METHODS FOR SOLVING INTERNATIONAL MATHEMATICAL OLYMPIAD

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ABSTRACT

This article deals with the mathematical Olympiad. This article provides solutions to a number of issues. Wonderful issues were solved using a variety of non-standard methods.

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INTRODUCTION, RESULTS AND DISCUSSION

1. If $a+b+c+d=6$ and $a^2+b^2+c^2+d^2=12$
prove $36 \le 4(a^3+b^3+c^3+d^3)-(a^4+b^4+c^4+d^4) \le 48$
Method 1. $4(a^3+b^3+c^3+d^3)-(a^4+b^4+c^4+d^4)$

$$4(a^3+b^3+c^3+d^3)-(a^4+b^4+c^4+d^4)=-((a-1)^4+(b-1)^4)-$$
$$-((c-1)^4+(d-1)^4)+6(a^2+b^2+c^2+d^2)-4(a+b+c+d)+4.$$

We will now enter the following values and use the following:

$$a-1=x,\ b-1=y,\ c-1=z,\ d-1=t,\ 36 \le -(x^4+y^4+z^4+t^4)+52 \le 48.$$

Suffice it now to prove this inequality $16 \ge x^4+y^4+z^4+t^4 \ge 4$.

$$x^2+y^2+z^2+t^2=a^2+b^2+c^2+d^2-2(a+b+c+d)+4=4.$$

$$x^4+y^4+z^4+t^4 \ge \frac{(x^2+y^2+z^2+t^2)^2}{4}=4.$$

$4 \le x^4+y^4+z^4+t^4=(x^2+y^2+z^2+t^2)^2-A=16-A \le 16$, proof is over.

Method 2. $a+b+c+d=6 \Rightarrow a+b+c=6-d$.

$$a^2+b^2+c^2+d^2=12 \Rightarrow a^2+b^2+c^2=12-d^2.$$

We use the following inequality:

$$a^2+b^2+c^2 \ge \frac{(a+b+c)^2}{3} \Rightarrow 12-d^2 \ge \frac{(6-d)^2}{3}.$$

$36-3d^2 \ge 36-12d+d^2$, that $d \in [0;3]$.

We use this inequality: $x^2(x-2)^2 \ge 0$.

$x^2(x^2-4x+4) \ge 0$ va $4x^3-x^4 \le 4x^2$.

$$4(a^3+b^3+c^3+d^3)-(a^4+b^4+c^4+d^4) \le 4(a^2+b^2+c^2+d^2)=48$$

Now to prove the left side of the double inequality $(\mathbf{x}+\mathbf{1})(\mathbf{x}-\mathbf{1})^2(\mathbf{x}-\mathbf{3}) \le \mathbf{0}$,

$$\mathbf{4x^3-x^4 \ge 2x^2+4x-3}$$

$$\mathbf{4(a^3+b^3+c^3+d^3)-(a^4+b^4+c^4+d^4)=4a^3-a^4+4b^3-b^4+4c^3-c^4+4d^3-d^4\geq 2a^2+4a-3+2b^2+4b-3+2c^2+4c-3+2d^2+4d-3=2(a^2+b^2+c^2+d^2)+4(a+b+c+d)-12=24+24-12=36}$$

Method 3. The right side of the double inequality is written as follows:

$$4\left(a^3+b^3+c^3+d^3\right)\leq 48+\left(a^4+b^4+c^4+d^4\right).$$

$$a^2+b^2+c^2+d^2=12$$

$$48+\left(a^4+b^4+c^4+d^4\right)=\left(a^4+4a^2\right)+\left(b^4+4b^2\right)+\left(c^4+4c^2\right)+\left(d^4+4d^2\right)\geq$$

$\geq 4\left(a^3+b^3+c^3+d^3\right)$. The same is true of the left-hand side of the inequality.

2. $A_n=\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\ldots+\frac{1}{2}\sqrt{\frac{1}{2}}}}}$ A_n Express trigonometrically.

$$A_1=\sqrt{\frac{1}{2}}=\cos 45^0,\ A_2=\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}=\sqrt{\frac{1+\cos 45^0}{2}}=\cos\frac{45^0}{2},\ \ldots,$$

$$A_n=\cos\frac{45^0}{2^{n-1}}.$$

3. Calculate the sum. $6+66+666+\ldots+\underbrace{66\ldots6}_{n\ ta}$

$$6+66+666+\ldots+\underbrace{66\ldots6}_{n\ ta}=\frac{6}{9}\left(10^1-1+10^2-1+\ldots+10^n-1\right)=$$

$$=\frac{2}{3}\left(10+10^2+\ldots+10^n-n\right)=\frac{2}{3}\left(\frac{10\left(10^n-1\right)}{9}-n\right).$$

4. $A=\frac{1}{1\cdot2}+\frac{1}{3\cdot4}+\frac{1}{5\cdot6}+\ldots+\frac{1}{2019\cdot2020}$ and

$B=\frac{1}{1011\cdot2020}+\frac{1}{2012\cdot2019}+\ldots+\frac{1}{2020\cdot2011}$, $\frac{A}{B}=?$

$$A=\frac{1}{1\cdot2}+\frac{1}{3\cdot4}+\frac{1}{5\cdot6}+\ldots+\frac{1}{2019\cdot2020}=\frac{1}{1}-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{1}{2019}-\frac{1}{2020}=$$

$$=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{2019}+\frac{1}{2020}-2\cdot\left(\frac{1}{2}+\frac{1}{4}+\ldots+\frac{1}{2020}\right)=$$

$$=\frac{1}{1011}+\frac{1}{1012}+\ldots+\frac{1}{2020}$$

$$B=\frac{1}{1011\cdot2020}+\frac{1}{1012\cdot2019}+\ldots+\frac{1}{2020\cdot1011}=$$

$$=\frac{1}{3031}\left(\frac{1}{1011}+\frac{1}{2020}+\ldots+\frac{1}{2020}+\frac{1}{1011}\right)=\frac{2}{3031}\left(\frac{1}{1011}+\frac{1}{1012}+\ldots+\frac{1}{2020}\right)$$

.

so, $\frac{A}{B}=\frac{3031}{2}$.

5. $\frac{8a^2}{a^2+9}=b,\ \frac{10b^2}{b^2+16}=c,\ \frac{6c^2}{c^2+25}=a\ ,\ a+b+c=?$

In reverse order we have both elements of the equations:

$$\begin{cases}\frac{a^2+9}{8a^2}=\frac{1}{b}\\ \frac{b^2+16}{10b^2}=\frac{1}{c}\\ \frac{c^2+25}{6c^2}=\frac{1}{a}\end{cases}\Rightarrow\begin{cases}1+\frac{9}{a^2}=\frac{8}{b}\\ 1+\frac{16}{b^2}=\frac{10}{c}\\ 1+\frac{25}{c^2}=\frac{6}{a}\end{cases}$$

$$\left(1-\frac{3}{a}\right)^2+\left(1-\frac{4}{b}\right)^2+\left(1-\frac{5}{c}\right)^2=0.$$

$a+b+c=12.$

6. $\frac{log_3^5\ log_2^5}{log_3^5+log_2^5}=a\,log_b^c\ ,\ a+b+c=?$

$$\frac{log_3^5\ log_2^5}{log_3^5+log_2^5}=\frac{1}{\frac{1}{log_3^5}+\frac{1}{log_2^5}}=\frac{1}{log_5^3+log_5^2}=\frac{1}{log_5^6}=1\cdot log_6^5.$$

$a+b+c=12$

7. If $a^2+b^2+c^2+d^2=4$, prove $(a+2)(b+2)\geq cd$.

$$(a+2)(b+2)=ab+2(a+b)+4=ab+2(a+b)+\frac{a^2+b^2}{2}+\frac{c^2+d^2}{2}+2=$$

$$=\frac{(a+b)^2}{2}+2(a+b)+2+\frac{c^2+d^2}{2}=\frac{1}{2}(a+b+2)^2+\frac{c^2+d^2}{2}\geq cd.$$

8. If $3\sqrt{2+\sqrt{2+\sqrt{3}}}=a\cos\frac{\pi}{b}$, find a and b .

$$3\sqrt{2+\sqrt{2+\sqrt{3}}}=3\sqrt{2+\sqrt{2\left(1+\frac{\sqrt{3}}{2}\right)}}=3\sqrt{2+\sqrt{2(1+\cos30^0)}}=$$

$$=3\sqrt{2+\sqrt{4\cos^2 15^0}}=3\sqrt{2(1+\cos15^0)}=3\sqrt{4\cos^2 7{,}5^0}=6\cos7{,}5^0.$$

$a=6,\ b=24,\ a+b=30$

9. Calculate $(1+tg1^0)(1+tg2^0)...(1+tg44^0)$

We'll do the following:

$$1+tg44^0=1+\frac{tg45^0-tg1^0}{1+tg45^0\cdot tg1^0}=1+\frac{1-tg1^0}{1+tg1^0}=\frac{2}{1+tg1^0}$$, buni

$$1+tg23^0=\frac{2}{1+tg22^0}$$

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$$(1+tg1^0)(1+tg2^0)\ldots(1+tg22^0)\frac{2}{1+tg22^0}\cdots\frac{2}{1+tg2^0}\frac{2}{1+tg1^0}=2^{22}.$$

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