

THE GLOBAL ATTRACTOR FOR THE HIGHER-ORDER COUPLED KIRCHHOFF –TYPE EQUATIONS WITH NONLINEAR STRONG DAMPING AND SOURCE TERMS

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ABSTRACT

In this paper, we investigate the global well-posedness and the longtime behavior of the solution for a class of generalized Higher-order coupled kirchhoff–type equations with nonlinear strongly damping and source terms. At first, under the proper assumptions, the existence and uniqueness of solution are demonstrated by using priori estimate and Galerkin method. Then, the existence of the global attractor is acquired by applying some of the attractor theorems.

Keywords: Higher-order coupled kirchhoff equations, The existence and uniqueness, Galerkin method, The Global attractor.

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1. INTRODUCTION

We consider the existence of the global attractor for the following Hinder-order coupled Kirchhoff equations:

$$u_{tt} + M(\|\nabla u\|^2 + \|\nabla v\|^2)(-\Delta)^m u + \beta(-\Delta)^m u_t + g_1(u, v) = f_1(x), \quad (1.1)$$

$$v_{tt} + M(\|\nabla u\|^2 + \|\nabla v\|^2)(-\Delta)^m v + \beta(-\Delta)^m v_t + g_2(u, v) = f_2(x), \quad (1.2)$$

$$u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x), \quad x \in \Omega, \quad (1.3)$$

$$v(x, 0) = v_0(x), \quad v_t(x, 0) = v_1(x), \quad x \in \Omega, \quad (1.4)$$

$$u|_{\partial\Omega} = 0, \quad \frac{\partial^i u}{\partial \mu^i} = 0, \quad i = 1, 2, 3 \dots m-1, \quad (1.5)$$

$$v|_{\partial\Omega} = 0, \quad \frac{\partial^i v}{\partial \nu^i} = 0, \quad i = 1, 2, 3 \dots m-1, \quad (1.6)$$

where $m > 1$ is an integer constant and Ω is a bounded domain of R^n with a smooth Dirichlet boundary $\partial\Omega$ and initial value. μ_i and ν_i are the unit outward normal on $\partial\Omega$, $M(s)$ is a nonnegative C^1 function, $(-\Delta)^m u_t$ and $(-\Delta)^m v_t$ are strongly damping, $g_1(u, v)$ and $g_2(u, v)$ are nonlinear source terms, $f_1(x)$ and $f_2(x)$ are given forcing function.

To better carry out our work, let us go back to G. Kirchhoff [1] and recall some results regarding wave equations and some coupled equations of Kirchhoff type.

In 1883, Kirchhoff posed model: $u_{tt} - \alpha \Delta u - M(\|\nabla u\|^2) \Delta u = f(x, u)$, he investigated the elastic string free vibration.

Ruijin Lou, Penghui Lv and Guoguang Lin [2] studied the existence and uniqueness of the solution for a class of generalized nonlinear Kirchhoff-sin-Gordon equation:

$$u_{tt} - \beta \Delta u_t + au_t - \phi(\|\nabla u\|^2) \Delta u + g(\sin u) = f(x), \quad (1.7)$$

$$u(x, t) = 0, \quad x \in \partial\Omega, \quad t > 0, \quad (1.8)$$

$$u(x, 0) = u_0(x), u_t(x, 0) = u_1(x), x \in \Omega. \quad (1.9)$$

In this case, they also investigated the global attractors of the equation.

Yuting Sun, Yunlong Gao and Guoguang Lin [3] studied the following Higher-order Kirchhoff-type wave equation with nonlinear strongly damping:

$$u_{tt} + \sigma(\|\nabla^m u\|^2)(-\Delta)^m u_t + \phi(\|\nabla^m u\|^2)(-\Delta)^m u = f(x), \quad (1.10)$$

$$u(x, t) = 0, \frac{\partial^i u}{\partial v^i} = 0, i = 1, \dots, m-1, x \in \partial\Omega, t \in (0, +\infty), \quad (1.11)$$

$$u(x, 0) = u_0(x), u_t(x, 0) = u_1(x), x \in \Omega. \quad (1.12)$$

They obtained some main results that are existence and uniqueness of the solution in $H^{2m}(\Omega) \times H_0^m(\Omega)$ and global attractors.

Ling Chen, Wei Wang and Guoguang Lin [4] investigated the following Higher-order Kirchhoff-type equation with nonlinear strongly dissipation and source term:

$$u_{tt} + (-\Delta)^m u_t + M(\left\|(-\Delta)^{\frac{m}{2}} u\right\|^2)(-\Delta)^m u + g(u) = f(x), x \in \Omega, t > 0, m > 1, \quad (1.13)$$

$$u(x, t) = 0, \frac{\partial^i u}{\partial v^i} = 0, i = 1, \dots, m-1, x \in \partial\Omega, x \in \Omega, t > 0, \quad (1.14)$$

$$u(x, 0) = u_0(x), u_t(x, 0) = u_1(x). \quad (1.15)$$

They gained existence and uniqueness of the solution, global attractors and estimation Hausdorff and fractal dimension of the global attractor.

Guoguang Lin and Yunlong Gao [5] investigated a class of strongly damped Higher-order Kirchhoff type equation:

$$u_{tt} + (-\Delta)^m u_t + (\alpha + \beta \|\nabla^m u\|^2)^q (-\Delta)^m u + g(u) = f(x), (x, t) \in \Omega \times [0, +\infty), \quad (1.16)$$

$$u(x, 0) = u_0(x), u_t(x, 0) = u_1(x), x \in \Omega, \quad (1.17)$$

$$u(x, t) = 0, \frac{\partial^i u}{\partial v^i} = 0, i = 1, \dots, m-1, x \in \partial\Omega, t \in (0, +\infty). \quad (1.18)$$

They obtained existence and uniqueness of the solution, global attractors and estimation of the upper bounds of Hausdorff for the global attractors and the existence of a fractal exponential attractor with non-supercritical and critical cases. Here, $g(u)$ of lemma 2.5. satisfying $g(u) \leq 1 + |u|^p$ and $|g'(u)| \leq 1 + |u|^{p-1}$.

Shun-Tang Wu [6] studied a system of viscoelastic wave equations of Kirchhoff type with the nonlinear damping and the source terms in bounded domain:

$$u_{tt} - M(\|\nabla u\|^2 + \|\nabla v\|^2) \Delta u + \int_0^t g(t-s) \Delta u(s) ds + |u_t|^{p-1} u_t = f_1(u, v) \text{ in } \Omega \times [0, \infty), \quad (1.19)$$

$$v_{tt} - M(\|\nabla u\|^2 + \|\nabla v\|^2) \Delta v + \int_0^t h(t-s) \Delta v(s) ds + |v_t|^{p-1} v_t = f_2(u, v) \text{ in } \Omega \times [0, \infty), \quad (1.20)$$

$$u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x), x \in \Omega, \quad (1.21)$$

$$v(x, 0) = v_0(x), \quad v_t(x, 0) = v_1(x), x \in \Omega, \quad (1.22)$$

$$u(x, t) = v(x, t) = 0, x \in \partial\Omega, t > 0. \quad (1.23)$$

He proved that, under suitable conditions on the nonlinearity of the damping and the source terms and certain initial data in the stable set and for a wider class of relaxation functions, the decay estimates of the energy function was exponential or polynomial depending on the exponents of the damping terms in both equations and for certain initial data in the unstable

set, they obtained the blow up of solutions in finite time when the initial energy was nonnegative conversely.

Yaojun Ye [7] studied the global existence and energy decay of solutions for coupled system of higher-order Kirchhoff-type equations with nonlinear dissipative and source terms in a bounded domain:

$$u_{tt} + \Phi(\|D^m u\|^2 + \|D^m v\|^2)(-\Delta)^{m_1} u + a|u_t|^{q-2} u_t = f_1(u, v), \quad x \in \Omega, t > 0, \quad (1.24)$$

$$v_{tt} + \Phi(\|D^m u\|^2 + \|D^m v\|^2)(-\Delta)^{m_2} v + a|v_t|^{q-2} v_t = f_2(u, v), \quad x \in \Omega, t > 0, \quad (1.25)$$

$$u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x), \quad x \in \Omega, \quad (1.26)$$

$$v(x, 0) = v_0(x), \quad v_t(x, 0) = v_1(x), \quad x \in \Omega, \quad (1.27)$$

$$\frac{\partial^i u}{\partial \mu^i} = 0, \quad i = 1, 2, 3 \dots m_1 - 1, \quad x \in \Omega, t > 0, \quad (1.28)$$

$$\frac{\partial^j v}{\partial \nu^j} = 0, \quad j = 1, 2, 3 \dots m_2 - 1, \quad x \in \Omega, t > 0. \quad (1.29)$$

He proved the existence of the global solutions for this problem by constructing a stable set in $H_0^{m_1}(\Omega) \times H_0^{m_2}(\Omega)$ and gave the decay estimate of the global solutions by applying a lemma of V. Komornik.

Motivated by previous researches, it is interesting to investigate the global existence and uniqueness of the solutions, the existence of the global attractor and the finite-dimension Hausdorff for the global attractors. At first, we prove that, under suitable assumptions on the functions, $M(s), g_i(u, v)$ and $f_i(x), i = 1, 2$, the solutions are existence and uniqueness. After that, the existence of the global attractor is obtained. In this case, the estimation of the upper bounds of Hausdorff for the global attractors is obtained.

The main details of this paper are scheduled as follow:

In section 2, under some assumptions of Lemma 2.4. and Lemma 2.5., we prove the existence of solutions; In section 3 we get the global attractor of the problem (1.1)-(1.6).

2. PRELIMINARIES

We denote the some simple symbol, $\|\cdot\|$ stands for norm, (\cdot) represents inner product and $H^m = H^m(\Omega), H_0^m = H^m(\Omega) \cap H_0^1(\Omega), H_0^{2m} = H^{2m}(\Omega) \cap H_0^1(\Omega), f_i = f_i(x) (i = 1, 2)$, $H = L^2, \|\cdot\| = \|\cdot\|_{L^2}, \|\cdot\|_\infty = \|\cdot\|_{L^\infty}, \nu = \|\nabla u\|^2 + \|\nabla v\|^2, c_i (i = 1, \dots), \mu_i (i = 0, 1)$ are constants. λ_1^m is the first eigenvalue of the operator $(-\Delta)^m$.

Next, we give some assumptions needed for problem (1.1)-(1.6).

(H1) $M(v) : R^+ \rightarrow R^+$ is a differentiable function;

$$(H2) \quad g_1(u, v)u + g_2(u, v)v \geq \varepsilon G(u, v) + \xi u^2 + \eta v^2, \quad (2.1)$$

$$J(u, v) = \int_{\Omega} G(u, v) dx, \text{ where } (G(u, v))_t = g_1(u, v)u_t + g_2(u, v)v_t, G(u, v) \geq 0. \quad (2.2)$$

$$(H3) \quad 0 \leq \mu_0 \leq M(v) \leq \mu_1, \quad \mu = \begin{cases} \mu_0, & \frac{d}{dt} (\|(-\Delta)^\gamma u\|^2 + \|(-\Delta)^\gamma v\|^2) \geq 0, \\ \mu_1, & \frac{d}{dt} (\|(-\Delta)^\gamma u\|^2 + \|(-\Delta)^\gamma v\|^2) < 0, \end{cases} \quad (2.3)$$

where $\gamma = \frac{m}{2}$ or $\gamma = m$.

$$(H4) |g_i(u, v)| \leq C(1 + |u|^{r_1} + |v|^{r_2}), \quad (2.4)$$

$$|g_{iu}(u, v)| \leq C(1 + |u|^{r_1-1}), \quad (2.5)$$

$$|g_{iv}(u, v)| \leq C(1 + |v|^{r_2-1}), \quad (2.6)$$

where $1 \leq r_i \leq \frac{n}{n-2m}$ ($n > 2m$) and $1 \leq r_i \leq +\infty$ ($n \leq 2m$), ($i = 1, 2$).

$$(H5) 0 \leq S_0 \leq \frac{dM(v)}{dv} \leq S. \quad (2.7)$$

Lemma 2.1. (Young's Inequality^(Lin,G.G.,2011)) For any $\varepsilon > 0$ and $a, b \geq 0$, then

$$ab \leq \frac{\varepsilon^p}{p} a^p + \frac{1}{q\varepsilon^q}, \quad (2.8)$$

where $\frac{1}{p} + \frac{1}{q} = 1$, $p > 1$, $q > 1$.

Lemma 2.2. (Gronwall's inequality^(Lin,G.G.,2011)) If $\forall t \in [t_0, +\infty)$, $y(t) \geq 0$ and $\frac{dy}{dt} + gy \leq h$, such that

$$y(t) \leq y(t_0)e^{-g(t-t_0)} + \frac{h}{g}, \quad t \geq t_0, \quad (2.9)$$

where $g \geq 0$, $h \geq 0$ are constants.

Lemma 2.3. Assume (H1)-(H3) hold, and $(u_0, u_1), (v_0, v_1) \in H^m(\Omega) \times L^2(\Omega)$, $f_i(x) \in L^2(\Omega)$, ($i = 1, 2$). Then the solutions (u, p) and (v, q) of the problem (1.1)-(1.6) satisfy $(u, p), (v, q) \in L^\infty((0, +\infty); H_0^m(\Omega) \times L^2(\Omega))$ and

$$\|(u, p)\|_{H^m \times L^2}^2 = \|\nabla^m u\|^2 + \|p\|^2 \leq \frac{1}{\min\{1, \mu_0\}} \varphi(0) e^{-at} + \frac{C_1}{a \min\{1, \mu_0\}}, \quad (2.10)$$

$$\|(v, q)\|_{H^m \times L^2}^2 = \|\nabla^m v\|^2 + \|q\|^2 \leq \frac{1}{\min\{1, \mu_0\}} \varphi(0) e^{-at} + \frac{C_1}{a \min\{1, \mu_0\}}, \quad (2.11)$$

where $p = u_t + \varepsilon u$, $q = v_t + \varepsilon v$.

$$\varphi(0) = \|p_0\|^2 + \|q_0\|^2 + \mu(\|\nabla^m u_0\|^2 + \|\nabla^m v_0\|^2) + 2J(u_0, v_0). \quad (2.12)$$

Thus, there exist $t = t_0 > 0$ and R_0 , such that

$$\overline{\lim}_{t \rightarrow \infty} \|(u, p)\|^2 = \|\nabla^m u\|^2 + \|p\|^2 \leq R_0^2, \quad t > t_0. \quad (2.13)$$

$$\overline{\lim}_{t \rightarrow \infty} \|(v, q)\|^2 = \|\nabla^m v\|^2 + \|q\|^2 \leq R_0^2, \quad t > t_0. \quad (2.14)$$

Proof. Let $p = u_t + \varepsilon u$, we use p to multiply both sides of equation (1.1) in H and obtain

$$(u_t + M(v)(-\Delta)^m u + \beta(-\Delta)^m u_t + g_1(u, v), p) = (f_1(x), p). \quad (2.15)$$

By using Poincare's inequality, Holder inequality and Young's inequality, after a computation in (2.13) one by one, as follows:

$$\begin{aligned}
(u_{tt}, p) &= (p_t - \varepsilon u_t, p) \\
&= \frac{1}{2} \frac{d}{dt} \|p\|^2 - \varepsilon \|p\|^2 + \varepsilon^2 (u, p) \\
&\geq \frac{1}{2} \frac{d}{dt} \|p\|^2 - \varepsilon \|p\|^2 - \frac{\varepsilon^2}{2} \|p\|^2 - \frac{\varepsilon^2}{2} \|u\|^2 \\
&\geq \frac{1}{2} \frac{d}{dt} \|p\|^2 - \frac{2\varepsilon + \varepsilon^2}{2} \|p\|^2 - \frac{\varepsilon^2}{2\lambda_1^m} \|\nabla^m u\|^2. \tag{2.16}
\end{aligned}$$

$$\begin{aligned}
(M(v)(-\Delta)^m u, p) &= M(v)((-\Delta)^m u, u_t + \varepsilon u) \\
&\geq \frac{1}{2} M(v) \frac{d}{dt} \|\nabla^m u\|^2 + \varepsilon \mu_0 \|\nabla^m u\|^2. \tag{2.17}
\end{aligned}$$

$$\begin{aligned}
(\beta(-\Delta)^m u_t, p) &= \beta((-\Delta)^m p, p) - \beta \varepsilon ((-\Delta)^m u, p) \\
&\geq \frac{\beta}{2} \lambda_1^m \|p\|^2 - \frac{\beta \varepsilon^2}{2} \|\nabla^m u\|^2. \tag{2.18}
\end{aligned}$$

$$\begin{aligned}
(g_1(u, v), p) &= (g_1(u, v), u_t + \varepsilon u) \\
&= \int_{\Omega} g_1(u, v) u_t dx + \varepsilon \int_{\Omega} g_1(u, v) u dx. \tag{2.19}
\end{aligned}$$

Substituting (2.17)-(2.20) into (2.16), we obtain

$$\begin{aligned}
&\frac{d}{dt} \|p\|^2 + M(v) \frac{d}{dt} \|\nabla^m u\|^2 + 2 \int_{\Omega} g_1(u, v) u_t dx + (\beta \lambda_1^m - 2\varepsilon - 2\varepsilon^2) \|p\|^2 \\
&\quad + (2\varepsilon \mu_0 - \varepsilon^2 \lambda_1^{-m} - \beta \varepsilon^2) \|\nabla^m u\|^2 + 2\varepsilon \int_{\Omega} g_1(u, v) u dx \leq \frac{1}{\varepsilon^2} \|f_1(x)\|^2. \tag{2.20}
\end{aligned}$$

Let $q = v_t + \varepsilon v$, we use q to multiply both sides of equation (1.2) in H and obtain

$$\begin{aligned}
&\frac{d}{dt} \|q\|^2 + M(v) \frac{d}{dt} \|\nabla^m v\|^2 + 2 \int_{\Omega} g_2(u, v) v_t dx + (\beta \lambda_1^m - 2\varepsilon - 2\varepsilon^2) \|q\|^2 \\
&\quad + (2\varepsilon \mu_0 - \varepsilon^2 \lambda_1^{-m} - \beta \varepsilon^2) \|\nabla^m v\|^2 + 2\varepsilon \int_{\Omega} g_2(u, v) v dx \leq \frac{1}{\varepsilon^2} \|f_2(x)\|^2. \tag{2.21}
\end{aligned}$$

Summing up (2.21) and (2.22), we obtain

$$\begin{aligned}
&\frac{d}{dt} (\|p\|^2 + \|q\|^2) + M(v) \frac{d}{dt} (\|\nabla^m u\|^2 + \|\nabla^m v\|^2) + 2 \int_{\Omega} g_1(u, v) u_t + g_2(u, v) v_t dx \\
&\quad + (\beta \lambda_1^m - 2\varepsilon - 2\varepsilon^2) (\|p\|^2 + \|q\|^2) + (2\varepsilon \mu_0 - \varepsilon^2 \lambda_1^{-m} - \beta \varepsilon^2) (\|\nabla^m u\|^2 + \|\nabla^m v\|^2) \\
&\quad + 2\varepsilon \int_{\Omega} g_1(u, v) u + g_2(u, v) v dx \leq C_1. \tag{2.22}
\end{aligned}$$

According to (H2)-(H3), we have

$$\begin{aligned}
&\frac{d}{dt} (\|p\|^2 + \|q\|^2 + \mu (\|\nabla^m u\|^2 + \|\nabla^m v\|^2) + 2J(u, v)) \\
&\quad + (\beta \lambda_1^m - 2\varepsilon - 2\varepsilon^2) (\|p\|^2 + \|q\|^2) \\
&\quad + (2\varepsilon \mu_0 - \varepsilon^2 \lambda_1^{-m} - \beta \varepsilon^2) (\|\nabla^m u\|^2 + \|\nabla^m v\|^2) + 2\varepsilon^2 J(u, v) \leq C_1. \tag{2.23}
\end{aligned}$$

There exists ε , such that

$$\begin{cases} a_1 = \beta\lambda_1^m - 2\varepsilon - 2\varepsilon^2 > 0, \\ a_2 = \frac{2\varepsilon\mu_0 - \varepsilon^2\lambda_1^{-m} - \beta\varepsilon^2}{\mu} > 0. \end{cases} \quad (2.24)$$

$$a = \min\{a_1, a_2, \varepsilon^2\}. \quad (2.25)$$

Then we obtain

$$\frac{d}{dt}\varphi(t) + a\varphi(t) \leq C_1, \quad (2.26)$$

where

$$\varphi(t) = \|p\|^2 + \|q\|^2 + \mu(\|\nabla^m u\|^2 + \|\nabla^m v\|^2) + 2J(u, v). \quad (2.27)$$

By using Gronwall's inequality, we have

$$\varphi(t) \leq \varphi(0)e^{-at} + \frac{C_1}{a}, \quad \forall t \geq 0, \quad (2.28)$$

where

$$\varphi(0) = \|p_0\|^2 + \|q_0\|^2 + \mu(\|\nabla^m u_0\|^2 + \|\nabla^m v_0\|^2) + 2J(u_0, v_0). \quad (2.29)$$

So, we have

$$\|(u, p)\|_{H_0^m \times L^2}^2 = \|\nabla^m u\|^2 + \|p\|^2 \leq \frac{1}{\min\{1, \mu_0\}} \varphi(0)e^{-at} + \frac{C_1}{a \min\{1, \mu_0\}}. \quad (2.30)$$

$$\|(v, q)\|_{H_0^m \times L^2}^2 = \|\nabla^m v\|^2 + \|q\|^2 \leq \frac{1}{\min\{1, \mu_0\}} \varphi(0)e^{-at} + \frac{C_1}{a \min\{1, \mu_0\}}. \quad (2.31)$$

Then

$$\overline{\lim}_{t \rightarrow \infty} \|(u, p)\|_{H_0^m \times L^2}^2 = \|\nabla^m u\|^2 + \|p\|^2 \leq \frac{C_1}{a \min\{1, \mu_0\}}. \quad (2.32)$$

$$\overline{\lim}_{t \rightarrow \infty} \|(v, q)\|_{H_0^m \times L^2}^2 = \|\nabla^m v\|^2 + \|q\|^2 \leq \frac{C_1}{a \min\{1, \mu_0\}}. \quad (2.33)$$

Thus, there exists t_0 and R_0 , such that

$$\|(u, p)\|_{H_0^m \times L^2}^2 = \|\nabla^m u\|^2 + \|p\|^2 \leq R_0^2, \quad t > t_0. \quad (2.34)$$

$$\|(v, q)\|_{H_0^m \times L^2}^2 = \|\nabla^m v\|^2 + \|q\|^2 \leq R_0^2, \quad t > t_0. \quad (2.35)$$

Lemma 2.4. Assume (H1), (H3)-(H5) hold, $(u_0, u_1), (v_0, v_1) \in H^{2m}(\Omega) \times H^m(\Omega)$ and $f_i(x) \in H^m(\Omega), (i=1, 2)$. Then the solutions (u, p) and (v, q) of the problem (1.1)-(1.6) satisfy $(u, p), (v, q) \in L^\infty((0, +\infty); H_0^{2m}(\Omega) \times H_0^m(\Omega))$ and

$$\|(u, p)\|_{H_0^{2m} \times H_0^m}^2 = \|(-\Delta)^m u\|^2 + \|\nabla^m p\|^2 \leq \frac{W(0)e^{-bt}}{\min\{1, \mu_0\}} + \frac{C_{13}}{b \min\{1, \mu_0\}}, \quad (2.36)$$

$$\|(v, q)\|_{H_0^{2m} \times H_0^m}^2 = \|(-\Delta)^m v\|^2 + \|\nabla^m q\|^2 \leq \frac{W(0)e^{-bt}}{\min\{1, \mu_0\}} + \frac{C_{13}}{\min\{1, \mu_0\}}, \quad (2.37)$$

where $p = u_t + \varepsilon u$, $q = v_t + \varepsilon v$ and

$$W(0) = \|\nabla^m p_0\|^2 + \|\nabla^m q_0\|^2 + \mu(\|(-\Delta)^m u_0\|^2 + \|(-\Delta)^m v_0\|^2). \quad (2.38)$$

Thus, there exist t_1 and R_1 , such that

$$\|(u, p)\|_{H_0^{2m} \times H_0^m} = \|(-\Delta)^m u\|^2 + \|\nabla^m p\|^2 \leq R_1^2, t > t_1. \quad (2.39)$$

$$\|(v, q)\|_{H_0^{2m} \times H_0^m} = \|(-\Delta)^m v\|^2 + \|\nabla^m q\|^2 \leq R_1^2, t > t_1. \quad (2.40)$$

Proof. Let $(-\Delta)^m p = (-\Delta)^m u_t + \varepsilon(-\Delta)^m u$, we use $(-\Delta)^m p$ to multiply both sides of equation (1.1) in H and obtain

$$(u_t + M(v)(-\Delta)^m u + \beta(-\Delta)^m u_t + g_1(u, v), (-\Delta)^m p) = (f_1(x), (-\Delta)^m p). \quad (2.41)$$

By using Poincare's inequality, Holder inequality and Young's inequality, after a computation in (2.14) one by one, as follows:

$$\begin{aligned} (u_t, (-\Delta)^m p) &= (p_t - \varepsilon u_t, (-\Delta)^m p) \\ &= (\nabla^m p_t, \nabla^m p) - \varepsilon(\nabla^m p, \nabla^m p) + \varepsilon^2(\nabla^m u, \nabla^m p) \quad (2.42) \\ &\geq \frac{1}{2} \frac{d}{dt} \|\nabla^m p\|^2 - (\varepsilon + \frac{\varepsilon^2}{2}) \|\nabla^m p\|^2 - C_2(R_0). \end{aligned}$$

$$\begin{aligned} (M(v)(-\Delta)^m u, (-\Delta)^m p) &= M(v)((-\Delta)^m u, (-\Delta)^m u_t + \varepsilon(-\Delta)^m u) \\ &\geq \frac{1}{2} M(v) \frac{d}{dt} \|(-\Delta)^m u\|^2 + \varepsilon \mu_0 \|(-\Delta)^m u\|^2. \quad (2.43) \end{aligned}$$

$$\begin{aligned} (\beta(-\Delta)^m u_t, (-\Delta)^m p) &= \beta((- \Delta)^m (p - \varepsilon u), (-\Delta)^m p) \\ &= \beta \|(-\Delta)^m p\|^2 - \varepsilon \beta ((-\Delta)^m u, (-\Delta)^m p) \quad (2.44) \\ &\geq \frac{\beta}{2} \|(-\Delta)^m p\|^2 - \frac{\beta \varepsilon^2}{2} \|(-\Delta)^m u\|^2. \end{aligned}$$

$$(g_1(u, v), (-\Delta)^m p) \geq -\frac{\|g_1(u, v)\|^2}{2\varepsilon^2} - \frac{\varepsilon^2 \|(-\Delta)^m p\|^2}{2}. \quad (2.45)$$

Next, in order to estimate $\|g_1(u, v)\|^2$ in (2.46). By (H4) and Young's inequality, we have

$$\begin{aligned} \|g_1(u, v)\|^2 &= \int_{\Omega} |g_1(u, v)|^2 dx \\ &\leq \int_{\Omega} C_3 (1 + |u|^{2r_1} + |v|^{2r_2}) \\ &= C_4 + C_3 \|u\|_{L^{2r_1}}^{2r_1} + C_3 \|v\|_{L^{2r_2}}^{2r_2}. \end{aligned} \quad (2.46)$$

By $1 \leq r_i \leq \frac{n}{n-2m}$ ($n > 2m$) and $1 \leq r_i \leq +\infty$ ($n \leq 2m$), ($i = 1, 2$). So, there exists

$c_i > 0$ ($i = 5, 6$), such that

$$\|u\|_{L^{2r_1}}^{2r_1} \leq C_5 \|\nabla^m u\|^{\frac{m(r_1-1)}{m}} \|u\|^{\frac{2mr_1-nr_1+n}{m}}, \|v\|_{L^{2r_2}}^{2r_2} \leq C_6 \|\nabla^m v\|^{\frac{m(r_2-1)}{m}} \|v\|^{\frac{2mr_2-nr_2+n}{m}}. \quad (2.47)$$

Thus

$$\begin{aligned} (g_1(u, v), (-\Delta)^m p) &\geq -\frac{C_4 + C_3 \|u\|_{L^{2r_1}}^{2r_1} + C_3 \|v\|_{L^{2r_2}}^{2r_2}}{2\varepsilon^2} - \frac{\varepsilon^2 \|(-\Delta)^m p\|^2}{2} \\ &\geq -\frac{C_4 + C_5 \|\nabla^m u\|^{\frac{m(r_1-1)}{m}} \|u\|^{\frac{2mr_1-nr_1+n}{m}} + C_6 \|\nabla^m v\|^{\frac{m(r_2-1)}{m}} \|v\|^{\frac{2mr_2-nr_2+n}{m}}}{2\varepsilon^2} - \frac{\varepsilon^2 \|(-\Delta)^m p\|^2}{2} \\ &= C_7(R_0) - \frac{\varepsilon^2 \|(-\Delta)^m p\|^2}{2}. \quad (2.48) \end{aligned}$$

From above, we obtain

$$\begin{aligned} & \frac{d}{dt} \|\nabla^m p\|^2 + M(v) \frac{d}{dt} \|(-\Delta)^m u\|^2 + (\beta - \varepsilon^2) \|(-\Delta)^m p\|^2 - (2\varepsilon + 2\varepsilon^2) \|\nabla^m p\|^2 \\ & + (2\varepsilon\mu_0 - \varepsilon^2\beta) \|(-\Delta)^m u\|^2 \leq C_8. \end{aligned} \quad (2.49)$$

Let $(-\Delta)^m q = (-\Delta)^m v_t + \varepsilon(-\Delta)^m v$, we use $(-\Delta)^m q$ to multiply both sides of equation (1.2) in H and obtain

$$\begin{aligned} & \frac{d}{dt} \|\nabla^m q\|^2 + M(v) \frac{d}{dt} \|(-\Delta)^m v\|^2 + (\beta - \varepsilon^2) \|(-\Delta)^m q\|^2 - (2\varepsilon + 2\varepsilon^2) \|\nabla^m q\|^2 \\ & + (2\varepsilon\mu_0 - \varepsilon^2\beta) \|(-\Delta)^m v\|^2 \leq C_9. \end{aligned} \quad (2.50)$$

Summing up (2.50) and (2.51), we obtain

$$\begin{aligned} & \frac{d}{dt} (\|\nabla^m p\|^2 + \|\nabla^m q\|^2 + \mu(\|(-\Delta)^m u\|^2 + \|(-\Delta)^m v\|^2)) + ((\beta - \varepsilon^2)\lambda_1^m - 2\varepsilon \\ & - 2\varepsilon^2)(\|\nabla^m p\|^2 + \|\nabla^m q\|^2) + (2\varepsilon\mu_0 - \varepsilon^2\beta)(\|(-\Delta)^m u\|^2 + \|(-\Delta)^m v\|^2) \leq C_{10}. \end{aligned} \quad (2.51)$$

There exists ε , such that

$$\begin{cases} b_1 = (\beta - \varepsilon^2)\lambda_1^m - 2\varepsilon - 2\varepsilon^2 \geq 0, \\ b_2 = \frac{2\varepsilon\mu_0 - \varepsilon^2\beta}{\mu}. \end{cases} \quad (2.52)$$

$$b = \min\{b_1, b_2\}. \quad (2.53)$$

Then, we gain

$$\frac{d}{dt} W(t) + bW(t) \leq C_{10}, \quad (2.54)$$

where

$$W(t) = \|\nabla^m p\|^2 + \|\nabla^m q\|^2 + \mu(\|(-\Delta)^m u\|^2 + \|(-\Delta)^m v\|^2). \quad (2.55)$$

By using Gronwall's inequality, we can obtain

$$W(t) \leq W(0)e^{-bt} + \frac{C_{10}}{b}, \quad (2.56)$$

where

$$W(0) = \|\nabla^m p_0\|^2 + \|\nabla^m q_0\|^2 + \mu(\|(-\Delta)^m u_0\|^2 + \|(-\Delta)^m v_0\|^2). \quad (2.57)$$

Therefore, we can get

$$\|\nabla^m p\|^2 + \|(-\Delta)^m u\|^2 \leq \frac{W(0)e^{-bt}}{\min\{1, \mu_0\}} + \frac{C_{10}}{b \min\{1, \mu_0\}}. \quad (2.58)$$

$$\|\nabla^m q\|^2 + \|(-\Delta)^m v\|^2 \leq \frac{W(0)e^{-bt}}{\min\{1, \mu_0\}} + \frac{C_{10}}{b \min\{1, \mu_0\}}. \quad (2.59)$$

Then

$$\overline{\lim}_{t \rightarrow \infty} \|(u, p)\|_{H_0^{2m} \times H_0^m}^2 = \|(-\Delta)^m u\|^2 + \|\nabla^m p\|^2 \leq \frac{C_{10}}{b \min\{1, \mu_0\}}. \quad (2.60)$$

$$\overline{\lim}_{t \rightarrow \infty} \|(v, q)\|_{H_0^{2m} \times H_0^m}^2 = \|(-\Delta)^m v\|^2 + \|\nabla^m q\|^2 \leq \frac{C_{10}}{b \min\{1, \mu_0\}}. \quad (2.61)$$

So, there exists t_1 and R_1 , such that

$$\|(u, p)\|_{H_0^{2m} \times H_0^m}^2 = \|(-\Delta)^m u\|^2 + \|\nabla^m p\|^2 \leq R_1^2, t > t_2. \quad (2.62)$$

$$\|(v, q)\|_{H_0^{2m} \times H_0^m}^2 = \|(-\Delta)^m v\|^2 + \|\nabla^m q\|^2 \leq R_1^2, t > t_1. \quad (2.63)$$

3. GLOBAL ATTRACTOR

3.1. The Existence And Uniqueness Of Solution

Theorem 3.1. Assume (H1), (H4)-(H5) hold, and $(u_0, u_1), (v_0, v_1) \in H^{2m}(\Omega) \times H^m(\Omega)$,

$f_i(x) \in H^m(\Omega), (i=1, 2), p = u_t + \varepsilon u, q = v_t + \varepsilon v$. So Equations exist a unique smooth

Solution:

$$(u, p), (v, q) \in L^\infty((0, +\infty); H_0^{2m}(\Omega) \times H_0^m(\Omega)). \quad (3.1)$$

Proof. According to Galerkin method, Lemma2.4. and 2.5., we obtain the existence of solutions, the procedure is omitted. Next, we prove the uniqueness of solutions in detail.

Assume (u_1, v_1) and (u_2, v_2) are two solutions of the problem (1.1)-(1.6). Let $u = u_1 - u_2, v = v_1 - v_2$, then $u(x, 0) = u_0(x) = 0, v(x, 0) = v_0(x) = 0, u_t(x, 0) = u_1(x) = 0,$

$v_t(x, 0) = v_1(x) = 0$ and two equations subtract respectively and obtain

$$u_{tt} + M(v_1)(-\Delta)^m u_1 - M(v_2)(-\Delta)^m u_2 + \beta(-\Delta)^m u_t + g_1(u_1, v_1) - g_1(u_2, v_2) = 0, \quad (3.2)$$

$$v_{tt} + M(v_1)(-\Delta)^m v_1 - M(v_2)(-\Delta)^m v_2 + \beta(-\Delta)^m v_t + g_2(u_1, v_1) - g_2(u_2, v_2) = 0, \quad (3.3)$$

where $\nu_1 = \|\nabla u_1\|^2 + \|\nabla v_1\|^2, \nu_2 = \|\nabla u_2\|^2 + \|\nabla v_2\|^2$.

By multiplying (3.2) by u_t in H , we get

$$(u_{tt} + M(v_1)(-\Delta)^m u_1 - M(v_2)(-\Delta)^m u_2 + \beta(-\Delta)^m u_t + g_1(u_1, v_1) - g_1(u_2, v_2), u_t) = 0. \quad (3.4)$$

After a computation in (3.4) one by one, as follows

$$\begin{aligned} & (M(v_1)(-\Delta)^m u_1 - M(v_2)(-\Delta)^m u_2, u_t) \\ &= (M(v_1)(-\Delta)^m u_1 - M(v_1)(-\Delta)^m u_2 + M(v_1)(-\Delta)^m u_2 - M(v_2)(-\Delta)^m u_2, u_t) \\ &\quad = M(v_1)((-\Delta)^m u, u_t) + (M(v_1)(-\Delta)^m u_2 - M(v_2)(-\Delta)^m u_2, u_t) \\ &\quad = M(v_1) \frac{1}{2} \frac{d}{dt} \|\nabla^m u\|^2 + (M(v_1)(-\Delta)^m u_2 - M(v_2)(-\Delta)^m u_2, u_t). \end{aligned} \quad (3.5)$$

By (H5), Holder inequality and Young's inequality, we have

$$\begin{aligned} & (M(v_1)(-\Delta)^m u_2 - M(v_2)(-\Delta)^m u_2, u_t) \\ &= M'(\xi)(v_1 - v_2)((-\Delta)^m u_2, u_t) \\ &= M'(\xi)(\|\nabla u_1\| - \|\nabla u_2\|)(\|\nabla u_1\| + \|\nabla u_2\|) + (\|\nabla v_1\| - \|\nabla v_2\|)(\|\nabla v_1\| + \|\nabla v_2\|)((-\Delta)^m u_2, u_t) \\ &\leq M'(\xi)(\|\nabla u\|(\|\nabla u_1\| + \|\nabla u_2\|) + \|\nabla v\|(\|\nabla v_1\| + \|\nabla v_2\|))\|(-\Delta)^m u_2\| \|u_t\| \\ &= M'(\xi)\|\nabla u\|(\|\nabla u_1\| + \|\nabla u_2\|)\|(-\Delta)^m u_2\| \|u_t\| + M'(\xi)\|\nabla v\|(\|\nabla v_1\| + \|\nabla v_2\|)\|(-\Delta)^m u_2\| \|u_t\|. \quad \text{By} \\ &\leq M'(\xi)\|(-\Delta)^m u_2\|(\|\nabla u_1\| + \|\nabla u_2\|) \frac{1}{2} (\|\nabla u\|^2 + \|u_t\|^2) \\ &\quad + M'(\xi)\|(-\Delta)^m u_2\|(\|\nabla v_1\| + \|\nabla v_2\|) \frac{1}{2} (\|\nabla v\|^2 + \|u_t\|^2) \\ &\leq C_{11}(\|\nabla u\|^2 + \|u_t\|^2) + C_{12}(\|\nabla v\|^2 + \|u_t\|^2) \\ &\leq C_{13}(\|\nabla u\|^2 + \|\nabla v\|^2 + \|u_t\|^2). \end{aligned} \quad (3.6)$$

using Poincare inequality, (H5), Lemma2.4. and Lemma2.5., we can get

$$\begin{aligned}
& (M(v_1)(-\Delta)^m u_1 - M(v_2)(-\Delta)^m u_2, u_t) \\
& \geq M(v_1) \frac{1}{2} \frac{d}{dt} \|\nabla^m u\|^2 - C_{13} (\|\nabla u\|^2 + \|\nabla v\|^2 + \|u_t\|^2) \\
& \geq \frac{1}{2} \frac{d}{dt} M(v_1) \|\nabla^m u\|^2 - M'(v_1) \|\nabla^m u\|^2 ((\nabla u_1, \nabla u_{1t}) + (\nabla v_1, \nabla v_{1t})) \\
& \quad - C_{13} \lambda_1^{1-m} (\|\nabla^m u\|^2 + \|\nabla^m v\|^2) - C_{13} \|u_t\|^2 \\
& \geq \frac{1}{2} \frac{d}{dt} M(v_1) \|\nabla^m u\|^2 - M'(v_1) \|\nabla^m u\|^2 (\|\nabla u_1\| \|\nabla u_{1t}\| + \|\nabla v_1\| \|\nabla v_{1t}\|) \\
& \quad - C_{13} \lambda_1^{1-m} (\|\nabla^m u\|^2 + \|\nabla^m v\|^2) - C_{13} \|u_t\|^2 \\
& \geq \frac{1}{2} \frac{d}{dt} M(v_1) \|\nabla^m u\|^2 - C_{14} \|\nabla^m u\|^2 - C_{13} \lambda_1^{1-m} (\|\nabla^m u\|^2 + \|\nabla^m v\|^2) - C_{13} \|u_t\|^2
\end{aligned} \tag{3.7}$$

By (H4), Lemma2., Lemma2.4. and Lemma2.5., we can get

$$\begin{aligned}
& (g_1(u_1, v_1) - g_1(u_2, v_2), u_t) \\
& = (g_1(u_1, v_1) - g_1(u_1, v_2) + g_1(u_1, v_2) - g_1(u_2, v_2), u_t) \\
& \leq \|g_1(u_1, v_1) - g_1(u_1, v_2) + g_1(u_1, v_2) - g_1(u_2, v_2)\| \|u_t\| \\
& \leq \|g_{1v}(u_1, sv_1 + (1-s)v_2)(v_1 - v_2) + g_{1u}(su_1 + (1-s)u_2, v_2)(u_1 - u_2)\| \|u_t\| \\
& \leq \|C(1 + |sv_1 + (1-s)v_2|^{r_2-1})(v_1 - v_2) + C(1 + |su_1 + (1-s)u_2|^{r_1-1})(u_1 - u_2)\| \|u_t\| \tag{3.8} \\
& \leq C \| [1 + (|v_1| + |v_2|)^{r_2-1}] |v| + [1 + (|u_1| + |u_2|)^{r_1-1}] |u| \| \|u_t\| \\
& \leq C_{15} \|u\| \|u_t\| + C_{16} \|v\| \|u_t\| \\
& \leq C_{17} (\|u_t\|^2 + \|\nabla^m u\|^2 + \|\nabla^m v\|^2),
\end{aligned}$$

where $s \in [0, 1]$.

Thus

$$\begin{aligned}
& (g_1(u_1, v_1) - g_1(u_2, v_2), u_t) \\
& \geq -C_{17} (\|u_t\|^2 + \|\nabla^m u\|^2 + \|\nabla^m v\|^2).
\end{aligned} \tag{3.9}$$

From (3.5)-(3.9), we obtain

$$\begin{aligned}
& \frac{d}{dt} (\|u_t\|^2 + M(v_1) \|\nabla^m u\|^2) + 2(\beta \lambda^m - C_{13} - C_{17}) \|u_t\|^2 \\
& - 2(C_{13} \lambda_1^{1-m} + C_{14} + C_{17}) \|\nabla^m u\|^2 - 2(C_{13} \lambda_1^{1-m} + C_{17}) \|\nabla^m v\|^2 \leq 0.
\end{aligned} \tag{3.10}$$

By multiplying (3.3) by v_t in H , we can get

$$\begin{aligned}
& \frac{d}{dt} (\|v_t\|^2 + M(v_1) \|\nabla^m v\|^2) + 2(\beta \lambda^m - C_{13} - C_{17}) \|v_t\|^2 \\
& - 2(C_{13} \lambda_1^{1-m} + C_{14} + C_{17}) \|\nabla^m v\|^2 - 2(C_{13} \lambda_1^{1-m} + C_{17}) \|\nabla^m u\|^2 \leq 0.
\end{aligned} \tag{3.11}$$

Summing up (3.10) and (3.11), we have

$$\begin{aligned}
& \frac{d}{dt} (\|u_t\|^2 + \|v_t\|^2 + M(v_1) (\|\nabla^m u\|^2 + \|\nabla^m v\|^2)) + 2(\beta \lambda^m - C_{13} - C_{17}) (\|u_t\|^2 + \|v_t\|^2) \\
& - (4C_{13} \lambda_1^{1-m} + 2C_{14} + 4C_{17}) (\|\nabla^m u\|^2 + \|\nabla^m v\|^2) \leq 0.
\end{aligned} \tag{3.12}$$

Let

$$\begin{cases} n_1 = 2(\beta\lambda^m - C_{13} - C_{17}), \\ n_2 = -\frac{(4C_{13}\lambda_1^{1-m} + 2C_{14} + 4C_{17})}{M(v_1)}. \end{cases} \quad (3.13)$$

$$n = \min\{n_1, n_2\}. \quad (3.14)$$

Thus, we have

$$\frac{d}{dt}\phi(t) + n\phi(t) \leq 0, \quad (3.15)$$

where

$$\phi(t) = \|u_t\|^2 + \|v_t\|^2 + M(v_1)(\|\nabla^m u\|^2 + \|\nabla^m v\|^2). \quad (3.16)$$

According to *Gronwall* inequality, we obtain

$$\phi(t) \leq \phi(0)e^{-nt} = 0, \quad (3.17)$$

where

$$\phi(0) = \|u_t(0)\|^2 + \|v_t(0)\|^2 + M(v_1(0))(\|\nabla^m u(0)\|^2 + \|\nabla^m v(0)\|^2). \quad (3.18)$$

So

$$u_1 = u_2, \quad v_1 = v_2. \quad (3.19)$$

We get the uniqueness of the solution.

3.2. The Existence Of The Global Attractor

Theorem3. 2. (Lin,G.G,2011) Let E_0 be a Banach space, and $\{S(t)\}(t \geq 0)$ are the semigroup operator on E_0 . $S(t) : E_0 \rightarrow E_0$, $S(t+\tau) = S(t)S(\tau)$ ($\forall t, \tau \geq 0$), $S(0) = I$, where I is a unit operator. Set $S(t)$ satisfies the follow condition:

(1) $S(t)$ is uniformly bounded, namely $\forall R > 0$, $\|u\|_{E_0} \leq R$, it exists a constant $C(R)$, so that

$$\|S(t)u\|_{E_0} \leq C(R), (t \in [0, +\infty)); \quad (3.20)$$

(2) It exists a bounded absorbing set $B_0 \subset E_0$, namely, $\forall B \subset E_0$, it exists a constant t_0 , so that

$$S(t)B \subset B_0 (t \geq t_0), \quad (3.21)$$

where B_0 and B are bounded sets.

(3) When $t > 0$, $S(t)$ is a completely continuous operator. Therefore, the semigroup operator $S(t)$ exists a compact global attractor A .

Theorem 3.3. under the assume of Lemma2.4., Lemma 2.5. and Theorem 3.1., equations have global attractor

$$A = \omega(B_0) = \overline{\bigcup_{\tau \geq 0} \bigcup_{t \geq \tau} S(t)B_0}, \quad (3.22)$$

here $B_0 = \{(u, v, p, q) \in E_1 : \|(u, v, p, q)\|_{E_1} = \|u\|_{H^{2m}} + \|v\|_{H^{2m}} + \|p\|_{H^m} + \|q\|_{H^m} \leq 2R_0 + 2R_1\}$ and

$E_1 = H^{2m} \times H^{2m} \times H^m \times H^m$. B_0 is the bounded absorbing set of E_1 and Satisfies:

(1) $S(t)A = A, t > 0$;

(2) $\lim_{t \rightarrow \infty} \text{dist}(S(t)B, A) = 0$, here $B \subset E_1$ and it is a bounded set,

$$\text{dist}(S(t)B, A) = \sup_{x \in B} (\inf_{y \in A} \|S(t)x - y\|_{E_1}) \rightarrow 0, t \rightarrow \infty. \quad (3.23)$$

Proof. Under the conditions of Theorem 3.1., it exists the solution Semigroup $S(t)$,

$S(t) : E_1 \rightarrow E_1$.

- (1) From Lemma 2.3. to Lemma 2.4., we can obtain that $\forall B \subset E_1$ is a bounded set that includes in the ball $\{(u, v, p, q) \in E_1 \mid \|u, v, p, q\|_{E_1} \leq R\}$,

$$\begin{aligned} \|S(t)(u_0, v_0, p_0, q_0)\|_{E_1}^2 &= \|u\|_{H^{2m}}^2 + \|v\|_{H^{2m}}^2 + \|p\|_{H^m}^2 + \|q\|_{H^m}^2 \\ &\leq \|u_0\|_{H^{2m}}^2 + \|v_0\|_{H^{2m}}^2 + \|p_0\|_{H^m}^2 + \|q_0\|_{H^m}^2 + C \\ &\leq 2R_1^2 + C, (t \geq 0, (u_0, v_0, p_0, q_0) \in B). \end{aligned} \quad (3.24)$$

This shows that $S(t)(t \geq 0)$ is uniformly bounded in E_1 .

- (2) Furthermore, for any $(u_0, v_0, p_0, q_0) \in E_1$, when $t \geq \max\{t_0, t_1\}$, we get

$$\|S(t)(u_0, v_0, p_0, q_0)\|_{E_1}^2 = \|u\|_{H^{2m}}^2 + \|v\|_{H^{2m}}^2 + \|p\|_{H^m}^2 + \|q\|_{H^m}^2 \leq 2(R_0^2 + R_1^2). \quad (3.25)$$

So we get B_0 is the bounded absorbing set.

- (3) Since $E_1 \rightarrow E_0$ is compact embedded, which means that the bounded set in E_1 is the compact set in E_0 , so the semigroup operator $S(t)$ exists a compact global attractor A , where $E_0 = H^m(\Omega) \times H^m(\Omega) \times L^2(\Omega) \times L^2(\Omega)$.

4. CONCLUSIONS

The main details of this paper handle with the global attractors of the generalized Higher-order coupled kirchhoff –type equations (1.1)-(1.6) with nonlinear strongly damping and source terms. In section 2, under some assumptions, we prove the existence and uniqueness of equations ; In section 3, we get the global attractor of the problem ; In section 4, according to (Yaojun Ye.,2013), finite-dimension Hausdorff for global attractors are obtained.

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