# A MATHEMATICAL MODEL TO PREDICT BACK PRESSURE USING CONSTANT BOTTOM HOLE PRESSURE TECHNIQUE IN MANAGED PRESSURE DRILLING

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## ABSTRACT

Drilling through narrow mud window sections using conventional drilling method has been very challenging as it could easily lead to drilling hazards such as; lost circulation, kick, borehole instability etc, thereby causing an increase in Non Productive time (NPT). Managed Pressure Drilling (MPD) is a drilling technology that can be used to precisely control the wellbore annular pressure profile so as to mitigate drilling hazards and eliminate NPT. In this study, back pressure was estimated using the pore pressure, hydrostatic pressure and the Annular Frictional Pressure Loss (AFPL) at various hole intervals using the Constant Bottom Hole Pressure (CBHP) technique of MPD. A Mathematical model was developed to predict backpressure as a function of the Bottom Hole Circulating Pressure (BHCP). Three regression models (linear, quadratic and cubic) were developed for the 12 1/4" and 8 1/2" hole sections respectively from the initial accurately estimated values of back pressure for these intervals. The models were validated with actual field data from a typical MPD well in West Africa. The quadratic regression model gave the best approximation for the two hole sections with an 81% accuracy for the 12 1/4" hole section and a 91% accuracy for the 8 1/2" hole section. These developed models provide an easy and efficient means of predicting back pressure from the BHCP and also the Equivalent Circulating Density (ECD) for MPD operations.

**Keywords:** Managed Pressure Drilling, Rheological Models, Backpressure, Regression Analysis.

## INTRODUCTION

Discoveries have shown that Non Productive Time (NPT) account for approximately 20% of total rig time and can be much higher in difficult and complex terrains. Rig rates are on the high, with some rigs going for as high as 1 million USD per day. During drilling, a range of mud weight is always given, when the mud weight is higher than the window, there is all tendency that there will be a higher overbalance pressure which will result to lost circulation that may ultimately lead to stuck pipe. Also, when the mud weight is outside the window, it results to a negative overbalance which also leads to drilling problems. To drill safely it is advisable to operate within the mud window. Most deep water formations have a very small drilling window because of the abnormally high formation pressure and a low fracture pressure which is caused by rapid sedimentation, lack of compaction and the low overburden due to the large column of water which is less dense than solid sediments. Hence, drilling deep water prospects by conventional method is almost not feasible (Malloy 2007). Drilling in deepwater formations using the conventional drilling technique requires setting of numerous casing strings at relatively shallow depths in order to prevent lost circulation. Managed Pressure Drilling (MPD) helps in controlling this problem by drilling with a controlled BHP. In mature fields, the formation pore pressure, the fracture pressure, the collapse pressure and the overburden pressure profiles are constantly changing due to production and depletion. This makes the pressure window narrower, thereby making drilling within the boundary more challenging without experiencing kicks or lost circulation (Malloy and MacDonald 2008). MPD is very effective in reducing NPT that are drilling related as it combines new technology with older principles and techniques to manage common drilling problems. According to the International Association of Drilling Contractors (IADC), Managed Pressure Drilling is an adaptive drilling process that is used to precisely control the annular pressure profile throughout the wellbore. It's an advanced form of well control where a closed and pressurized mud system is applied to enable a more precise control of wellbore pressure profiles than just mud hydrostatic and the pump pressure adjustments. Estimating the required back pressure term is very important to achieve a successful MPD operation. Hence, it is very necessary to have a very reliable tool for estimating the required back pressure. This study aimed at applying the concepts of Constant Bottom hole Pressure (CBHP) method of managed pressure drilling to develop and validate mathematical models that can accurately estimate the backpressure required to mitigate drilling hazards for various hole sections using regression analysis.

## LITERATURE REVIEW

According to Rehm et al (2008) new drilling techniques simply combine new methods and the historical methods to effectively mitigate drilling hazards. Demirdal and Cunha (2007) carried out some experimental analysis to ascertain the best fluid rheological model for HPHT condition (40 to 280$^0$F and 500 to 12000psi) using un-weighted n-paraffin base drilling fluid (synthetic mud) in MPD operations. They compared the Bingham plastic, Power law and Yield power law models to their experimental findings. The yield power law gave the most accurate result when compared to the experimental result. They modeled the effect of temperature and pressure on drilling fluid density. Malloy and MacDonald (2008) compared and contrasted conventional, underbalanced and managed pressure drilling. Their comparison was based on planning objectives, equipment, operation and well control. They stated that MPD was mainly used to drill wells that are impossible or uneconomical to drill using the conventional overbalanced drilling technique and that MPD is a technology for the mitigation of drilling hazards. They concluded that the MPD and underbalanced drilling are quite different technologies as against the misconception that they are the same. Glen-Ole et al (2012) stated that the automation of the choke manifold for an MPD system was achieved with a control system that consists of two main parts, namely; a hydraulic model which was used for computing real time downhole pressure which in turn controls the choke pressure and a feedback control algorithm which automatically controls the choke manifold to enable it maintain the desired choke pressure. They stated further that the hydraulics model determines the accuracy of the MPD system. They developed a simplified hydraulics model called the fit-for-purpose hydraulics model for computing the downhole pressures and to provide a choke pressure set point for automated MPD systems. Jan et al (2014) carried a study to know how pressure control was affected and sometimes limited by the actual available data and its quality, equipment, hydraulic models, control algorithm and downhole condition during MPD operations in ERD wells. They carried out some simulations and showed how the sensor response and bandwidth affected the ability to accurately control downhole pressures in ERD wells. They concluded that special care should be taken when applying MPD in ERD wells because, ERD wells are more complex and challenging when compared to shorter wells. Fan et al (2014) carried out a study on Herschel Buckley model and came up with a new model by modifying the Herschel Buckley model. They obtained an explicit equation between the wall shear stress and the volumetric flow rate for pipe and annular flow from Herschel Buckley fluid model. They were also able to establish a new relation for pipe and annular Reynolds number and frictional pressure drop. They validated the new model using well data from Sichuan Basin and they concluded that the new model predicted and calculated hydraulics more accurately than the other traditional models previously used in MPD operations. Kinik et al (2015) carried out a simulation analysis for kick detection, control and circulation using MPD. They were able to highlight the benefits of automated influx detection and control using MPD system compared to a conventional well control method. Their simulations successfully detected and controlled a gas influx in oil based mud while drilling in onshore western Canada. They concluded that the current MPD system has the potential for drilling formations with narrow pressure margins through their accuracy and precision in pressure control and early kick detection. Hannegan (2010) stated that in reactive MPD, a conventional-wisdom well construction and fluids program is planned, but the rig is equipped with at least an RCD, choke, and drillstring float(s) as a means to more safely and efficiently deal with unexpected downhole pressure environment limits. Medley and Reynolds (2006) stated that the reactive MPD has been implemented on potential problem wells for years, but very few proactive applications were seen until recently, as the need for drilling alternatives increased. Aadnoy (2009) stated that the shift from reactive to proactive MPD requires that the wells be preplanned more thoroughly, but the benefits to the drilling program typically more than offset the cost of the additional MPD engineering and project management. Hannegan (2006) stated that MPD has been proven to enable drilling of what might otherwise be economically un-drillable prospects and that MPD was well on its way to becoming the status quo technology over the next decade due to the fact that it increased recoverable assets. He discussed the following variations of MPD; Constant Bottom Hole Pressure (CBHP) technique, Pressurized Mud Cap Drilling (PMCD) technique, Dual Gradient (DG) technique, Return Flow Control (RFC) technique and the Reverse Circulation (RC) technique. Some other application of MPD includes; depleted reservoir drilling, methane hydrates drilling, High pressure High temperature drilling and extended reach drilling. Hannegan (2009) stated that PMCD method of MPD should be utilized in deep water where some depleted zones may be encountered before reaching a deeper productive target zone with a virgin pressure. Once the depleted zone above the target zone has the rock properties capable of taking in the sacrificial fluid and drill cuttings, safe drilling with PMCD variation would be a good option. Syltoy et al (2008) stated that it is required that an accurate automated choke control be used so as to compensate for the variations in BHP that results from change in downhole temperature, pipe rotation, surge and swab, and other situations that results in variaton of BHP in HPHT wells. He further stated that it is very important to calibrate the model with downhole measured pressure so as obtain accuracy. Elieff (2006) stated that methane hydrates cannot be formed at temperatures greater than or equal to $68^0$F as they can only be formed when the temperature is below $68^0$F with adequate pressure. With oil and gas exploration getting into deep waters, the presence of methane hydrates is now constantly reported. However, when MPD technique is been used, the wellbore conditions would be properly managed and the hydrate dissociation in the wellbore can then be avoided.

## METHODOLOGY

The data used for this study were obtained from an MPD field in West Africa. It contained the pore pressure, fracture gradient, rheological properties of the fluid used, hole size and depth, drill string components, sizes and lengths. The three well intervals used are; 17 1/2" hole, 12 1/4" hole and the 8 1/2" hole sections. The 17 1/2" hole section was drilled using the conventional drilling technique. But the 12 1/4" and 8 1/2" hole sections were drilled using Managed Pressure Drilling (MPD) techniques. The data are shown in table 5 of the appendix.

## ANALYSIS METHOD

A computer software program was developed to compute the back pressures by utilizing three different AFPL models. The Marc.Soft program was developed using visual basic.Net programming language in order to estimate the back pressure from the pore pressure, hydrostatic pressure and the AFPL models. The three different AFPL models that were utilized include; the Bingham Plastic AFPL model, the power law AFPL model and the Herschel Bulkley AFPL model. Regression models were developed using the most accurate back pressure estimate for each hole section.

## MATHEMATICAL MODEL DEVELOPMENT

In general, a mathematical model describes the relationship between a dependent variable and an independent variable. The Utilization of the appropriate AFPL model together with the mud hydrostatic pressure and the pore pressure allowed for an accurate estimation of the Back Pressure. The mathematical model developed used simple linear, quadratic and cubic regression analysis to investigate the relationship between the Bottom Hole Circulating Pressure, BHCP and the Back pressure. The regression Model that gave the best fit with the actual data from the three regression models (linear, quadratic and cubic) was taken as the best. The regression analysis was done for each hole section and it was done for just the Back pressure estimate with the least percentage error.

## THE LINEAR REGRESSION

A linear regression model relates a dependent variable to just a single independent variable to a degree of just the first order.

$$y = \alpha_0 + \alpha_1 x \qquad 1$$

The normal equations to get the solution of the linear regression model are given below as:

$$\sum y = \propto_0 n + \propto_1 \sum x \qquad 2$$

$$\sum xy = \propto_0 \sum x + \propto_2 \sum x^2 \qquad 3$$

Equations 2 and 3 would be solved simultaneously to get the regression constants ($\alpha_0$ and $\alpha_1$). n is the number of sample points.

## THE QUADRATIC REGRESSION

A Quadratic regression model relates a dependent variable to just a single independent variable to a degree of the second order.

$$y = \propto_0 + \propto_1 x + \propto_2 x^2 \qquad 4$$

The normal equations to get the solution of the quadratic regression model are given below as:

$$\sum y = \propto_0 n + \propto_1 \sum x + \propto_2 \sum x^2 \qquad 5$$

$$\sum xy = \propto_0 \sum x + \propto_1 \sum x^2 + \propto_2 \sum x^3 \qquad 6$$

$$\sum x^2 y = \propto_0 \sum x^2 + \propto_1 \sum x^3 + \propto_2 \sum x^4 \qquad 7$$

Equations 5, 6 and 7 would be solved simultaneously to get the regression constants ($\propto_0, \propto_1$ *and* $\propto_2$). n is the number of sample points.

## THE CUBIC REGRESSION

A Cubic regression model relates a dependent variable to just a single independent variable to a degree of the third order.

$$y = \propto_0 + \propto_1 x + \propto_2 x^2 + \propto_3 x^3 \qquad 8$$

The normal equations to get the solution of the cubic regression model are given below as:

$$\sum y = \propto_0 n + \propto_1 \sum x + \propto_2 \sum x^2 + \propto_3 \sum x^3 \qquad 9$$

$$\sum xy = \propto_0 \sum x + \propto_1 \sum x^2 + \propto_2 \sum x^3 + \propto_3 \sum x^4 \qquad 10$$

$$\sum x^2 y = \propto_0 \sum x^2 + \propto_1 \sum x^3 + \propto_2 \sum x^4 + \propto_3 \sum x^5 \qquad 11$$

$$\sum x^3 y = \propto_0 \sum x^3 + \propto_1 \sum x^4 + \propto_2 \sum x^5 + \propto_3 \sum x^6 \qquad 12$$

Equations 9, 10, 11 and 12 would be solved simultaneously to get the regression constants. n is the number of sample points.

## RESULTS AND DISCUSSION

The summary of the results gotten from utilizing the three rheological models were presented in tables 1a and 1b. The well data as shown in table 5 in the appendix are for the two different hole sections (12.25" and 8.5"). The 17 1/2" hole section was drilled by normal conventional drilling method. MPD was used in this well because of the narrow mud window between the pore pressure and the fracture gradient. The results are presented according to individual hole sections.

**Table 1a - The estimated back pressures using each of the three AFPL models and their error % estimate (3930 to 7600 ft) for 12 1/4" section**

MPD Optimisation

File Edit View Operations

Input Data
Evaluated Data
Back Pressure
Chart

| Depth (ft) | Field Back Pressure (Psi) | Bingham Plastic (BP) (Psi) | Power Law (BP) (Psi) | Herschel Buckley (BP) (Psi) | Bingham Plastic (% Error) | Power Law (% Error) | Herschel Buckley (% Error) |
|---|---|---|---|---|---|---|---|
| 3930 | 449.3 | 425.98 | 453.78 | 452.9 | -5.19 | 1 | 0.8 |
| 3980 | 443.32 | 382.82 | 447.62 | 434.15 | -13.65 | 0.97 | -2.07 |
| 4000 | 444.49 | 384.75 | 449.87 | 436.33 | -13.44 | 1.21 | -1.84 |
| 4100 | 454.55 | 394.37 | 461.12 | 447.24 | -13.24 | 1.44 | -1.61 |
| 4200 | 465.55 | 403.99 | 472.36 | 458.14 | -13.22 | 1.46 | -1.59 |
| 4300 | 476.33 | 413.6 | 483.61 | 469.05 | -13.17 | 1.53 | -1.53 |
| 4400 | 487.8 | 423.22 | 494.86 | 479.96 | -13.24 | 1.45 | -1.61 |
| 4500 | 498.97 | 432.84 | 506.1 | 490.87 | -13.25 | 1.43 | -1.62 |
| 4600 | 510.16 | 442.46 | 517.35 | 501.78 | -13.27 | 1.41 | -1.64 |
| 4700 | 521.11 | 452.08 | 528.6 | 512.69 | -13.25 | 1.44 | -1.62 |
| 4800 | 532.53 | 461.7 | 539.84 | 523.59 | -13.3 | 1.37 | -1.68 |
| 4900 | 543.44 | 471.32 | 551.09 | 534.5 | -13.27 | 1.41 | -1.64 |
| 5000 | 554.68 | 480.93 | 562.34 | 545.41 | -13.3 | 1.38 | -1.67 |
| 5200 | 577.44 | 500.17 | 584.83 | 567.23 | -13.38 | 1.28 | -1.77 |
| 5400 | 600.18 | 519.41 | 607.32 | 589.04 | -13.46 | 1.19 | -1.86 |
| 5600 | 621.61 | 538.65 | 629.82 | 610.86 | -13.35 | 1.32 | -1.73 |
| 5800 | 643.69 | 557.88 | 652.31 | 632.68 | -13.33 | 1.34 | -1.71 |
| 6000 | 665.93 | 577.12 | 674.8 | 654.49 | -13.34 | 1.33 | -1.72 |
| 6300 | 699.72 | 605.98 | 708.54 | 687.22 | -13.4 | 1.26 | -1.79 |
| 6600 | 731.12 | 634.83 | 742.28 | 719.94 | -13.17 | 1.53 | -1.53 |
| 6900 | 764.47 | 663.69 | 776.02 | 752.67 | -13.18 | 1.51 | -1.54 |
| 7000 | 775.71 | 673.31 | 787.27 | 763.57 | -13.2 | 1.49 | -1.56 |
| 7100 | 787.17 | 682.93 | 798.52 | 774.48 | -13.24 | 1.44 | -1.61 |
| 7200 | 798.07 | 692.55 | 809.76 | 785.39 | -13.22 | 1.47 | -1.59 |
| 7300 | 809.08 | 702.16 | 821.01 | 796.3 | -13.21 | 1.47 | -1.58 |
| 7400 | 820.11 | 711.78 | 832.26 | 807.21 | -13.21 | 1.48 | -1.57 |
| 7500 | 831 | 721.4 | 843.5 | 818.11 | -13.19 | 1.5 | -1.55 |
| 7600 | 842.23 | 731.02 | 854.75 | 829.02 | -13.2 | 1.49 | -1.57 |

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**Table 1b - The estimated back pressures using each of the three AFPL models and their error % estimate (7600 to 8300 ft) for 121/4" section**

MPD Optimisation

File Edit View Operations

Input Data
Evaluated Data
Back Pressure
Chart

| Depth (ft) | Field Back Pressure (Psi) | Bingham Plastic (BP) (Psi) | Power Law (BP) (Psi) | Herschel Buckley (BP) (Psi) | Bingham Plastic (% Error) | Power Law (% Error) | Herschel Buckley (% Error) |
|---|---|---|---|---|---|---|---|
| 7600 | 842.23 | 731.02 | 854.75 | 829.02 | -13.2 | 1.49 | -1.57 |
| 7700 | 479.26 | 428.16 | 488.83 | 450.99 | -10.66 | 2 | -5.9 |
| 7800 | 482.69 | 433.72 | 495.18 | 456.85 | -10.15 | 2.59 | -5.35 |
| 7900 | 486.94 | 439.28 | 501.52 | 462.71 | -9.79 | 3 | -4.98 |
| 8000 | 493.8 | 444.84 | 507.87 | 468.56 | -9.92 | 2.85 | -5.11 |
| 8020 | 881.43 | 757.51 | 897.18 | 864.71 | -14.06 | 1.79 | -1.9 |
| 8040 | 881.2 | 759.4 | 899.41 | 866.86 | -13.82 | 2.07 | -1.63 |
| 8060 | 892.5 | 770.87 | 904.97 | 876.06 | -13.63 | 1.4 | -1.84 |
| 8080 | 894.18 | 772.78 | 907.22 | 878.23 | -13.58 | 1.46 | -1.78 |
| 8100 | 898.76 | 775.03 | 909.53 | 881.32 | -13.77 | 1.2 | -1.94 |
| 8120 | 900.53 | 776.94 | 911.78 | 883.5 | -13.72 | 1.25 | -1.89 |
| 8140 | 901.78 | 778.85 | 914.02 | 885.68 | -13.63 | 1.36 | -1.79 |
| 8160 | 857.53 | 699.03 | 887.59 | 819.88 | -18.48 | 3.51 | -4.39 |
| 8170 | 857.83 | 699.89 | 888.68 | 820.89 | -18.41 | 3.6 | -4.31 |
| 8180 | 856.97 | 700.75 | 889.77 | 821.89 | -18.23 | 3.83 | -4.09 |
| 8190 | 858.94 | 701.6 | 890.86 | 822.9 | -18.32 | 3.72 | -4.2 |
| 8200 | 859.75 | 702.46 | 891.94 | 823.9 | -18.29 | 3.74 | -4.17 |
| 8300 | 511.32 | 467.59 | 520.45 | 476.57 | -8.55 | 1.79 | -6.8 |

Since the power law model AFPL gave the best estimation of back pressure (least average error percent) for this hole section (121/4"), a mathematical model was developed based on the solution gotten from the power law model AFPL using regression analysis. The model showed a mathematical relationship between the Bottom Hole Circulating Pressure, BHCP (equivalent to the ECD) and the back pressure.

## Linear Regression Model for 12 1/4" Section

From tables 1a to 1b in combination with the hydrostatic pressure for the depth interval (hole section) and utilizing equations 2 and 3, the normal equation for the linear model was gotten

$$29924.15 = 43 \propto_0 + 136007.45 \propto_1 \quad (13)$$

$$98641164.41 = 136007.45 \propto_0 + 455646615.2 \propto_1 \quad (14)$$

Solving equations 13 and 14 simultaneously, the following values were obtained for $\alpha_0$ and $\alpha_1$:
$\propto_0 = 199.89$ and $\propto_1 = 0.1568$

Hence the Linear regression model is given as
$Y = 199.89 + 0.1568x$ 15
$\text{Back pressure} = 199.89 + 0.1568\ \text{BHCP}$ 16
Equation 16 is the linear regression model for back pressure in terms of BHCP for this hole section.
Note: BHCP = AFPL + HP

**Quadratic Regression Model for 12 1/4" Section**

From tables 1a to 1b in combination with the hydrostatic pressure for the depth interval and utilizing equations 5, 6 and 7, the quadratic model was gotten as shown below:
$\propto_0 = -1371.63789$, $\propto_1 = 1.263369$ and $\propto_2 = -1.8199 \times 10^{-4}$
Hence the quadratic regression model is given as:
$y = -1371.63789 + 1.263369x - 1.8199 \times 10^{-4}x^2$ 17
$Back\ pressure = -1371.63789 + 1.263369BHCP - 1.8199 \times 10^{-4}BHCP^2$ 18
Equations 18 is the quadratic regression model for back pressure in terms of BHCP for this hole section.

**Cubic Regression Model for 12 1/4" Section**

From tables 1a to 1b in combination with the hydrostatic pressure for the depth interval and utilizing equations 9, 10, 11 and 12, the cubic model was gotten as shown below:
$\propto_0 = -2137.650155$, $\propto_1 = 2.042631821$, $\propto_2 = -0.000436238$ and $\propto_3 = 2.68172 \times 10^{-8}$
Hence the cubic regression model is given as shown below:
$y = -2137.650155 + 2.042631821x - 0.000436238x^2 + 2.68172 \times 10^{-8}x^3$ 19
$Back\ Pressure = -2137.650155 + 2.04263182BHCP - 0.000436238BHCP^2 + 2.68172 \times 10^{-8}BHCP^2$

20

Equation 20 is the cubic model regression model for back pressure in terms of BHCP for this hole section.

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**Table 2a - The estimated back pressures using each of the three AFPL models and their error % estimate (8300 to 12500 ft) for 81/2" section**

MPD Optimisation

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| Depth (ft) | Field Back Pressure (Psi) | Bingham Plastic (BP) (Psi) | Power Law (BP) (Psi) | Herschel Buckley (BP) (Psi) | Bingham Plastic (% Error) | Power Law (% Error) | Herschel Buckley (% Error) |
|---|---|---|---|---|---|---|---|
| 8300 | 511.32 | 467.59 | 520.45 | 476.57 | -8.55 | 1.79 | -6.8 |
| 8400 | 510.88 | 473.22 | 526.72 | 482.31 | -7.37 | 3.1 | -5.59 |
| 8500 | 512.06 | 478.85 | 532.99 | 488.05 | -6.48 | 4.09 | -4.69 |
| 8600 | 554.11 | 512.94 | 562.73 | 521.95 | -7.43 | 1.56 | -5.8 |
| 8700 | 553.82 | 518.91 | 569.27 | 528.02 | -6.3 | 2.79 | -4.66 |
| 8800 | 562.33 | 524.87 | 575.82 | 534.08 | -6.66 | 2.4 | -5.02 |
| 8900 | 579.29 | 540.42 | 590.29 | 549.72 | -6.71 | 1.9 | -5.1 |
| 9000 | 582.89 | 546.49 | 596.92 | 555.9 | -6.24 | 2.41 | -4.63 |
| 9200 | 593.58 | 558.64 | 610.19 | 568.25 | -5.89 | 2.8 | -4.27 |
| 9400 | 619.17 | 580.94 | 631.87 | 590.6 | -6.18 | 2.05 | -4.61 |
| 9600 | 630.64 | 593.3 | 645.31 | 603.17 | -5.92 | 2.33 | -4.36 |
| 9800 | 640.45 | 605.66 | 658.76 | 615.73 | -5.43 | 2.86 | -3.86 |
| 10000 | 652.8 | 618.02 | 672.2 | 628.3 | -5.33 | 2.97 | -3.75 |
| 10200 | 656.15 | 619.36 | 676.51 | 630.02 | -5.61 | 3.1 | -3.98 |
| 10400 | 658.89 | 620.3 | 680.51 | 631.19 | -5.86 | 3.28 | -4.2 |
| 10600 | 688.01 | 655.1 | 712.53 | 666 | -4.78 | 3.56 | -3.2 |
| 10800 | 705.73 | 667.46 | 725.98 | 678.56 | -5.42 | 2.87 | -3.85 |
| 11000 | 734.13 | 691.47 | 749.09 | 702.76 | -5.81 | 2.04 | -4.27 |
| 11200 | 737.6 | 704.04 | 762.71 | 715.53 | -4.55 | 3.4 | -2.99 |
| 11400 | 753.76 | 716.61 | 776.33 | 728.31 | -4.93 | 2.99 | -3.38 |
| 11600 | 771.98 | 729.18 | 789.95 | 741.09 | -5.54 | 2.33 | -4 |
| 11800 | 782.52 | 741.76 | 803.57 | 753.86 | -5.21 | 2.69 | -3.66 |
| 12000 | 796.19 | 754.33 | 817.19 | 766.64 | -5.26 | 2.64 | -3.71 |
| 12100 | 801.96 | 760.61 | 824 | 773.03 | -5.16 | 2.75 | -3.61 |
| 12200 | 805.92 | 766.9 | 830.81 | 779.42 | -4.84 | 3.09 | -3.29 |
| 12300 | 813.5 | 773.19 | 837.62 | 785.81 | -4.96 | 2.96 | -3.4 |
| 12400 | 819.56 | 779.47 | 844.43 | 792.2 | -4.89 | 3.03 | -3.34 |
| 12500 | 826.91 | 785.76 | 851.24 | 798.59 | -4.98 | 2.94 | -3.43 |

**Table 2b - The estimated back pressures using each of the three AFPL models and their error % estimate (12700 to 13500 ft) for 81/2" section**

MPD Optimisation

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| Depth (ft) | Field Back Pressure (Psi) | Bingham Plastic (BP) (Psi) | Power Law (BP) (Psi) | Herschel Buckley (BP) (Psi) | Bingham Plastic (% Error) | Power Law (% Error) | Herschel Buckley (% Error) |
|---|---|---|---|---|---|---|---|
| 12700 | 839.58 | 798.33 | 864.86 | 811.36 | -4.91 | 3.01 | -3.36 |
| 12800 | 845.04 | 804.62 | 871.67 | 817.75 | -4.78 | 3.15 | -3.23 |
| 12900 | 721.05 | 615.74 | 946.94 | 735.69 | -14.6 | 31.33 | 2.03 |
| 13000 | 722.42 | 620.52 | 954.28 | 741.39 | -14.11 | 32.09 | 2.63 |
| 13100 | 719.68 | 625.29 | 961.62 | 747.09 | -13.12 | 33.62 | 3.81 |
| 13200 | 710.97 | 630.06 | 968.96 | 752.8 | -11.38 | 36.29 | 5.88 |
| 13220 | 698.2 | 631.02 | 970.43 | 753.94 | -9.62 | 38.99 | 7.98 |
| 13240 | 685.56 | 631.97 | 971.9 | 755.08 | -7.82 | 41.77 | 10.14 |
| 13260 | 685.9 | 632.93 | 973.36 | 756.22 | -7.72 | 41.91 | 10.25 |
| 13280 | 685.91 | 633.88 | 974.83 | 757.36 | -7.59 | 42.12 | 10.42 |
| 13300 | 732.66 | 597.97 | 1034.73 | 872.8 | -18.38 | 41.23 | 19.13 |
| 13350 | 743.78 | 609.55 | 1053.88 | 899.96 | -18.05 | 41.69 | 21 |
| 13400 | 748.95 | 611.84 | 1057.83 | 903.34 | -18.31 | 41.24 | 20.61 |
| 13420 | 746.9 | 612.75 | 1059.41 | 904.68 | -17.96 | 41.84 | 21.13 |
| 13440 | 744.68 | 613.66 | 1060.98 | 906.03 | -17.59 | 42.47 | 21.67 |
| 13460 | 951.24 | 770.8 | 1189.44 | 1087.07 | -18.97 | 25.04 | 14.28 |
| 13480 | 948.9 | 771.95 | 1191.21 | 1088.68 | -18.65 | 25.53 | 14.73 |
| 13490 | 944.82 | 772.52 | 1192.09 | 1089.49 | -18.24 | 26.17 | 15.31 |
| 13500 | 936.8 | 773.09 | 1192.97 | 1090.3 | -17.48 | 27.35 | 16.39 |

For this hole section, the Herschel Bulkley model AFPL performed best on the average. Hence the estimated back pressure using the Herschel Bulkley model AFPL was the basis for the mathematical model developed for this hole section. Just as for the 121/4" hole section, a mathematical model was developed for this 81/2" hole section using regression analysis. The model gave a mathematical relationship between the Bottom Hole Circulating Pressure, BHCP (which is like the ECD but in Psi) and the back pressure.

## Linear Regression Model for 8 1/2" Section

From tables 2a to 2b in combination with the hydrostatic pressure for this hole section and utilizing equations 2 and 3, the normal equation for the linear model was gotten and shown below:

$$35371.84605 = 48 \propto_0 + 291393.7539 \propto_1 \quad (21)$$

$$220484566.1 = 291393.7539 \propto_0 + 1809080411 \propto_1 \quad (22)$$

Solving equations 21 and 22 simultaneously, the values of $\alpha_0$ and $\alpha_1$ were obtained.
$\propto_0 = -133.6308849$ And $\propto_1 = 0.143400906$
Hence the linear regression model is given as shown below:

$$y = -133.6308849 + 0.143400906x \quad 23$$

$$Back\ Pressure = -133.6308849 + 0.143400906BHCP \quad 24$$

Equation 24 is the linear regression model for back pressure in terms of the BHCP for this 8 1/2" hole section.

**Quadratic Regression Model for 8 1/2" Section**

From tables 2a to 2b in combination with the hydrostatic pressure for this hole section and utilizing equations 5,6 and 7, the quadratic model was gotten and it is shown below:
$\propto_0 = -704.9642994$, $\propto_1 = 0.344513113$ and $\propto_3 = -1.723463295 \times 10^{-5}$
Hence the quadratic model is given as shown below:

$$y = -704.9642994 + 0.344513113x - 1.723463295 \times 10^{-5}x^2 \quad 25$$

$$Back\ Pressure = -704.9642994 + 0.344513113BHCP - 1.723463295 \times 10^{-5}BHCP^2 \quad 26$$

Equation 26 is the quadratic regression model for back pressure in terms of the BHCP for this 8 1/2" hole section.

**Cubic Regression Model for 8 1/2" Section**

From tables 2a to 2b in combination with the hydrostatic pressure for this hole section and the normal equation (equations 9, 10,11 and 12), the cubic regression model was gotten and it is shown below:
$\propto_0 = -2434.76709$, $\propto_1 = 1.252329698$, $\propto_3 = -0.000173873$ and $\propto_4 = 8.89652 \times 10^{-9}$
Hence, the cubic model is given as shown below;

$$y = -2434.76709 + 1.252329698x - 0.000173873x^2 + 8.89652 \times 10^{-9}x^3 \quad 27$$

$$Back\ Pressure = -2434.76709 + 1.252329698BHCP - 0.000173873BHCP^2 + 8.89652 \times 10^{-9}BHCP^2 \quad 28$$

Equation 28 is the cubic regression model for back pressure in terms of the BHCP for this 8 1/2" hole section.

**Model Validation Using Actual Field Data for the various Intervals**

The developed regression models were validated with the actual field data. The various models (linear, quadratic and cubic models) were used to estimate the back pressures, the estimated values were plotted against the actual field back pressure, the correlation coefficient between the respective regression models and the actual field data were estimated, the model with the highest correlation coefficient ($R^2$) value was taken as the best.

**For the 12 1/4" Hole Section**

From equations 16, 18 and 20 the back pressures were estimated for linear, quadratic and cubic models respectively. (See estimated results in table 7 of the appendix)

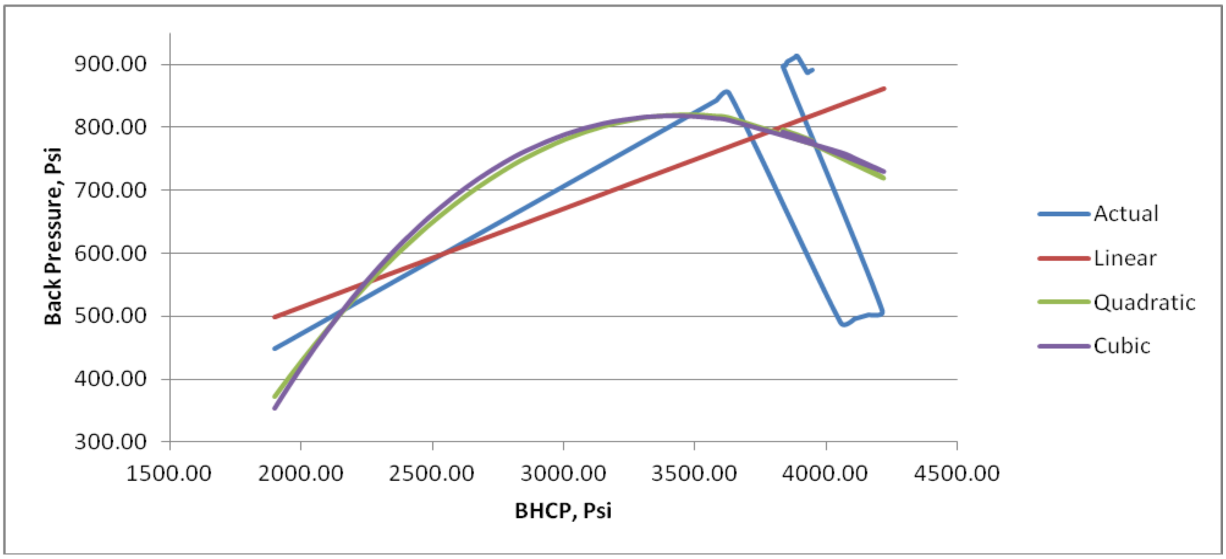


**Figure 1 - Plots of back pressure against the BHCP for the 121/4" section**

The correlation coefficients for the results from the regression models are shown below with their rank:

**Table 3 - Rank of the regression models**

| Models | Correlation Coefficient | Rank |
|---|---|---|
| Linear | 0.7007 | 3rd |
| Quadratic | 0.8058 | 1st |
| Cubic | 0.7862 | 2nd |

Hence the Quadratic model (equation 18) from the three regression models compared gave the best approximation for back pressure for the 121/4" hole section. This means that the model gives 80.58% representation of the desirable data.

**For the 8 1/2" hole section**

From equations 24, 26 and 28 the back pressures were estimated for linear, quadratic and cubic models respectively. (See estimated results in table 8 of the appendix)

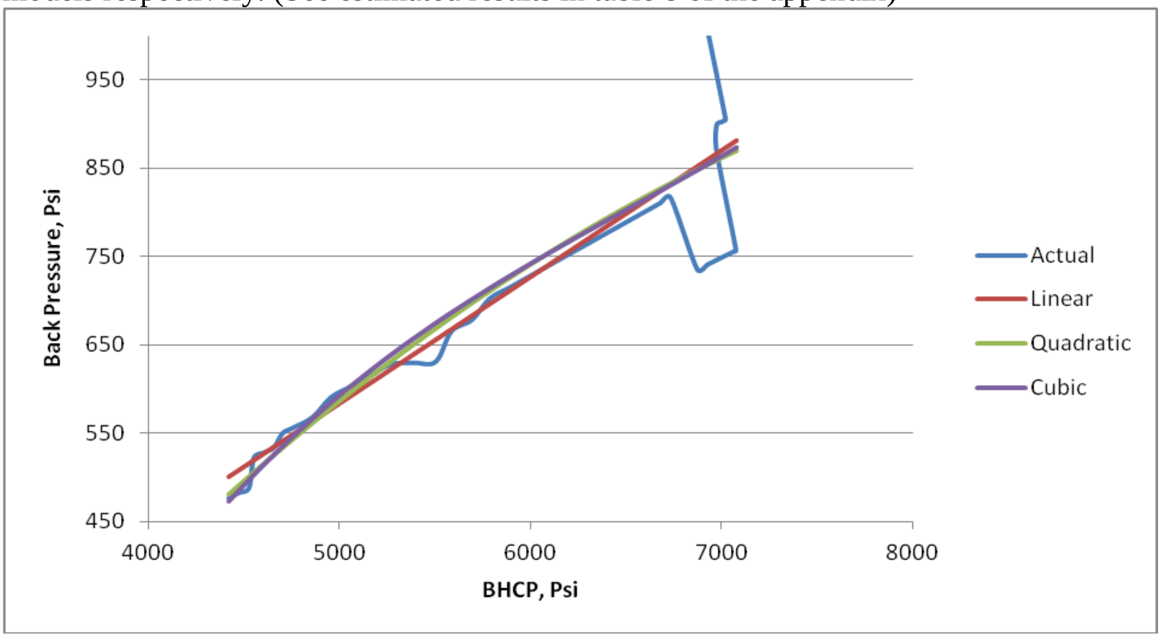

**Figure 2 - Plot of Back Pressure against BHCP for the 81/2" hole section**

The correlation coefficients for the results from the regression models are shown below with their rank:

**Table 4 - Rank of the regression models**

| Models | Correlation Coefficient | Rank |
|---|---|---|
| Linear | 0.8907 | 3rd |
| Quadratic | 0.9128 | 1st |
| Cubic | 0.8962 | 2nd |

The result showed that the Quadratic model (equation 26) from the three regression models compared gave the best approximation for back pressure estimation for the 81/2" hole section. This means that the model gives 91.28% representation of the desirable data.

## CONCLUSIONS

The ability to precisely control the pressures in the wellbore will go a long way in helping us to eradicate and minimize drilling problems such as; lost circulation, borehole instability, kick and stuck pipe. Based on this study, it is concluded that an accurate estimation of the required back pressure is very necessary for a successful MPD operation when using the CBHP technique. The mathematical models developed for back pressure estimation based on the bottom hole circulating pressure (ECD) is reliable and efficient. The quadratic model showed the best approximation for the actual field back pressure for the two hole sections analyzed in this study. Hence with the ECD, the required back pressure for a CBHP MPD operation can be confidently predicted using the quadratic regression models developed in this study.

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## APPENDIX

**Table 5 - Drilling data from a Well X in West Africa**

| Depth, ft | Densityppg | V, ft/s | hole size, inch | O.D, inches | PV, Cp | YP, lb/100ft$^2$ | Q,gpm | HP, Psi | PP, Psi | Back Pressure, psi |
|---|---|---|---|---|---|---|---|---|---|---|
| 3980 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 1883.34 | 2348.20 | 443.32 |
| 4000 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 1892.80 | 2360.00 | 444.49 |
| 4100 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 1940.12 | 2419.00 | 454.55 |
| 4200 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 1987.44 | 2478.00 | 465.55 |
| 4300 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 2034.76 | 2537.00 | 476.33 |
| 4400 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 2082.08 | 2596.00 | 487.80 |
| 4500 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 2129.40 | 2655.00 | 498.97 |
| 4600 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 2176.72 | 2714.00 | 510.16 |
| 4700 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 2224.04 | 2773.00 | 521.11 |
| 4800 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 2271.36 | 2832.00 | 532.53 |
| 4900 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 2318.68 | 2891.00 | 543.44 |
| 5000 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 2366.00 | 2950.00 | 554.68 |
| 5200 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 2460.64 | 3068.00 | 577.44 |
| 5400 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 2555.28 | 3186.00 | 600.18 |
| 5600 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 2649.92 | 3304.00 | 621.61 |
| 5800 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 2744.56 | 3422.00 | 643.69 |
| 6000 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 2839.20 | 3540.00 | 665.93 |
| 6300 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 2981.16 | 3717.00 | 699.72 |
| 6600 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 3123.12 | 3894.00 | 731.12 |
| 6900 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 3265.08 | 4071.00 | 764.47 |
| 7000 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 3312.40 | 4130.00 | 775.71 |
| 7100 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 3359.72 | 4189.00 | 787.17 |
| 7200 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 3407.04 | 4248.00 | 798.07 |
| 7300 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 3454.36 | 4307.00 | 809.08 |
| 7400 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 3501.68 | 4366.00 | 820.11 |
| 7500 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 3549.00 | 4425.00 | 831.00 |
| 7600 | 9.10 | 3.85 | 12.25 | 6.625 | 16 | 21 | 1000 | 3596.32 | 4484.00 | 842.23 |
| 7700 | 9.10 | 6.83 | 12.25 | 9.5 | 16 | 21 | 1000 | 3643.64 | 4543.00 | 479.26 |
| 7800 | 9.10 | 6.83 | 12.25 | 9.5 | 16 | 21 | 1000 | 3690.96 | 4602.00 | 482.69 |
| 7900 | 9.10 | 6.83 | 12.25 | 9.5 | 16 | 21 | 1000 | 3738.28 | 4661.00 | 486.94 |
| 8000 | 9.10 | 6.83 | 12.25 | 9.5 | 16 | 21 | 1000 | 3785.60 | 4720.00 | 493.80 |
| 8020 | 9.10 | 4.04 | 12.25 | 7 | 16 | 21 | 1000 | 3795.06 | 4731.80 | 881.43 |
| 8040 | 9.10 | 4.04 | 12.25 | 7 | 16 | 21 | 1000 | 3804.53 | 4743.60 | 881.20 |
| 8060 | 9.10 | 3.91 | 12.25 | 6.75 | 16 | 21 | 1000 | 3813.99 | 4755.40 | 892.50 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 8080 | 9.10 | 3.91 | 12.25 | 6.75 | 16 | 21 | 1000 | 3823.46 | 4767.20 | 894.18 |
| 8100 | 9.10 | 3.83 | 12.25 | 6.75 | 16 | 21 | 980 | 3832.92 | 4779.00 | 898.76 |
| 8120 | 9.10 | 3.83 | 12.25 | 6.75 | 16 | 21 | 980 | 3842.38 | 4790.80 | 900.53 |
| 8140 | 9.10 | 3.83 | 12.25 | 6.75 | 16 | 21 | 980 | 3851.85 | 4802.60 | 901.78 |
| 8160 | 9.10 | 4.88 | 12.25 | 8.25 | 16 | 21 | 980 | 3861.31 | 4814.40 | 857.53 |
| 8170 | 9.10 | 4.88 | 12.25 | 8.25 | 16 | 21 | 980 | 3866.04 | 4820.30 | 857.83 |
| 8180 | 9.10 | 4.88 | 12.25 | 8.25 | 16 | 21 | 980 | 3870.78 | 4826.20 | 856.97 |
| 8190 | 9.10 | 4.88 | 12.25 | 8.25 | 16 | 21 | 980 | 3875.51 | 4832.10 | 858.94 |
| 8200 | 9.10 | 4.88 | 12.25 | 8.25 | 16 | 21 | 980 | 3880.24 | 4838.00 | 859.75 |
| 8300 | 9.10 | 8.22 | 8.50 | 5 | 14 | 20 | 950 | 3927.56 | 4897.00 | 511.32 |
| 8400 | 9.10 | 8.22 | 8.50 | 5 | 14 | 20 | 950 | 3974.88 | 4956.00 | 510.88 |
| 8500 | 9.10 | 8.22 | 8.50 | 5 | 14 | 20 | 950 | 4022.20 | 5015.00 | 512.06 |
| 8600 | 9.10 | 7.96 | 8.50 | 5 | 14 | 20 | 920 | 4069.52 | 5074.00 | 554.11 |
| 8700 | 9.10 | 7.96 | 8.50 | 5 | 14 | 20 | 920 | 4116.84 | 5133.00 | 553.82 |
| 8800 | 9.10 | 7.96 | 8.50 | 5 | 14 | 20 | 920 | 4164.16 | 5192.00 | 562.33 |
| 8900 | 9.10 | 7.87 | 8.50 | 5 | 14 | 20 | 910 | 4211.48 | 5251.00 | 579.29 |
| 9000 | 9.10 | 7.87 | 8.50 | 5 | 14 | 20 | 910 | 4258.80 | 5310.00 | 582.89 |
| 9200 | 9.10 | 7.87 | 8.50 | 5 | 14 | 20 | 910 | 4353.44 | 5428.00 | 593.58 |
| 9400 | 9.10 | 7.78 | 8.50 | 5 | 14 | 20 | 900 | 4448.08 | 5546.00 | 619.17 |
| 9600 | 9.10 | 7.78 | 8.50 | 5 | 14 | 20 | 900 | 4542.72 | 5664.00 | 630.64 |
| 9800 | 9.10 | 7.78 | 8.50 | 5 | 14 | 20 | 900 | 4637.36 | 5782.00 | 640.45 |
| 10000 | 9.10 | 7.78 | 8.50 | 5 | 14 | 20 | 900 | 4732.00 | 5900.00 | 652.80 |
| 10200 | 9.10 | 7.87 | 8.50 | 5 | 14 | 20 | 910 | 4826.64 | 6018.00 | 656.15 |
| 10400 | 9.10 | 7.96 | 8.50 | 5 | 14 | 20 | 920 | 4921.28 | 6136.00 | 658.89 |
| 10600 | 9.10 | 7.78 | 8.50 | 5 | 14 | 20 | 900 | 5015.92 | 6254.00 | 688.01 |
| 10800 | 9.10 | 7.78 | 8.50 | 5 | 14 | 20 | 900 | 5110.56 | 6372.00 | 705.73 |
| 11000 | 9.10 | 7.70 | 8.50 | 5 | 14 | 20 | 890 | 5205.20 | 6490.00 | 734.13 |
| 11200 | 9.10 | 7.70 | 8.50 | 5 | 14 | 20 | 890 | 5299.84 | 6608.00 | 737.60 |
| 11400 | 9.10 | 7.70 | 8.50 | 5 | 14 | 20 | 890 | 5394.48 | 6726.00 | 753.76 |
| 11600 | 9.10 | 7.70 | 8.50 | 5 | 14 | 20 | 890 | 5489.12 | 6844.00 | 771.98 |
| 11800 | 9.10 | 7.70 | 8.50 | 5 | 14 | 20 | 890 | 5583.76 | 6962.00 | 782.52 |
| 12000 | 9.10 | 7.70 | 8.50 | 5 | 14 | 20 | 890 | 5678.40 | 7080.00 | 796.19 |
| 12100 | 9.10 | 7.70 | 8.50 | 5 | 14 | 20 | 890 | 5725.72 | 7139.00 | 801.96 |
| 12200 | 9.10 | 7.70 | 8.50 | 5 | 14 | 20 | 890 | 5773.04 | 7198.00 | 805.92 |
| 12300 | 9.10 | 7.70 | 8.50 | 5 | 14 | 20 | 890 | 5820.36 | 7257.00 | 813.50 |
| 12400 | 9.10 | 7.70 | 8.50 | 5 | 14 | 20 | 890 | 5867.68 | 7316.00 | 819.56 |
| 12500 | 9.10 | 7.70 | 8.50 | 5 | 14 | 20 | 890 | 5915.00 | 7375.00 | 826.91 |
| 12600 | 9.10 | 7.70 | 8.50 | 5 | 14 | 20 | 890 | 5962.32 | 7434.00 | 831.58 |
| 12700 | 9.10 | 7.70 | 8.50 | 5 | 14 | 20 | 890 | 6009.64 | 7493.00 | 839.58 |
| 12800 | 9.10 | 7.70 | 8.50 | 5 | 14 | 20 | 890 | 6056.96 | 7552.00 | 845.04 |
| 12900 | 9.10 | 5.45 | 8.50 | 6.5 | 14 | 20 | 400 | 6104.28 | 7611.00 | 721.05 |
| 13000 | 9.10 | 5.45 | 8.50 | 6.5 | 14 | 20 | 400 | 6151.60 | 7670.00 | 722.42 |
| 13100 | 9.10 | 5.45 | 8.50 | 6.5 | 14 | 20 | 400 | 6198.92 | 7729.00 | 719.68 |
| 13200 | 9.10 | 5.45 | 8.50 | 6.5 | 14 | 20 | 400 | 6246.24 | 7788.00 | 710.97 |
| 13220 | 9.10 | 5.45 | 8.50 | 6.5 | 14 | 20 | 400 | 6255.70 | 7799.80 | 698.20 |
| 13240 | 9.10 | 5.45 | 8.50 | 6.5 | 14 | 20 | 400 | 6265.17 | 7811.60 | 685.56 |
| 13260 | 9.10 | 5.45 | 8.50 | 6.5 | 14 | 20 | 400 | 6274.63 | 7823.40 | 685.90 |
| 13280 | 9.10 | 5.45 | 8.50 | 6.5 | 14 | 20 | 400 | 6284.10 | 7835.20 | 685.91 |
| 13300 | 9.10 | 3.22 | 8.50 | 6.75 | 14 | 20 | 210 | 6293.56 | 7847.00 | 732.66 |
| 13350 | 9.10 | 3.06 | 8.50 | 6.75 | 14 | 20 | 200 | 6317.22 | 7876.50 | 743.78 |
| 13400 | 9.10 | 3.06 | 8.50 | 6.75 | 14 | 20 | 200 | 6340.88 | 7906.00 | 748.95 |
| 13420 | 9.10 | 3.06 | 8.50 | 6.75 | 14 | 20 | 200 | 6350.34 | 7917.80 | 746.90 |
| 13440 | 9.10 | 3.06 | 8.50 | 6.75 | 14 | 20 | 200 | 6359.81 | 7929.60 | 744.68 |
| 13460 | 9.10 | 2.72 | 8.50 | 6.50 | 14 | 20 | 200 | 6369.27 | 7941.40 | 951.24 |
| 13480 | 9.10 | 2.72 | 8.50 | 6.50 | 14 | 20 | 200 | 6378.74 | 7953.20 | 948.90 |
| 13490 | 9.10 | 2.72 | 8.50 | 6.50 | 14 | 20 | 200 | 6383.47 | 7959.10 | 944.82 |
| 13500 | 9.10 | 2.72 | 8.50 | 6.50 | 14 | 20 | 200 | 6388.20 | 7965.00 | 936.80 |

## Table 6 - Additional fluid data

| Powel law ($n_a$) | Powel law ($K_a$) | Herschel Buckley ($n_a$) | Herschel Buckley ($K_a$) |
|---|---|---|---|
| 0.61 | 290 | 0.51 | 22 |

**Table 7 - Results from the regression models used in the 121/4" hole section**

| BHCP, Psi | Actual Back Pressure, Psi | Linear Model Back Pressure, Psi | Quadratic Model Back Pressure, Psi | Cubic Model Back Pressure, Psi |
|---|---|---|---|---|
| 1900.58 | 447.62 | 497.90 | 372.11 | 352.86 |
| 1910.13 | 449.87 | 499.40 | 377.55 | 359.28 |
| 1957.88 | 461.12 | 506.89 | 404.27 | 390.62 |
| 2005.64 | 472.36 | 514.37 | 430.15 | 420.68 |
| 2053.39 | 483.61 | 521.86 | 455.21 | 449.49 |
| 2101.14 | 494.86 | 529.35 | 479.43 | 477.07 |
| 2148.90 | 506.10 | 536.84 | 502.83 | 503.42 |
| 2196.65 | 517.35 | 544.32 | 525.39 | 528.58 |
| 2244.40 | 528.60 | 551.81 | 547.13 | 552.55 |
| 2292.16 | 539.84 | 559.30 | 568.03 | 575.35 |
| 2339.91 | 551.09 | 566.79 | 588.10 | 597.01 |
| 2387.66 | 562.34 | 574.28 | 607.35 | 617.54 |
| 2483.17 | 584.83 | 589.25 | 643.35 | 655.26 |
| 2578.68 | 607.32 | 604.23 | 676.03 | 688.68 |
| 2674.18 | 629.82 | 619.20 | 705.39 | 717.92 |
| 2769.69 | 652.31 | 634.18 | 731.42 | 743.12 |
| 2865.20 | 674.80 | 649.15 | 754.14 | 764.44 |
| 3008.46 | 708.54 | 671.62 | 782.00 | 789.42 |
| 3151.72 | 742.28 | 694.08 | 802.38 | 806.42 |
| 3294.98 | 776.02 | 716.54 | 815.29 | 815.93 |
| 3342.73 | 787.27 | 724.03 | 817.94 | 817.51 |
| 3390.48 | 798.52 | 731.52 | 819.75 | 818.34 |
| 3438.24 | 809.76 | 739.01 | 820.73 | 818.41 |
| 3485.99 | 821.01 | 746.49 | 820.89 | 817.76 |
| 3533.74 | 832.26 | 753.98 | 820.21 | 816.40 |
| 3581.50 | 843.50 | 761.47 | 818.71 | 814.34 |
| 3629.25 | 854.75 | 768.96 | 816.37 | 811.61 |
| 4054.16 | 488.84 | 835.58 | 759.04 | 760.37 |
| 4106.81 | 495.19 | 843.84 | 747.35 | 751.00 |
| 4159.47 | 501.53 | 852.09 | 734.66 | 741.05 |
| 4212.12 | 507.88 | 860.35 | 720.97 | 730.53 |
| 3834.62 | 897.18 | 801.16 | 796.86 | 792.59 |
| 3844.19 | 899.41 | 802.66 | 795.58 | 791.43 |
| 3850.43 | 904.97 | 803.64 | 794.73 | 790.66 |
| 3859.98 | 907.22 | 805.14 | 793.39 | 789.46 |
| 3869.47 | 909.53 | 806.62 | 792.03 | 788.25 |
| 3879.02 | 911.78 | 808.12 | 790.63 | 787.01 |
| 3888.58 | 914.02 | 809.62 | 789.19 | 785.75 |
| 3926.81 | 887.59 | 815.61 | 783.12 | 780.46 |
| 3931.62 | 888.68 | 816.37 | 782.31 | 779.77 |
| 3936.43 | 889.77 | 817.12 | 781.50 | 779.07 |
| 3941.25 | 890.85 | 817.88 | 780.68 | 778.37 |
| 3946.06 | 891.94 | 818.63 | 779.85 | 777.67 |

**Table 8 Results from the regression models used in the 81/2" hole section**

| BHCP, Psi | Actual Back Pressure, Psi | Linear Model Back Pressure, Psi | Quadratic Model Back Pressure, Psi | Cubic Model Back Pressure, Psi |
|---|---|---|---|---|
| 4420.30 | 476.70 | 500.24 | 481.14 | 471.97 |
| 4473.55 | 482.45 | 507.88 | 491.32 | 484.42 |
| 4526.81 | 488.19 | 515.52 | 501.41 | 496.56 |
| 4551.90 | 522.10 | 519.12 | 506.13 | 502.17 |
| 4604.83 | 528.17 | 526.71 | 516.01 | 513.80 |
| 4657.76 | 534.24 | 534.30 | 525.79 | 525.14 |
| 4701.14 | 549.86 | 540.52 | 533.74 | 534.23 |
| 4753.96 | 556.04 | 548.09 | 543.33 | 545.05 |
| 4859.60 | 568.40 | 563.24 | 562.22 | 565.91 |
| 4955.24 | 590.76 | 576.96 | 578.99 | 583.95 |
| 5060.67 | 603.33 | 592.07 | 597.12 | 602.95 |
| 5166.10 | 615.90 | 607.19 | 614.86 | 621.08 |
| 5271.53 | 628.47 | 622.31 | 632.21 | 638.42 |
| 5387.82 | 630.18 | 638.99 | 650.91 | 656.69 |
| 5504.63 | 631.37 | 655.74 | 669.23 | 674.23 |
| 5587.82 | 666.18 | 667.67 | 681.98 | 686.26 |
| 5693.26 | 678.74 | 682.79 | 697.81 | 701.02 |

| | | | | |
|---|---|---|---|---|
| 5787.08 | 702.92 | 696.24 | 711.57 | 713.75 |
| 5892.30 | 715.70 | 711.33 | 726.64 | 727.62 |
| 5997.52 | 728.48 | 726.42 | 741.33 | 741.12 |
| 6102.74 | 741.26 | 741.51 | 755.63 | 754.31 |
| 6207.96 | 754.04 | 756.60 | 769.56 | 767.26 |
| 6313.18 | 766.82 | 771.68 | 783.10 | 780.03 |
| 6365.79 | 773.21 | 779.23 | 789.73 | 786.37 |
| 6418.40 | 779.60 | 786.77 | 796.26 | 792.68 |
| 6471.01 | 785.99 | 794.32 | 802.70 | 798.98 |
| 6523.62 | 792.38 | 801.86 | 809.04 | 805.27 |
| 6576.23 | 798.77 | 809.41 | 815.29 | 811.57 |
| 6628.83 | 805.17 | 816.95 | 821.44 | 817.87 |
| 6681.44 | 811.56 | 824.49 | 827.50 | 824.19 |
| 6734.05 | 817.95 | 832.04 | 833.46 | 830.54 |
| 6874.99 | 736.01 | 852.25 | 848.96 | 847.72 |
| 6928.29 | 741.71 | 859.89 | 854.64 | 854.31 |
| 6981.58 | 747.42 | 867.53 | 860.22 | 860.96 |
| 7034.88 | 753.12 | 875.18 | 865.71 | 867.68 |
| 7045.54 | 754.26 | 876.71 | 866.80 | 869.03 |
| 7056.20 | 755.40 | 878.23 | 867.88 | 870.39 |
| 7066.85 | 756.55 | 879.76 | 868.95 | 871.75 |
| 7077.51 | 757.69 | 881.29 | 870.03 | 873.11 |
| 6973.86 | 873.14 | 866.43 | 859.42 | 859.99 |
| 6976.23 | 900.27 | 866.77 | 859.67 | 860.28 |
| 7002.36 | 903.64 | 870.51 | 862.37 | 863.57 |
| 7012.81 | 904.99 | 872.01 | 863.45 | 864.88 |
| 7023.26 | 906.34 | 873.51 | 864.52 | 866.21 |
| 6854.13 | 1087.27 | 849.26 | 846.71 | 845.15 |
| 6864.32 | 1088.88 | 850.72 | 847.81 | 846.40 |
| 6869.41 | 1089.69 | 851.45 | 848.36 | 847.03 |
| 6874.50 | 1090.50 | 852.18 | 848.90 | 847.66 |