

PARAMETRIC COMPARISON OF CIRCULAR, TRIANGULAR AND RECTANGULAR DUAL-BAND MICROSTRIP ANTENNAS FOR WIRELESS COMMUNICATION

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ABSTRACT

This paper presents a parametric comparison of the three commonly use microstrip antenna geometries designed to resonate at dual frequencies for wireless communication. Both triangular and rectangular dual band antennas were designed using flame-resistant (FR-4) substrate while RT Duroid 5880 substrate was used in designing the circular dual band patch. All antennas were designed at 2.4 GHz and 5.2 GHz, respectively. From the results obtained, the rectangular dual band patch achieved the highest percentage bandwidth performance at 2.4 GHz while the triangular patch achieved the best bandwidth performance at 5.2 GHz with all the antennas presented resonating within the acceptable limit of $1 \leq VSWR \leq 2$.

Keywords: Comparison, dual band, microstrip, antenna, bandwidth.

INTRODUCTION

The advancement in communication in recent time is the driving force behind miniaturization of antenna for modern communication devices. Microstrip antennas have been at the forefront of research in the quest for this miniaturization because of its inherent characteristics (Balanis, 2005). These characteristics include small size, ease of fabrication, lightweight, compact nature, planar and cost effectiveness. Microstrip antennas exist in different shapes but among the popular shapes are rectangular, circular and triangular microstrip antennas (Kumar and Ray, 2003; Balanis, 2005; Huang and Boyle, 2008). Despite the endearing attributes of this type of antenna, they are limited by a number of factors such as narrow bandwidth, low power handling capacity, low gain and low antenna efficiency (James and Hall, 1989). However, to augment the performance of microstrip antennas in respect of the limitations, a compromise is usually made on an antenna parameter in order to increase the performance of a desired parameter e.g. in order to increase the gain of microstrip antenna, array configuration is preferred hence a compromise is made on the size of the antenna (Kwaha, Inyang, and Amalu, 2011). Also, the type of feeding technique adopted has been shown to slightly increase the achieved bandwidth of the patch antenna. Circular, rectangular and triangular microstrip antennas are designed and results of each is compared in the later section of this paper.

METHODOLOGY

A noticeable feature in the design of microstrip antennas is that each shape is designed with a unique set of equations after specifying the operating frequency, dielectric substrate and substrate thickness. For the circular patch, RT Duroid 5880 substrate with a dielectric constant, ϵ_r of 2.2, substrate height of 1.6 mm and at frequencies of 2.4 and 5.2 GHz, respectively was selected and designed with the following detailed equations from (Balanis,

2005; Kumar and Ray, 2003; Kumar and Ray, 2000; Stutzman and Thiele, 2013 and Garg *et al.*, 2001).

- a) Calculate the radius, a and effective radius, a_e of the patch using equations adopted from (Balanis, 2005) as follows:

$$a = \frac{F}{\left\{1 + \frac{2h}{\pi \epsilon_r F} \left[\ln \left(\frac{\pi F}{2h} \right) + 1.7726 \right] \right\}^{1/2}} \quad (1)$$

$$F = \frac{8.791 \times 10^9}{f_r \sqrt{\epsilon_r}} \quad (2)$$

In order to take care of the fringing effect which makes the patch electrically larger, the effective radius of the patch is therefore calculated to take care of this fringing thus;

$$a_e = a \left\{ 1 + \frac{2h}{\pi \epsilon_r a} \left[\ln \left(\frac{\pi a}{2h} \right) + 1.7726 \right] \right\}^{1/2} \quad (3)$$

- b) Calculate the radial distance, ρ of the patch thus:

$$\rho = \frac{2(2a)}{\lambda_{air}} \quad (4)$$

- c) Calculate the width of the transmission line, W_f using equation adopted from Pozar, 2012:

$$W_f = \left(\frac{2h}{\pi} \right) \times \left[\frac{\frac{60\pi^2}{Z_o \sqrt{\epsilon_r}} - 1 - \ln \left[2 \times \left[\frac{60\pi^2}{Z_o \sqrt{\epsilon_r}} \right] - 1 \right] + \left(\frac{\epsilon_r - 1}{2\epsilon_r} \right) \dots}{\dots \times \left(\ln \left[\left[\frac{60\pi^2}{Z_o \sqrt{\epsilon_r}} \right] - 1 \right] + 0.39 - \frac{0.61}{\epsilon_r} \right)} \right] \quad (5)$$

- d) Calculate the notch width, g using equation from Ramli *et al.*, 2012:

$$g = \frac{c f_r \times 10^{-9} \times 4.65 \times 10^{-9}}{\sqrt{2\epsilon_r}} \quad (6)$$

- e) Calculate the minimum permissible length of transmission line, L_f is using the equation from Ramli *et al.*, 2012:

$$L_f = \frac{6h}{2} \quad (7)$$

A summary of the dual-band antenna design parameters after optimization in CST MW Studio is given in Table 1.

Table 1. Summarized design parameters of circular patch antenna

Parameter	Value (mm)
Radius, a	16.8059
Effective radius, a_e	17.4067
Radial length, ρ	7.4086
Notch width, g	1.6438
Substrate height, h	1.6
Radius of slot on patch, b	5.0
Length of slot on ground plane, p	4.0
Width of slot on ground plane, v	2.0

The geometry of the proposed circular dual-band antenna is given in Figure 1 while the design of the dual-band antenna in CST Microwave Studio is given in Figure 1.

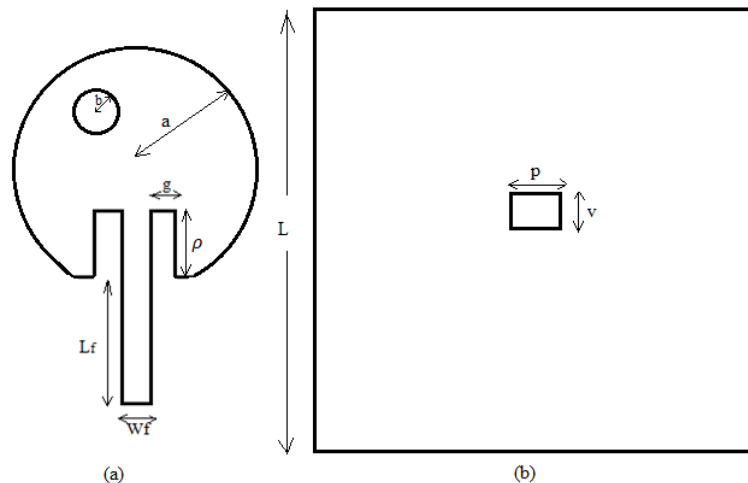


Figure 1. Schematic diagram of proposed dual band antenna (a) top view (b) bottom view

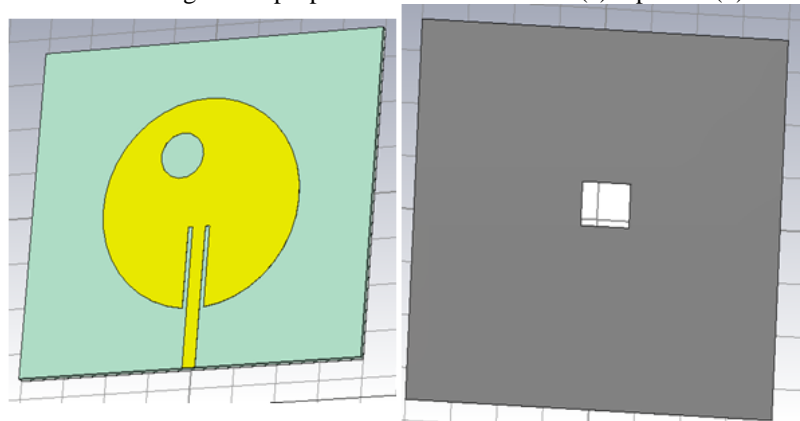


Figure 2. Top and bottom view of dual band CMSA antenna designed in CST MW studio

An FR-4 substrate with dielectric constant, ϵ_r of 4.3, substrate thickness of 1.6 mm and at the same operating frequencies as the circular patch earlier designed was selected for the triangular patch antenna. The design equations are given in the following steps.

Step one: Considering the dominant TM_{10} mode of an equilateral triangular microstrip antenna (ETMA), the sidelength, a , as stated by Ramesh *et al.*, 2001 is given as:

$$a = \frac{2c}{3f_r \sqrt{\epsilon_r}} \quad (8)$$

Step two: The effective sidelength which takes into consideration the effect of spurious radiation from the antenna is given in (9).

$$a_e = a \left[\begin{array}{l} 1 + 2.199 \frac{h}{a} - 12.853 \frac{h}{a \sqrt{\epsilon_r}} + 16.436 \frac{h}{a \epsilon_r} \dots \\ + 6.182 \left(\frac{h}{a} \right)^2 - 9.802 \frac{1}{\sqrt{\epsilon_r}} \left(\frac{h}{a} \right)^2 \end{array} \right] \quad (9)$$

Step three: The minimum length of ground plane, L_g as adopted from (Balanis, 2005; Sharma and Sharma, 2017) given for rectangular patch is calculated thus:

$$L_g = 6h + a \quad (10)$$

Step four: Applying a modified expression for calculating the edge impedance of rectangular patch adopted from (Sharma and Sharma, 2017; Huang and Boyle, 2008 and Nwalozie *et al.*, 2013) to estimate the edge impedance of ETMA given thus:

$$Z_{\text{edge}} = 90 \frac{\epsilon_r^2}{\epsilon_r + 1} \left[\frac{a_e}{L_g} \right]^2 \quad (11)$$

The edge impedance does not match well with 50 Ω standard microstrip line and therefore a quarter-wavelength transformer is used to connect the microstrip line to the resonating patch.

The Characteristic impedance of the transition section as given by Huang and Boyle, 2008 was calculated as follows:

$$Z_{qw} = \sqrt{Z_0 Z_{edge}} \quad (12)$$

Step five: The width of the 50 Ω transmission line, W_f , as given by (Pozar, 2012) is calculated thus:

$$Z_o = \frac{120\pi}{\sqrt{\epsilon_r}(W_f/h + 1.393 + 0.667 \ln(W_f/h + 1.44))} \text{ for } \frac{W_f}{h} \geq 1 \quad (13)$$

$$50 = \frac{120\pi}{\sqrt{4.3} \times \left[\frac{W_f}{1.6} + 1.393 + 0.667 \ln\left(\frac{W_f}{1.6} + 1.44\right) \right]}$$

$$\frac{W_f}{1.0672} + \ln\left(\frac{W_f}{1.6}\right) = 5.0867$$

Solving the above equation by iteration, the solution was at;

$$W_f = 0.004367 \text{ m} = 4.367 \text{ mm}$$

The width of the 94.64 Ω transition section, W_q , is calculated thus:

$$Z_o = \frac{60}{\sqrt{\epsilon_r}} \ln\left(\frac{8h}{W_q} + \frac{W_q}{4h}\right) \text{ for } \frac{W_q}{h} \leq 1 \quad (14)$$

$$94.64 = \frac{60}{\sqrt{4.3}} \ln\left(\frac{8 \times 1.6}{W_q} + \frac{W_q}{4 \times 1.6}\right)$$

$$\log\left(\frac{94.64 \times \sqrt{4.3}}{60}\right) = \frac{8 \times 1.6}{W_q} + \frac{W_q}{4 \times 1.6}$$

$$W_q^2 - 3.294W_q + 81.92 = 0$$

$$W_q = 0.001647 \text{ m} = 1.647 \text{ mm}$$

Step six: The length of the quarter-wave microstrip transmission line, L_q was calculated thus

$$L_q = \frac{\lambda}{4} = \frac{\lambda_{air}}{4\sqrt{\epsilon_r}} \quad [11] \quad (15)$$

$$\lambda_{air} = \frac{3 \times 10^8}{2.4 \times 10^9} = 0.125 \text{ m} = 125 \text{ mm}$$

$$\therefore L_q = \frac{125}{4 \times \sqrt{4.3}} = 0.01507 \text{ m} = 15.07 \text{ mm}$$

the length of the 50 Ω transmission line, L_f was calculated as follows (Ramli *et al.*, 2013):

$$L_f = \frac{6h}{2} \quad (16)$$

Summary of results of design calculation for various dimensions of the proposed dual band ETMA is given in Table 2.

The schematic diagram of the designed antenna is given in Figure 3 while Figure 4 shows the designed antenna in CST MW Studio.

Table 2. Equilateral triangular patch antenna computed design parameters

Parameter	Value (mm)
Patch dimensions:	
Sidelength, a	19.50
Effective sidelength, a_e	14.78
Dielectric constant, ϵ_r	4.30
Height of substrate, h	1.60
Quarter-wave fed dimensions:	
Width of transition section, W_q	1.57
Width of transmission line, W_f	4.37
Length of quarter wave, L_f	4.80
Length of quarter wave, L_q	7.32
Resonance Frequency, f_r	2.4 and 5.2 GHz
Ground plain dimensions:	
Length of ground plain, $L_g = W_g$	29.10

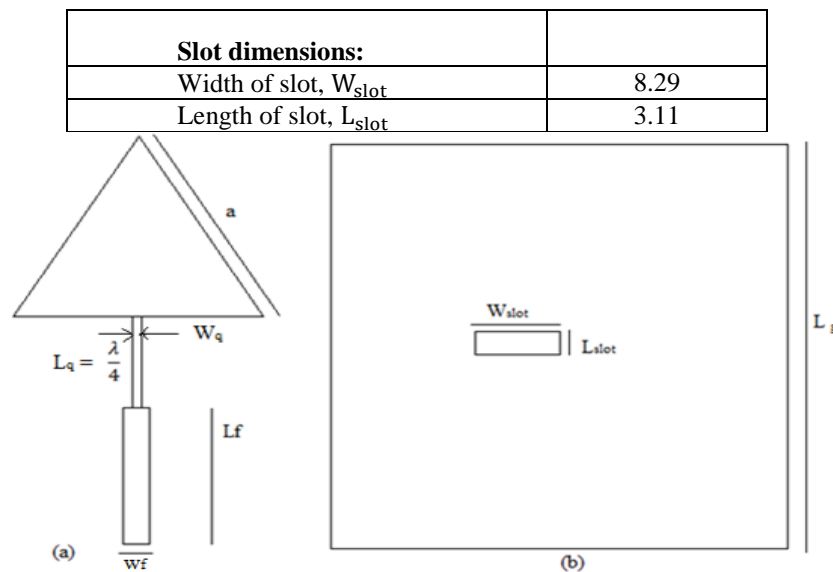


Figure 3. Schematic diagram of (a) top and (b) bottom view of the proposed antenna

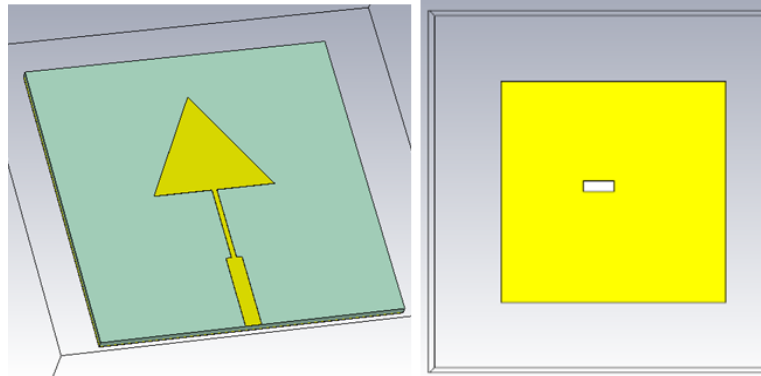


Figure 4. Top and bottom view of designed dual band ETMA in CST MW Studio

For design, the procedure of rectangular microstrip antenna it starts with the calculation of the length and width of conventional single band microstrip patch antenna using standard antenna design equations from (Subramanian and Prabhu, 2015; Ren, 2008) at 2.4 GHz. The first design step is to choose a suitable dielectric substrate of appropriate thickness (h). Here, Flame Resistant substrate with dielectric constant $\epsilon_r = 4.4$ having thickness (h) = 1.6mm is chosen as the substrate material for the patch antenna. Furthermore, the dimensions of the patch and feed line are determined and the feed line is placed properly to resonate at 2.4 GHz. Then the modifications of the antenna structure is carried out to create dual resonance. For efficient radiation, patch parameters are determined as illustrated in the following steps;

Step One: The resonant frequency (f_r), select substrate relative permittivity (ϵ_r) and a substrate thickness (h) are specified. The loss due to surface waves can be neglected when h satisfies the criterion in Equation (17) according to (Subramanian and Prabhu, 2015) given as:

$$h \leq 0.3 \times \frac{\lambda_{\text{air}}}{2\pi\sqrt{\epsilon_r}} \quad (17)$$

$$\lambda_{\text{air}} = \frac{c}{f_r} \quad (18)$$

Where λ_{air} = wavelength in free space (air), c = speed of light = 3×10^8 m/s, f_r = the selected resonant frequency = 2.4×10^9 GHz, loss tangent ($\tan \delta$) = 0.023

Step Two: Calculate the width of the patch W_p thus;

$$W_p = \frac{c}{2f_r} \sqrt{\frac{2}{\epsilon_r + 1}} \quad (19)$$

Step Three: The effective dielectric constant ϵ_{reff} is determined based on Equation (20);

$$\epsilon_{\text{reff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[1 - 12 \frac{h}{W_p} \right]^{-\frac{1}{2}} \quad \text{For } \frac{W_p}{h} > 1 \quad (20)$$

Step Four: calculate the normalized extension length ΔL given as;

$$\frac{\Delta L}{h} = 0.412 \frac{(\epsilon_{\text{reff}} + 0.3) \left[\frac{W_p}{h} + 0.264 \right]}{(\epsilon_{\text{reff}} - 0.258) \left[\frac{W_p}{h} + 0.8 \right]} \quad (21)$$

Step Five: The value of the length of the patch L_p is given by,

$$L = \frac{c}{2f_r \sqrt{\epsilon_r}} - 2\Delta L_p \quad (22)$$

Step Six: Calculate the notch width, g using the equation from (Ramli, *et al.*, 2011) thus:

$$g = \frac{c f_r \times 10^{-9} \times 4.65 \times 10^{-9}}{\sqrt{2\epsilon_{\text{reff}}}} \quad (23)$$

Step Seven: Compute the resonant input resistance R_{in} thus;

$$R_{\text{in}}(y=y_o) = \frac{1}{2(G_1 \pm G_{12})} \cos^2 \left(\frac{\pi y_o}{L_p} \right) \quad (24)$$

The equation for the characteristic impedance Z_o as given by (Vilaltella, 2013) as presented in Equations 25 and 25;

$$Z_o = \frac{60}{\sqrt{\epsilon_{\text{reff}}}} \ln \left[\frac{8h}{W_f} + \frac{W_f}{4h} \right] \quad \frac{W_f}{h} \leq 1 \quad (25)$$

$$Z_o = \frac{120\pi}{\sqrt{\epsilon_{\text{reff}}}} \left[\frac{W_f}{h} + 1.393 + 0.667 \ln \left(\frac{W_f}{h} + 1.444 \right) \right] \quad \frac{W_f}{h} \geq 1$$

In this design, the second expression in Equation (25) applies. Hence;

$$R_{\text{in(edge)}} = \frac{1}{2(G_1 \mp G_{12})} \quad (26)$$

$$k = \frac{2\pi}{\lambda_{\text{air}}} \quad (27)$$

$$I_1 = -2 + \cos(X) + X S_i(X) + \frac{\sin(X)}{X} \quad (28)$$

$$X = kW_p \quad (29)$$

$$G_1 = \frac{I_1}{120\pi^2} \quad (30)$$

$$G_{12} = \frac{1}{120\pi^2} \int_0^\pi \left[\frac{\sin(\frac{kW_p}{2} \cos\theta)}{\cos\theta} \right]^2 J_o(kL_p \sin\theta) \sin^3 \theta d\theta \quad (31)$$

Where J_o is the Bessel function of the first kind of order zero, G_1 is conductance of microstrip radiator and G_{12} is the mutual conductance. For this design, input impedance of 50 Ω was used.

Step Eight: Calculate the inset feed recessed distance y_o and the width of the transmission line W_f using the equation for resonant input impedance thus;

$$y_o = \frac{L_p}{\pi} \cos^{-1} \left[\sqrt{\frac{Z_o}{R_{\text{in(edge)}}}} \right] \quad (32)$$

According to (Mansour, 2014; Pozar, 2012), the width of the transmission line is calculated using Equation 5 restated as follows:

$$\text{For } \frac{W_f}{h} > 2;$$

$$W_f = \left(\frac{2h}{\pi} \right) \times \left[\frac{60\pi^2}{Z_o \sqrt{\epsilon_r}} - 1 - \ln \left[2 \times \left[\frac{60\pi^2}{Z_o \sqrt{\epsilon_r}} \right] - 1 \right] + \left(\frac{\epsilon_r - 1}{2\epsilon_r} \right) \times \left(\ln \left[\left[\frac{60\pi^2}{Z_o \sqrt{\epsilon_r}} \right] - 1 \right] + 0.39 - \frac{0.61}{\epsilon_r} \right) \right]$$

Step Nine: Calculate the ground plane dimensions thus;

The length (L_g) and width (W_g) of the ground plane is:

$$L_g = 6h + L_p \quad (33)$$

$$W_g = 6h + W_g \quad (34)$$

The summary of the rectangular patch design parameters is presented in Table 3.

Table 3. Rectangular dual patch computed design parameters

Variables	W _g	L _g	W _p	L _p	W _f	L _f	g	y _o	C	b	a	L1	W1	L2	W2	n	m
Value(mm)	44	41	24	21	2.96	10	2	5.2	10	13	2.3	20	4.5	22	12.3	12	26

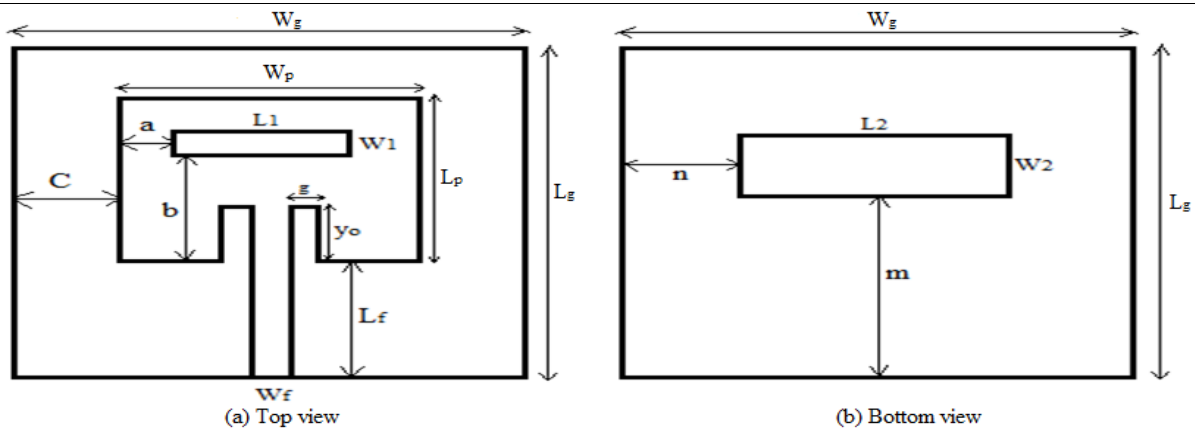


Figure 5. Geometry of the proposed dual-band antenna.

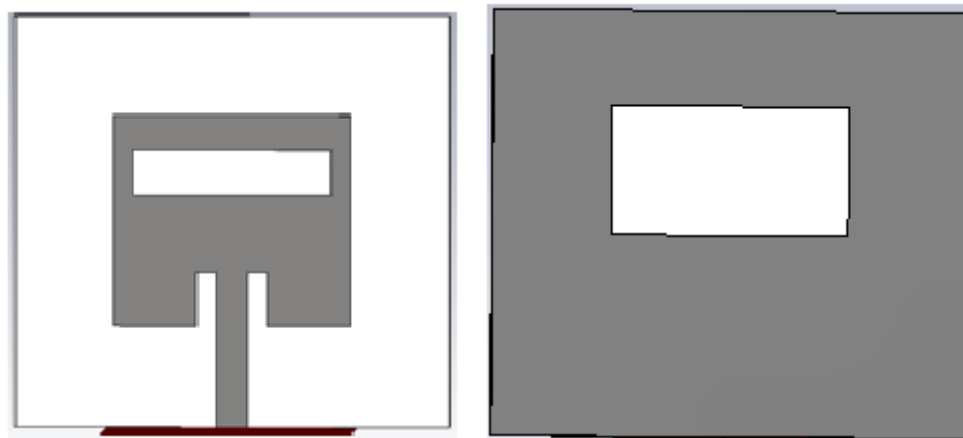


Figure 6. Designed rectangular dual-band antenna in CST Microwave Studio

RESULTS

The results of CST Studio simulation of all the antennas designed previously are presented in this section in form of antenna parameters. The focus of the result was on return loss and bandwidth of each antenna designed and appropriate comparison of the three shapes are also presented.

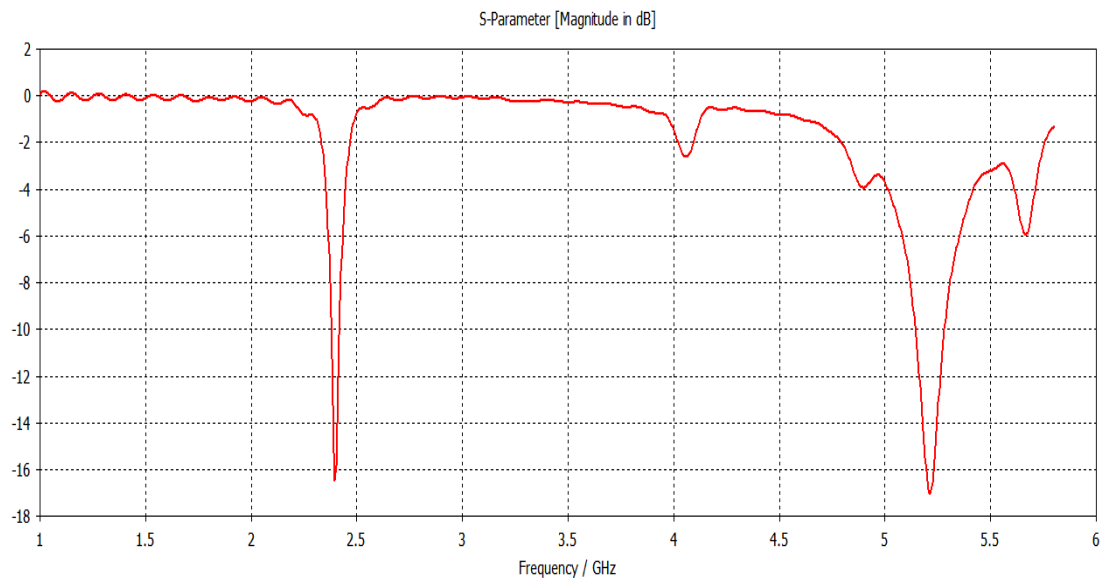


Figure 7. Return loss of dual band CMSA

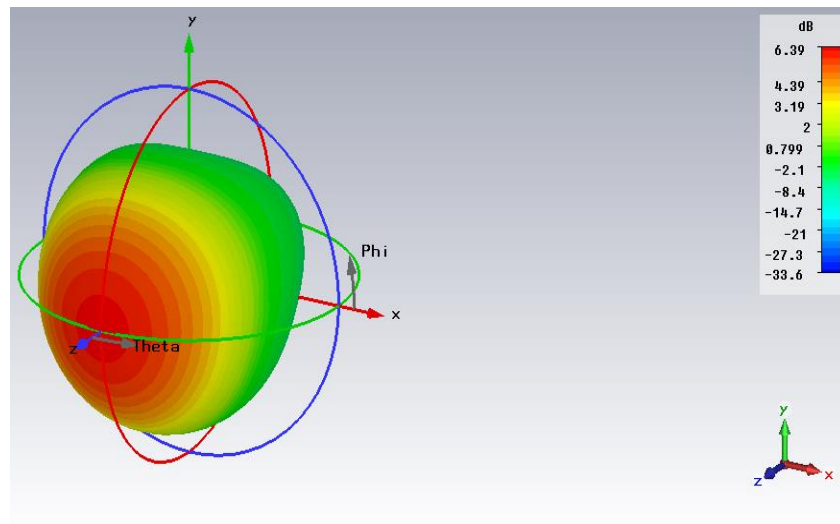


Figure 8. 3D gain of dual band CMSA at 2.4 GHz

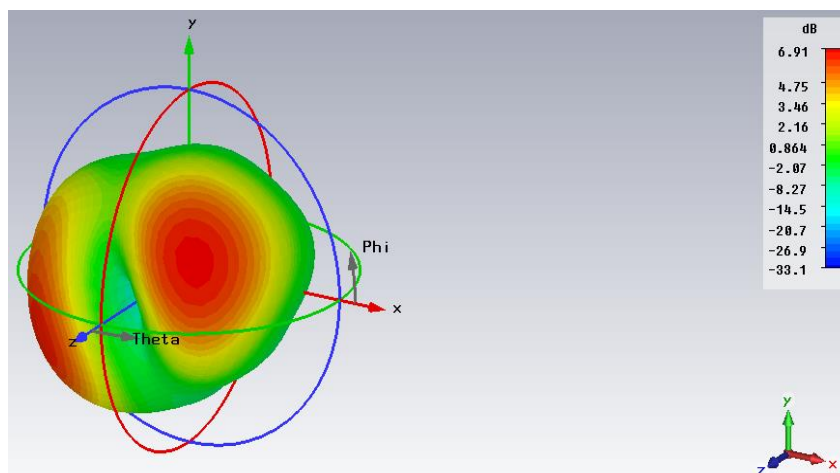


Figure 9. 3D gain of dual band CMSA at 5.2 GHz

The return loss of the proposed antenna as can be seen from Figure 7, shows that at -16.3608 dB, the resonance frequency is 2.4089 GHz and at -17.0214 dB the resonance frequency is 5.2129 GHz, respectively. An impedance bandwidth of 69.2 MHz at 2.4 GHz and 144.9 MHz at 5.2 GHz was realised.

The percentage bandwidth of the antenna was calculated using equation from Siakavara, 2010 thus:

$$\text{Bandwidth at 2.4 GHz} = \frac{2.445 - 2.3758}{2.4} \times 100\% = 3.30\%$$

$$\text{Bandwidth at 5.2 GHz} = \frac{5.2874 - 5.1425}{5.2} \times 100\% = 2.8\%$$

Figures 8 and 9 gives the illustration of the dual band CMSA gain at the designed frequencies.

For the dual band ETMSA, the return loss and antenna gain are presented in Figures 10 to 12. From the return loss plot of the proposed dual band ETMA given in Figure 10, it was observed that a maximum return loss of -14.4137 dB and -20.0915 dB were achieved at 2.4 GHz and 5.2 GHz respectively, and a bandwidth of 41.4 MHz and 774 MHz at 2.4 GHz and 5.2 GHz, respectively were achieved by the dual band antenna.

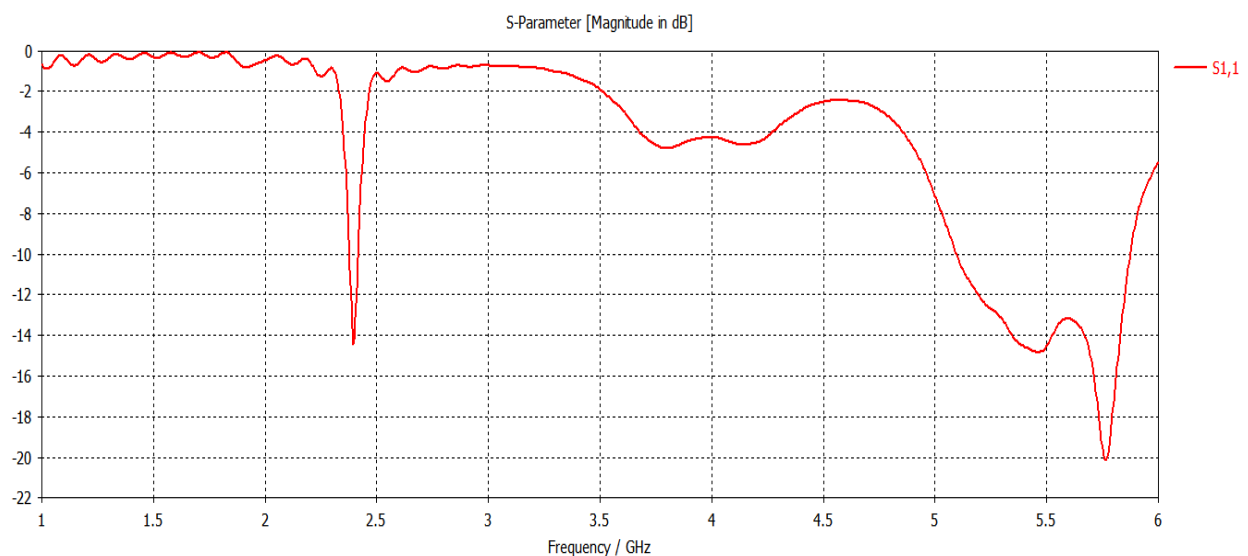


Figure 10. Return loss of dual band ETMSA

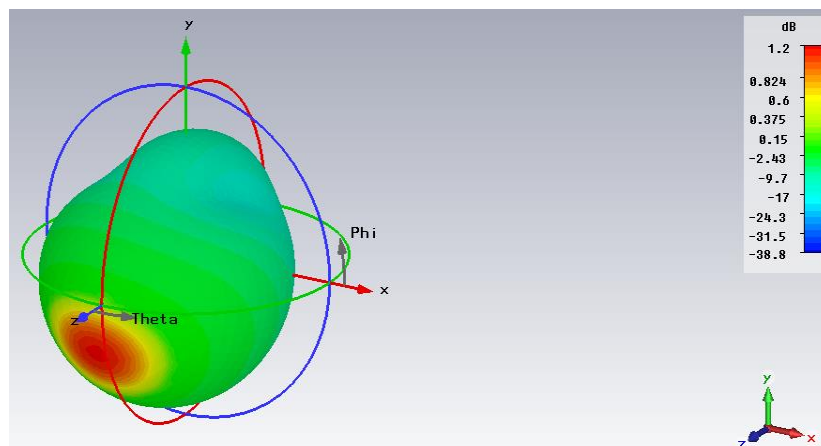


Figure 11. 3D gain of dual band ETMSA at 2.4 GHz

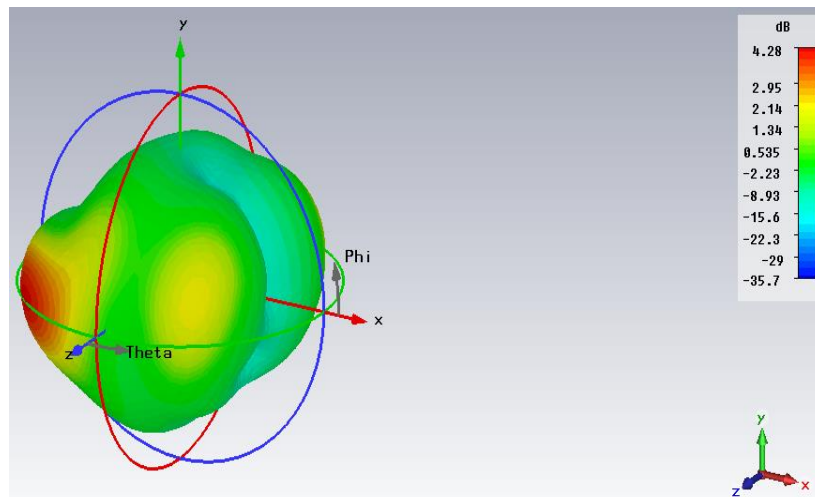


Figure 12. 3D gain of dual band ETMSA at 5.2 GHz

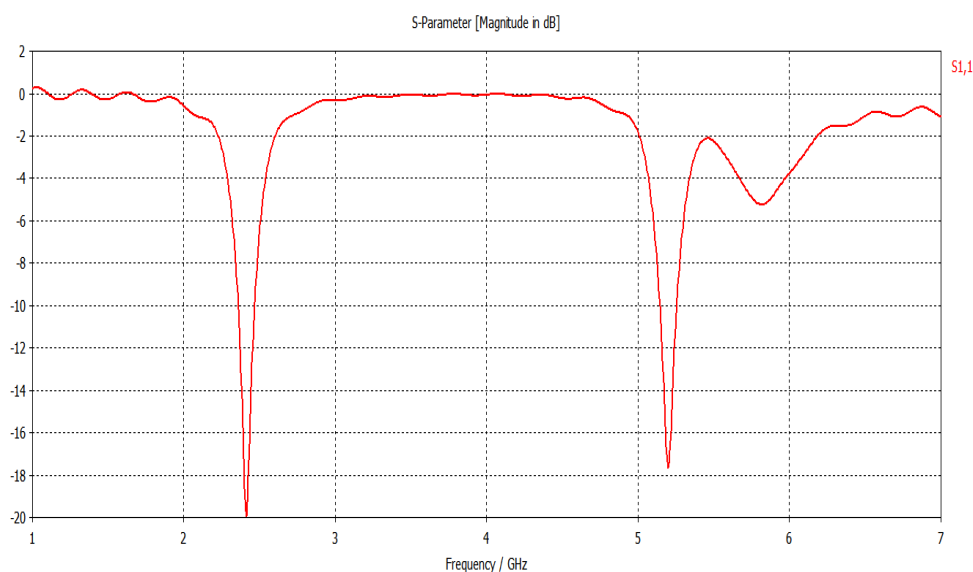


Figure 13. Return loss of dual band RMSA

The percentage bandwidth of the dual band ETMSA antenna was calculated thus:

$$\text{Bandwidth at 2.4 GHz} = \frac{2.4156 - 2.3742}{2.4} \times 100\% = 1.74\%$$

$$\text{Bandwidth at 5.2 GHz} = \frac{5.8717 - 5.0977}{5.2} \times 100\% = 14.88\%$$

Evidently, the proposed antenna exhibits broadband characteristics at 5 GHz band.

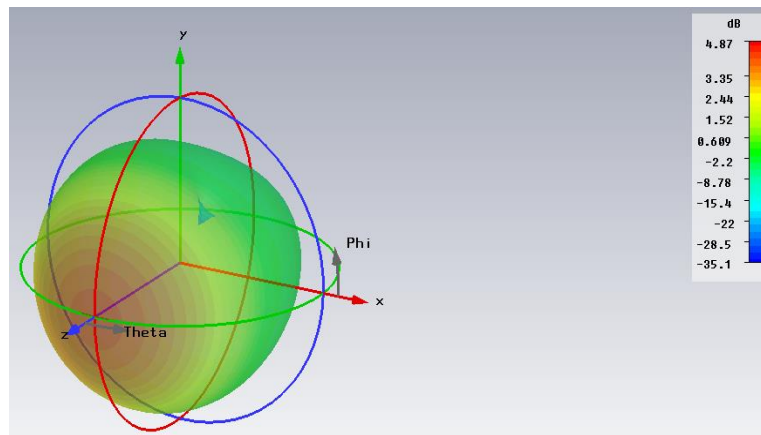


Figure 14. 3D gain of dual band RMSA at 2.4 GHz

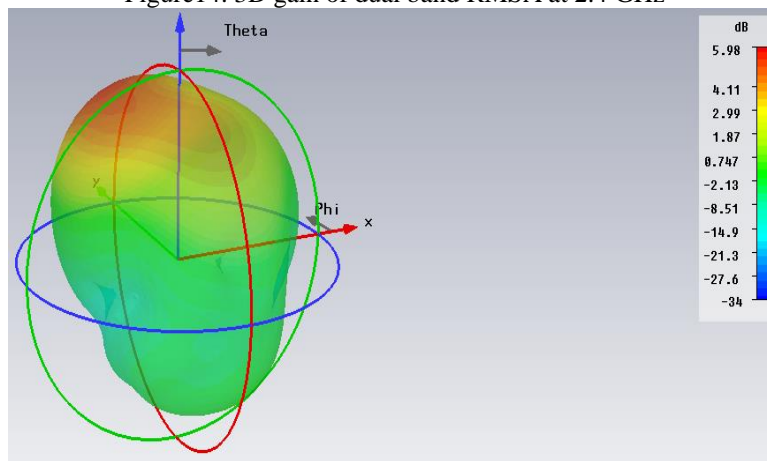


Figure 15. 3D gain of dual band RMSA at 5.2 GHz

The return loss of the dual band RMSA as could be seen from Figure 13, shows that at -9.619 dB, the resonance frequency is 2.4189 GHz and at -17.55 dB the resonance frequency is 5.2034 GHz. The 124.6 MHz at 2.4 GHz and 119.8 MHz at 5.2 GHz. The gain of the antenna at 2.4 GHz and 5.2 GHz are as illustrated in Figures 14 and 15, respectively.

The bandwidth of the dual band RMSA antenna was calculated thus;

$$\text{Bandwidth at 2.4 GHz} = \frac{2.48 - 2.35}{2.4} \times 100\% = 5.4\%$$

$$\text{Bandwidth at 5.2 GHz} = \frac{5.27 - 5.15}{5.2} \times 100\% = 2.3\%$$

DISCUSSION

The results presented show a clear distinction in performance in terms of bandwidth from the return loss plot and antenna gain. The dual band equilateral triangular microstrip antenna achieved by far, a greater combined impedance bandwidth of a little less than 1 GHz covering almost the entire 5 GHz band along with 2.4 GHz. RMSA achieved the highest impedance bandwidth at 2.4 GHz followed by CMSA. However, ETMSA has the least gain performance among the three antennas designed while the CMSA has the best gain performance at both frequency bands considered.

CONCLUSIONS

Circular, equilateral triangular and rectangular dual band microstrip antennas has been designed and results compared. The antennas resonated below standard – 10 dB and within allowable VSWR limits ($1 \leq \text{VSWR} \leq 2$) with an appreciable bandwidth and antenna.

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