A STUDY OF MASS TRANSFER ON HYDROMAGNETIC FREE CONVECTIVE NON-NEWTONIAN FLOW

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ABSTARCT

The problem concern with mass transfer on free convective two dimensional unsteady flow of a non-Newtonian incompressible fluid through a porous medium bounded by an infinite vertical porous plate subjected to a uniform suction is presented under the influence of a uniform transverse magnetic field. Approximate solutions for velocity distribution ,fluctuating parts of the velocity profiles, amplitude and phase lead of the skin friction at the plate has been found by using perturbation technique.The effects of various parameters has been studied, discussed numerically and shown graphically. And numerical values of coefficient of skin friction for various values of physical parameters are presented.

Key words : Mass transfer, Free convection ,Walters liquid model B.

INTRODUCTION AND LITERATURE

The study of free convective flow through porous medium to make heat insulation of surface more effective to estimate its effects in heat and mass transfer attracts by many researchers. Singh[1] studied MHD free convective and mass transfer flow with heat source and thermal diffusion. Bansood [2] discuss boundary layer solution of convective heat and mass transfer from a horizontal surface in a non-Darcy porous media. Sahoo.et.al[3] studied MHD unsteady free convective flow with constant suction and heat sink. Atul Kumar Singh .et.al [4] discuss heat and mass transfer in MHD flow of a viscous flid past a vertical plate under oscillatory suction. Unsteady effect on MHD free convective and mass transfer flow through porous medium with constant and constant heat flux in rotating system was studied by Sharma[5]. Two dimensional MHD oscillatory flow along a uniform moving infinite vertical porous plate bounded by porous medium studied by Ahmed & Ahmed[6]. Kim[7] studied heat and mass transfer in MHD micropolar flow over a vertical moving porous plate in a porous medium.

Alam et.al [8] studied the dufour and soret effects on unsteady MHD free convective and mass transfer flow past a vertical porous plate in porous medium. Sarangi and Jose[9] studied unsteady MHD free convective flow and mass transfer through a porous medium with constant suction and constant heat flux. Unsteady two dimensional flow and heat transfer through an elastic– viscous liquid along an infinite hot vertical porous medium studied by Sharma &Sharma[10]. Veena et.al[11] discussed the heat transfer in a viscoelastic fluid past a stretching sheet with viscous dissipation and internal heat generation. MHD convective flow of a micropolar fluid past a continuously moving vertical porous plate in the presence of heat generation/absorption was studied by Rahman and Sattar[12].·Singh et.al [13]discussed MHD free convective and mass transfer flow past a flat plate. Sharma and Yadav[14] studied the three dimensional flow and heat transfer through porous medium bounded by a porous vertical surface with variable permeability and heat source. Ramana[15] studied MHD unsteady free convective Walter's memory flow with constant suction and heat sink. Noushima et.al [16] extended the work of Sharma .et.al[17] to memory flow.[9] discussed with viscoelastic fluid by Noushima et.al [18].

In present problem we studied the mas transfer effects on hydromagnetic free convective non-newtonian fluid.

METHODOLOGY

Consider the flow of incompressible Walter's liquid model B [19][20]fluid through a porous medium bounded by an infinite vertical porous plate with constant suction, under the influence of uniform transverse magnetic field.The x – axis is taken along the plate in the upward direction anda straight line perpendicular to that as the y-axis.All the fluid properties are assumed constant except that the influence of the density variation with temperature is considered only in the body force term.The magnetic field of small intensity H_0 is introduced in the y - direction.Since the fluid is slightly conducting,the magnetic Reynolds number is much less than unity and hence the induced magnetic field is neglected in comparison with the applied magnetic field following. The permeability of the porous medium is assumed to be of the form

$K'(t') = K_0'(1+\varepsilon e^{i\omega' t'})$ where K_0' is the mean permeability of the medium,ω'

is the frequency of the fluctuation, t ′is the time and ε (<< 1) is a constant quantity.

In the absence of any input electric field, the equations governing the flow under Boussineqs approx. are

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = g\beta\,(T - T_\infty) + g\beta^*\,(C^* - C_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \beta_1\left(\frac{\partial^3 u}{\partial t\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right) + \nu\, u / K'(t')$$

$$-\ \sigma\mu e^2 H_0^2 u / \rho \qquad (1)$$

$$\frac{\partial u}{\partial y} = 0 \qquad (2)$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = (K_T/\rho\, C_p)\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

$$\frac{\partial C^*}{\partial t} + v\frac{\partial C^*}{\partial y} = D\frac{\partial^2 C^*}{\partial y^2} \qquad (4)$$

Equation (2) yields

$$v = -\,v_0 \qquad (5)$$

where $v_0 > 0$ is a constant and the negative sign indicates that the suction is towards the plate.

The boundary conditions of the problem are :

$$\left.\begin{array}{l} y = 0\ :\ u = 0,\ T = T_w,\ C^* = C^* \\ y \to \infty : u = 0,\ T = T_\infty,\ C^* \to C^*_\infty \end{array}\right\} \qquad (6)$$

Introducing the following non-dimensional quantities :

$$\bar{y} = y\, v_0 / \nu \qquad \bar{t} = t\, v_0^2 / 4\nu$$

$$\bar{\omega} = 4\nu\,\omega / v_0^2 \quad \bar{u} = u / v_0 \quad \bar{\theta} = T - T_\infty / T_\omega - T_\infty$$

$$C = C^* - C_\infty / C_\omega - C_\infty \quad Pr = \mu C_p / \kappa$$

$$Gr = \nu g\,\beta\,(T_\omega - T_\infty) / v_0^3$$

$$K_0 = K_0' v_0^2 / \nu^2 \quad Sc = \nu / D$$

$$Gm = \nu g\,\beta^*\,(C_\omega - C_\infty) / v_0^3$$

$$M = \sigma\mu e^2 H_0^2 \nu / \rho v_0^2 \quad R_m = \beta_1 v_0^2 / \nu^2 \qquad (7)$$

using (7), (1), (3) and (4) reduces to (dropping the bars)

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = Gr\,\theta + Gm\,C + \frac{\partial^2 u}{\partial y^2} - R_m\left(\frac{1}{4}\frac{\partial^3 u}{\partial t\partial y^2} - \frac{\partial^3 u}{\partial y^3}\right) -$$

$$u / K_0 (1 + \varepsilon e^{i\omega t}) - M u \qquad (8)$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = 1 / Pr\,\frac{\partial^2 \theta}{\partial y^2} \qquad (9)$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = 1/Sc\ \frac{\partial^2 C}{\partial y^2} \tag{10}$$

where subscripts denotes the differentiation

The non-dimensional boundary conditions are :

$$\left.\begin{array}{ll} y=0 : & u=0,\ \theta=1,\ C=1 \\ y\to\infty : & u=0,\ \theta=0,\ C=0 \end{array}\right\} \tag{11}$$

SOLUTION OF THE PROBLEM

In the neighbourhood of the plate, we take solution of the form

$$u(y,t) = u_0(y)+\varepsilon e^{i\omega t} u_1(y) \tag{12}$$

$$\theta(y,t) = \theta_0(y)+\varepsilon e^{i\omega t} \theta_1(y) \tag{13}$$

$$C(y,t) = C_0(y)+\varepsilon e^{i\omega t} C_1(y) \tag{14}$$

Substituting (12),(13) and(14) into (8),(9) and(10), and comparing the harmonic and non-harmonic terms we obtain

$$-R_m u_0^{111} + u_0^{11} - u_0^{1} - L_1 u_0 = Gr\,\theta_0 + Gm\,C_0 \tag{15}$$

$$-R_m u_1^{111} - u_1^{11} + u_1^{1}(1+R_m i\omega/4) + u_1 L_2 = Gr\,\theta_1 + Gm\,C_1 - u_0/k_0 \tag{16}$$

$$\theta_0^{11} + Pr\,\theta_0^{1} = 0 \tag{17}$$

$$C_0^{11} + Sc\,C_0^{1} = 0 \tag{18}$$

$$\theta_1^{11} + Pr\,\theta_1^{1} - Pr\,i\omega/4\,\theta_1 = 0 \tag{19}$$

$$C_1^{11} + Sc\,C_1^{1} - Sc\,i\omega/4\,C_1 = 0 \tag{20}$$

where the primes denotes differentiation with respect to y.

Now the boundary conditions(11) reduces to

$$\left.\begin{array}{l} y=0 : u_0=u_1=0,\ \theta_0=1,\ \theta_1=0,\ C_0=1,\ C_1=0 \\ y\to\infty : u_0=u_1=0,\ \theta_0=\theta_1=0,\ C_0=C_1=0 \end{array}\right\} \tag{21}$$

The equations (15) and (16) are third order differential equations, due to presence of elasticity. Since the viscoelasticity coefficient R_m is very small,therefore u_0, u_1 is expanded using Beard Walter rule [21]

$$\left.\begin{array}{l} u_0 = u_{00} + R_m u_{01} \\ u_1 = u_{11} + R_m u_{12} \end{array}\right\} \tag{22}$$

Zeroth – Order of R_m :

$$u_{00}^{11} + u_{00}^{1} + u_{00} L_1 = Gr\,e^{-Pry} + Gm\,e^{-Scy} \tag{23}$$

$$u_{11}^{11} - u_{11}^{1} - u_{11} L_2 = -u_{00}/k_0 \tag{24}$$

First - Order of R_m :

$$u_{01}^{11} + u_{01}^{1} + u_{01} L = - u_{00}^{111} \quad (25)$$

$$u_{12}^{11} - u_{12}^{1} - u_{12} L_2 = - u_{11}^{111} + i\omega / 4u_{11}^{11} - u_{01} / k_0 \quad (26)$$

The corresponding boundary conditions are :

$$\left.\begin{array}{l} y=0 \ : \ u_{00} = u_{01} = u_{11} = u_{12} = 0 \\ \\ y \to \infty : \ u_{00} = u_{01} = u_{11} = u_{12} = 0 \end{array}\right\} \quad (27)$$

solving (23) to (26) under the boundary conditions (27) and substituting the obtained solution into(15) and (16).Using this in equation(12),(13)&(14),velocity field can be expressed in terms of the fluctuating parts as

$$u(y,t) = [L_0(e^{-m_1 y} - e^{-Pry} - e^{-Sc\,y}) - R_m (G_1(e^{-Pry} + e^{-Scy} - e^{-m_1 y}) - G_2\, y\, e^{-m_1 y})] + \varepsilon(\cos \omega t\, M_r - \sin \omega t M_i) \quad (28)$$

where $M_r + iM_i = u_1(y)$.

$$M_r = L_6\, e^{-ay} \sin by - e^{-ay}(L_9 \sin by + L_8 \cos by) + L_8\, e^{-Pry} + L_{81} e^{-Scy} + R_m [e^{-ay} (L_{40} \cos by + L_{41} \sin by) - y\, e^{-ay} (L_{53} \sin by - L_{54} \cos by) + L_{49} e^{-m_1 y} - L_{51}\, e^{-Pry} - L_{515}\, e^{-Scy}]$$

$$M_i = L_6\, e^{-ay} \cos by - e^{-ay} (L_9 \cos by - L_8 \sin by) - L_6\, e^{-m_1 y} + L_9\, e^{-Pry} + L_{91} e^{-Scy} + R_m [e^{-ay} (L_{41} \cos by - L_{40} \sin by) - y\, e^{-ay} (L_{53} \cos by + L_{54} \sin by) + (L_{50} + y\, L_{17}) e^{-m_1 y} - L_{52}\, e^{-Pry} + L_{521}\, e^{-Scy}]$$

$$\theta (y) = e^{-Pry} \quad (29)$$

$$C(y) = e^{-Scy} \quad (30)$$

Hence, the expression for the transient velocity,for $\omega t = \pi / 2$ is given by

$$u(y, \pi/2\omega) = u_0(y) - \varepsilon M_i \quad (31)$$

The expression for the skin -friction at the plate in terms of its amplitude and phase as

$$C_f = Z' / f v_0^2 = (\partial u / \partial y)_{y=0}$$

$$Z = L_0 (Pr + Sc - m_1) + R_m [G_1(m_1 - Pr - Sc) - G_2] + \varepsilon\, e^{i\omega t} (N_r + iN_i) \quad (32)$$

where

$$N_r = b L_6 + a L_8 - b L_9 - Pr L_8 - Sc L_{81} + R_m [-a L_{40} + b L_{41} + L_{54} - m_1 L_{49} + Pr L_{51} + Sc L_{515}]$$

$$N_i = -a L_6 + a L_9 + b L_8 + m_1 L_6 - Pr L_9 + Sc L_{91} + R_m [-a L_{41} - b L_{40} - L_{53} - m_1 L_{50} + L_{17} + Pr L_{52} + Sc L_{521}]$$

$$Z = L_0 (Pr + Sc - m_1) + R_m[G_1 (m_1 - Pr - Sc) - G_2] + \varepsilon\, |N| \cos (\omega t + \alpha) \quad (33)$$

$$|N| = \sqrt{(N_r^2 + N_i^2)} \qquad \tan \alpha = N_i / N_r$$

where G_1, G_2, a ,b , L_1 to L_{52} are constants , their expressions are not presented here for sake of brevity. $R_m = 0.05$ has been taken throughout the computations.

DISCUSSION AND CONCLUSION

Figure 1 depicts the velocity profile for cool plate .It is observe that with increase in frequency parameter ω, permeability parameter K_0, Grashoff number Gr and modified Grashoff number Gm .But decrease with increase in Magnetic parameter M,Prandtl number Pr and Schimdt number Sc.

Figure 2 depicts the velocity profile for hot plate .It is observe that velocity decreases with increase in frequency parameter ω, Schimdt number Sc ,and permeability parameter K_0 .But increases with increase in Grashoff number Gr,modified Grashoff number Gm,Magnetic parameterM,andPrandtl number Pr.

Figure 3 depicts the fluctuating part M_r of velocity against y .It is observe that it increases with increase in Grashoff number Gr and modified Grashoff number Gm.But,decreases with increase in frequency parameter ω, permeability parameter K_0, Magnetic parameter M , Prandtl number Pr and Schimdt number Sc.

Figure 4 depicts the fluctuating part M_i of velocity against y .It is observe that it increases with increase in Grashoff number Gr and modified Grashoff number Gm,Prandtl number Pr and Schimdt number Sc..But,decreases with increase in frequency parameter ω, permeability parameter K_0 and Magnetic parameter M .

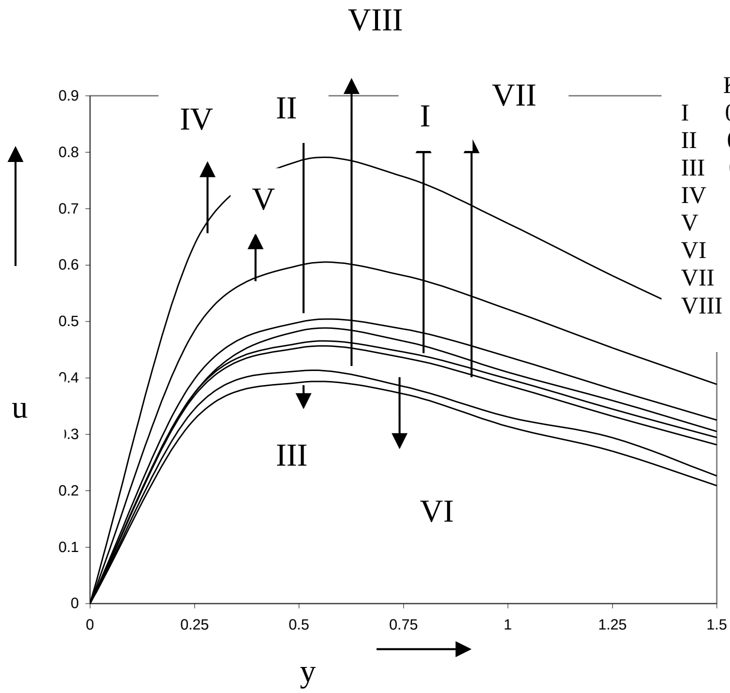
Figure 5 depicts the amplitude of the skin friction $|N|$ against frequency parameter ω.It is observe that amplitude of skin friction decreases with increase in permeability parameter K_0 ,Grashoff number Gr,modified Grashoff number Gm,.But increases with increase in Magnetic parameter M , Schimdt number Sc and Prandtl number Pr.

Table 1 , depicts that the skin friction coefficient .For cool plate skin friction coefficient increase with the increase in permeability parameter K_0 ,Grashoff number Gr ,modified Grashoff number Gm and frequency parameter ω, but decrease with increase in Schimdt number Sc, Magnetic parameter M and Prandtl number Pr. For hot plate skin friction coefficient increase with the increase in permeability parameter K_0 ,Grashoff number Gr ,modified Grashoff number Gm and Prandtl number Pr but decrease with increase in Schimdt number Sc, Magnetic parameter M and frequency parameter ω.

Table 1

Values of skin- friction coefficient at the plate when $\varepsilon = 0.2$

K_0	Gr	Gm	M	Pr	Sc	ω	C_f
0.3	5	2	5	0.71	0.60	4	-2.5240
0.4	5	2	5	0.71	0.60	4	-1.5021
0.3	5	2	7	0.71	0.60	4	-4.0544
0.3	10	2	5	0.71	0.60	4	-2.1699
0.3	5	4	5	0.71	0.60	4	-2.3559
0.3	5	2	5	0.71	0.60	8	-1.8651
0.3	5	2	5	0.71	0.70	4	-2.5374
0.3	5	2	5	1.0	0.60	4	-2.6962
0.3	-5	2	5	0.71	0.60	4	-3.2323
0.4	-5	2	5	0.71	0.60	4	-2.9620
0.3	-5	2	7	0.71	0.60	4	-4.5161
0.3	-10	2	5	0.71	0.60	4	-3.5865
0.3	-5	4	5	0.71	0.60	4	-3.0642
0.3	-5	2	5	0.71	0.60	8	-3.9140
0.3	-5	2	5	0.71	0.70	4	-3.2458
0.3	-5	2	5	1.0	0.60	4	-3.0601



	K_0	M	Gr	Gm	Pr	Sc	ω
I	0.3	5.0	5.0	2.0	0.71	0.60	4
II	0.4	5.0	5.0	2.0	0.71	0.60	4
III	0.3	7.0	5.0	2.0	0.71	0.60	4
IV	0.3	5.0	10.0	2.0	0.71	0.60	4
V	0.3	5.0	5.0	4.0	0.71	0.60	4
VI	0.3	5.0	5.0	2.0	1.0	0.60	4
VII	0.3	5.0	5.0	2.0	0.71	0.70	4
VIII	0.3	5.0	5.0	2.0	0.71	0.60	8

Figure 1 : Velocity distribution u against y for different values of different parameters for cool plate

	K_0	M	Gr	Gm	Pr	Sc	ω
I	0.3	5.0	-5.0	2.0	0.71	0.60	4
II	0.4	5.0	- 5.0	2.0	0.71	0.60	4
III	0.3	7.0	-5.0	2.0	0.71	0.60	4
IV	0.3	5.0	-10.0	2.0	0.71	0.60	4
V	0.3	5.0	- 5.0	4.0	0.71	0.60	4
VI	0.3	5.0	-5.0	2.0	1.0	0.60	4
VII	0.3	5.0	-5.0	2.0	0.71	0.70	4
VIII	0.3	5.0	- 5.0	2.0	0.71	0.60	8

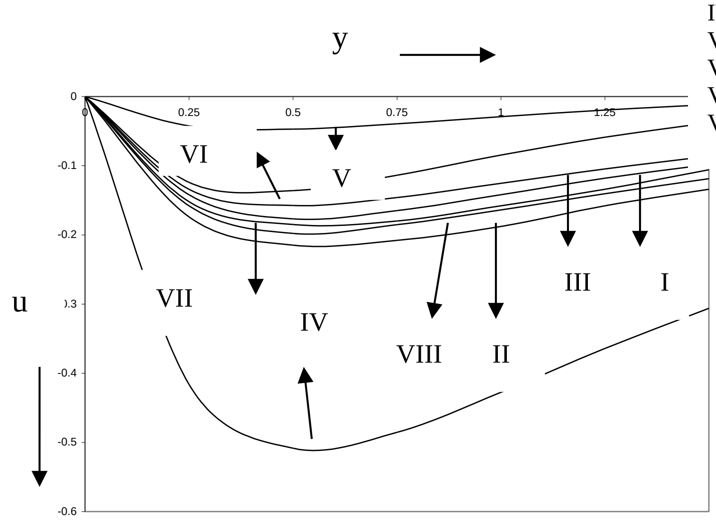
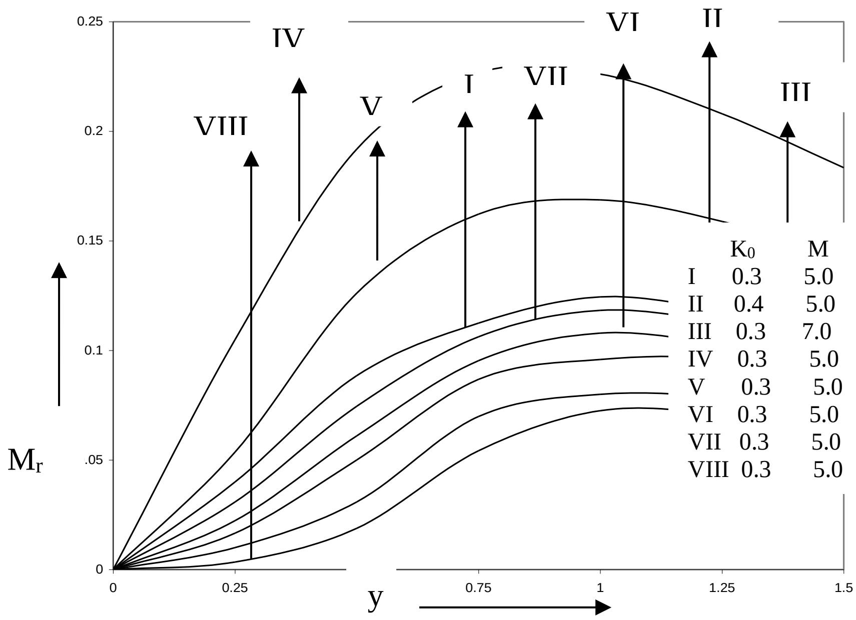
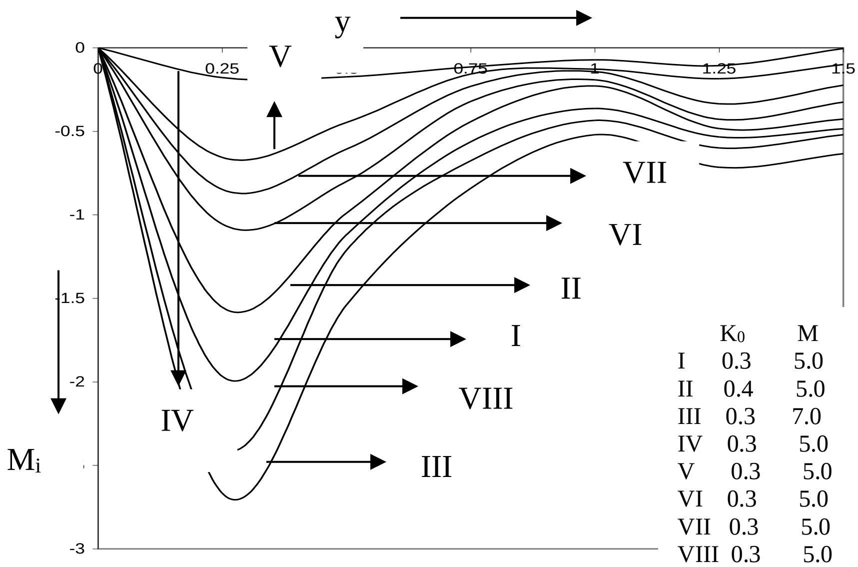


Figure 2 : Velocity distribution u against y for different values of different parameters for extremely hot plate



	K_0	M	Gr	Gm	Pr	Sc	ω
I	0.3	5.0	5.0	2.0	0.71	0.60	4
II	0.4	5.0	5.0	2.0	0.71	0.60	4
III	0.3	7.0	5.0	2.0	0.71	0.60	4
IV	0.3	5.0	10.0	2.0	0.71	0.60	4
V	0.3	5.0	5.0	4.0	0.71	0.60	4
VI	0.3	5.0	5.0	2.0	1.0	0.60	4
VII	0.3	5.0	5.0	2.0	0.71	0.70	4
VIII	0.3	5.0	5.0	2.0	0.71	0.60	8

Figure 3 : Fluctuating part of velocity M_r against y



	K_0	M	Gr	Gm	Pr	Sc	ω
I	0.3	5.0	5.0	2.0	0.71	0.60	4
II	0.4	5.0	5.0	2.0	0.71	0.60	4
III	0.3	7.0	5.0	2.0	0.71	0.60	4
IV	0.3	5.0	10.0	2.0	0.71	0.60	4
V	0.3	5.0	5.0	4.0	0.71	0.60	4
VI	0.3	5.0	5.0	2.0	1.0	0.60	4
VII	0.3	5.0	5.0	2.0	0.71	0.70	4
VIII	0.3	5.0	5.0	2.0	0.71	0.60	8

Figure 4 : Fluctuating part of velocity M_i against y

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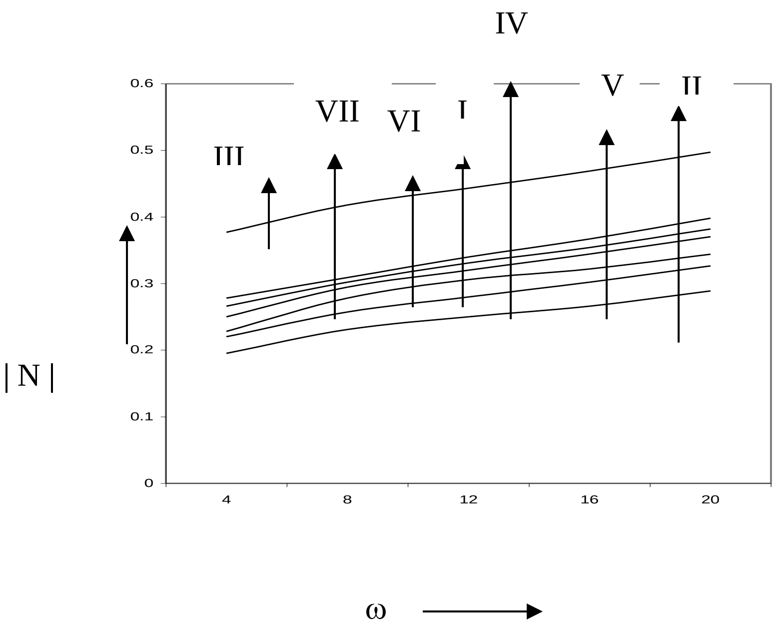

	K_0	M	Gr	Gm	Pr	Sc
I	0.3	5.0	5.0	2.0	0.71	0.60
II	0.4	5.0	5.0	2.0	0.71	0.60
III	0.3	7.0	5.0	2.0	0.71	0.60
IV	0.3	5.0	10.0	2.0	0.71	0.60
V	0.3	5.0	5.0	4.0	0.71	0.60
VI	0.3	5.0	5.0	2.0	1.0	0.60
VII	0.3	5.0	5.0	2.0	0.71	0.70

Figure 5 : Amplitude of the skin friction | N | against frequency parameter ω

Nomenclatures

u and v = the components of the velocity in the x and y direction respectively,

g = the acceleration due to gravity ,

β =the coefficient of volume expansion

β_1 = kinematic viscoelasticity,

ρ = density,

μ = viscosity ,

ν =kinematic viscosity ,

K_T =therma conductivity,

C_p = specific heat at constant pressure

t = time,

σ = electrical conductivity of thefluid,

μ_e = the magnetic permeability,

T_w & T_∞ = the temperature of the plate and temperature of the fluid far way from the plate

Gr = Grashoff Number,

Gm = modified Grashoff number,

Sc = Schimdt number,

Pr = Prandtl Number,

K_0 = Permeability Parameter,

M = Magnetic Parameter and

R_m = Magnetic Reynolds number

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