psi(\lambda z)=\frac{1}{2\pi}\int_0^{2\pi}\left\{\int_G U(\xi,z)\Delta_z\varphi dV\right\}\frac{\xi+\lambda}{\xi-\lambda}dt$

Holomorphic to λ , the function $\psi(\lambda z)$ is either identically equal to 0 or transfers the neighborhood of zero to some neighborhood of zero. The latter is impossible because of $\operatorname{Re}\psi(\lambda z)\geq 0$ which is in G. From it $\psi(\lambda z)\equiv 0$ for each $|\lambda|<1$ and $z\in G$.

Consequently $(U(l),\Delta_z\varphi)=0$ for any non-negative in G function φ of the category C^∞. This means that $U(z)$ is harmonic in G, and, partially U, infinitely smooth in the neighborhood of zero. This implies that U is pluraharmonic in D.

The theorem is proven.

The theorem 1 can be used to continue holomorphic function in the following pattern:

Corollary 1. Let the function f given in the entire circular domain $D\subset C^n$ satisfies the following conditions:

1) $\operatorname{Re} f$ M – subharmonic in the neighborhood $G\ni 0$.

2) Stiction f/l is holomorphic in $D\cap l$ for each complex line $l\ni 0$.

Then, f is holomorphic according to the set of variables in D.

In fact, according to Theorem 1. $\operatorname{Re} f$ is pluraharmonicin D.

We take the full sphere $B=B(o,r)\subset D$ and in it the function $F\in O(B)$ is found which is $\operatorname{Re}F=\operatorname{Re} f$. Then the difference $\varphi(z)=F(z)-f(z)$ has the following features: $\operatorname{Re}\varphi\equiv 0$ and stiction φ/l is holomorphic in the neighborhood $l\cap B$. Consequently, $\varphi/l\equiv C(l)$ is a constant depending perhaps on l. But $C(l)=\varphi(0)=F(0)-f(0)=\mathrm{const}$ from which $f=F-const$ and

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is a holomorphic function in B. Using the theorem of Forelli, we obtain holomorphicity f in the neighborhood D. Corollary is proven.

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