STUDY OF CEREBRAL BLOOD FLOW BY THE LUMPED PARAMETER MODEL TO PREDICT THE RUPTURE OF THE ARTERIAL WALL: APPLICATION TO STROKE

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ABSTRACT

The main purpose of this work is to control the evolution of lipid deposition size wall in order to prevent wall rupture, obstruction of cardiovascular circulation as well as the asphyxia of certain body members. The model of blood flow in the cerebral artery is described, using the lumped parameter method and taking into account the hematocrit and stenosis. The obtain model allows to understand non-invasively of the cardiovascular functionality and should deal with improve the blood circulation in the cerebral artery. Principal results in this paper showed that when the size of stenosis increases, the volume of hematocrit decreases. Also the radial growth of the stenosis is more critical than longitudinal growth and this radial growth reached faster rupture than the longitudinal. Finally, all the present results are in good agreement with the expected results of the literature.

INTRODUCTION

Cardiovascular disease is one of the major health problems in the Western world and the leading cause of death in the United States(Jonas Schollenberger 2015). According to the American Heart Association, stroke is the second leading cause of cardiovascular disease after coronary heart disease (Ghasemalizadeh et al 2014). A stroke refers to the event, in which certain brain functions are lost, because of an inadequate supply of oxygen to the brain tissue. Ischemia can be caused by either general hypoperfusion or blockage of the vessel due to thrombosis, arterial embolism or plaque buildup (Gragossian and Dacquel 2019). One of the main challenges of this procedure is to modeling the brain circulation toincrease important understanding of cerebral disease and to improve also clinical treatment (Pantoni 2010). Several theoretical models for the autoregulation of blood flow have been developed using a multicompartmental approach, in which blood is considered to flow through a number of compartments connected in series representing different types and sizes of vessels (Lawford and Hose 2011). Gao et al. (1998) developed a model for the cerebral circulation with four vascular regions, where the vessel size and response in each region were described by empirical equations based on in vivo experimental data (Carlson and Secomb2008). Although the model implicitly included all mechanisms acting in vivo, such as myogenic, shear-dependent, and metabolic responses, it did not allow analysis of the relative contributions of hematocrits and stenosis. In the model of (Cornelissen et al. 2002) for the coronary circulation, the myogenic response was included by assuming pressure-diameter relationships in vasoactive segments based on experimental observations on isolated segments, but recent progressed in imagining technologies as well as more powerful and less expensive computer, has open greater possibilities for developing more sophisticated modeling method of blood circulation. (Liao and Kuo 1997) considered the effects of only myogenic and shear-dependent responses in an

empirically based model. (Ursino and Lodi 1998 and Cornelissen et al 2002) employed pressure, flow and metabolic-dependent responses in their respective models of cerebral and skeletal muscle tissues; however, these three responses were not present in all of the resistance vessels. For example, in the latter model, vascular segments responded to either pressure and shear stress or metabolic state depending on their position in the vascular tree. (Spronck et al 2012) develop a physiologically based mathematical modelof cerebral blood flow regulation combining cerebral autoregulation and neurovascular coupling but, omit hematocrit and stenosis. This model is a 0D model that represents the main component of the system, is appropriate for the study of global distribution of pressure, flow rate and blood volume for specific physiological condition. So far, the 0D models are largely used in the clinical diagnosis and treatment with the collaboration of the medical community(Kokalari and Guerrisi2013).The stenosis of a 0D coronary model was presented by changing the parameter to mimic the reduction of cross-section area (insert Wang et al 1989). For the different models which have been investigated in literature, none of them did not considered the effect of hematocrit and stenosis.

In the present work, we are going to use the model of (Spronck et al 2012), by adding hematocrit and stenosis to better control the brain circulation. Furthermore, we shall analyze the partitioning effect of hematocrit in stenosis conduction. The proposed 0D model is based on Poiseuille's law and is able to reproduce the values reported in the literature for mean flow and mean pressure in the cerebrovascular system (Abdi et al 2014). Large cerebral arteries, including the proximal arteries are modeled via resistances, capacitor and inductance. This model of cerebral hemodynamics is complemented by a phenomenological model of self-regulation, which takes into account changes in cerebral blood flow (CBF) by adapting complacency and resistance to cerebral arteries (Liang and Takagi 2011). The objective of this work is to improve the model of Lumped parameter proposed by (Spronck et al 2012), including hematocrit control mechanisms and the presence of stenosis.

This 0D model simulates the transient behavior of the cerebrovascular system in response to a sudden change in hemodynamics due to change in hematocrit. Privatization of oxygen for several minutes can lead to irreversible damage to brain function (Kashif et al 2012). This process is mainly performed by remarkably deformable red blood cells, leading to a change in the rheological properties of vessel compliance and resistance, thus modifying the blood flow(Musielak2009). These vascular properties are adapted to the vascular smooth muscle located in the middle of the vessel wall. The results are valid and supported by the micro fluid experiments of the literature (Guibert et al 2010, pries et al 1989, Dellimore et al 1983).This result shows that the deformability of red blood cells is governed by several parameters such as membrane stiffness, reduction of traffic lumen and contrast of viscosity as we observe in figure (1).

In section 2, different materials and methods used for the cardiovascular circulation are presented. Section 3 is dedicated to numerical results and discussions. The conclusion is given in the last section.

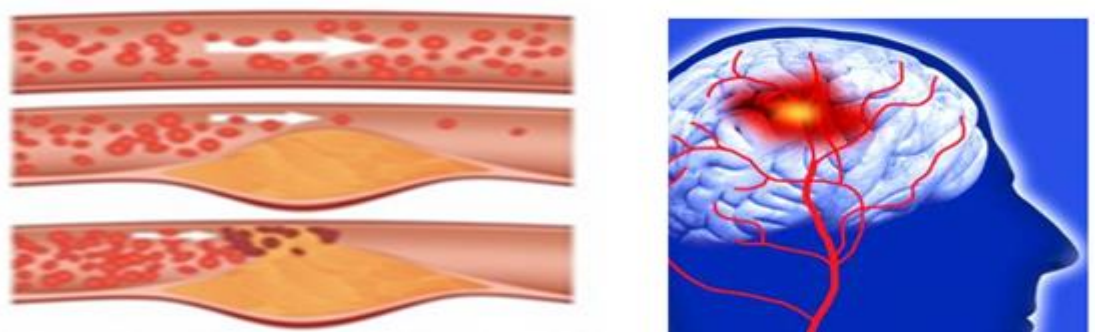


Figure (1): http://www.psychomedia.qc.ca/image/2016-10/37377-57967-image

II) MATERIALS AND METHODS

Model Definition

The modified Lumped parameter model (Spronck et al 2012) is used to study CBF regulation. A lumped parameter model was developed (Fig. 2), describing the segment of the posterior cerebral artery (PCA) and its distal vessels, up to the large cerebral veins. This segment was chosen since it supplies the primary visual cortex with blood (Aaslid R.1987). The model consists of three lumped parts: the PCA, the arteriolar circulation and microcirculation, and the venous circulation.

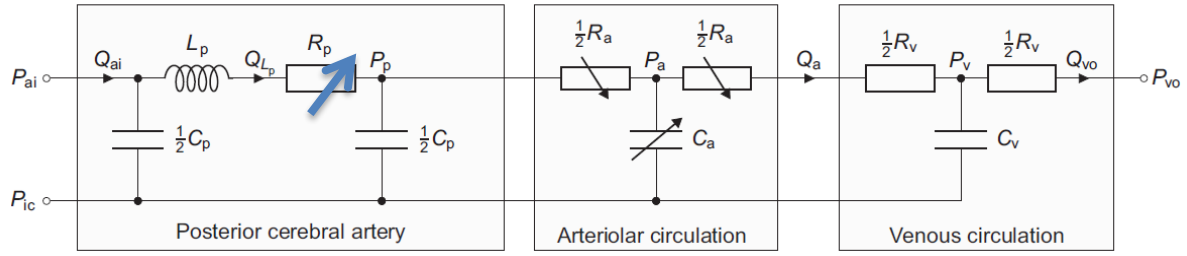


Figure (2): Lumped parameter model of the posterior cerebral artery (PCA) and its distal arteriolar and venous beds. P_{ai}, $P_{ic}$and P_{v0}, arterial entry, intracranial and venous exit blood pressure, respectively. P_P, output pressure of the PCA. $P_a$and P_v, central and venous blood pressure, respectively. Qai and QLp, PCA input and inertia flow, respectively. $Q_a$and Q_{v0}venous entry and exit streams, respectively. C_p, $L_p$andR_p, PCA compliance, inertia and resistance, respectively c_a and R_a, arteriolar compliance and resistance, respectively. c_v and R_v, venous compliance and resistance, respectively. Posterior cerebral artery (Spronck et al 2012)

II-1 MODELING OF THE CEREBRAL BLOOD FLOW

II-1-1 THE LAW OF POISEUILLE

The law of Poiseuille describes the pressure in an incompressible laminar flow passing through a cylindrical tube. The pressure P through a tube can be described as

$$\Delta P = R_p Q \qquad (1)$$

R_p is the resistance and Q the blood flow. This relationship is only valid for a perfectly developed and symmetric axial flow. The resistance stands for the viscous loss along the pipe and is calculated as follows:

$$R_p = \frac{8\mu L}{\pi r^4} \qquad (2)$$

Where μ is the dynamic viscosity of the blood and L, r are the length and radius of the vessel respectively. Due to the strong dependence of the resistance with the radius, a small change of the radius will significantly modified the resistance. This can be cause by the presence of the stenosis in the lumen of the artery.

II-2) MODELING OF STENOSIS

Fat accumulation, observed in the arterial stenosis, forms a rough and uneven surface, thus decreasing the effective diameter of the vessel. In a 0D representation of the vessel, a stenosis can be modeled with ad-hoc resistance. To calculate the resistance, the vessel is divided into two segments, one which is stenosis and the steroid. For simplicity, the radius of light in the stenosis portion is assumed to be of constant value along the stenosis. Figure (3) shows an idealized geometry of a stenosis.

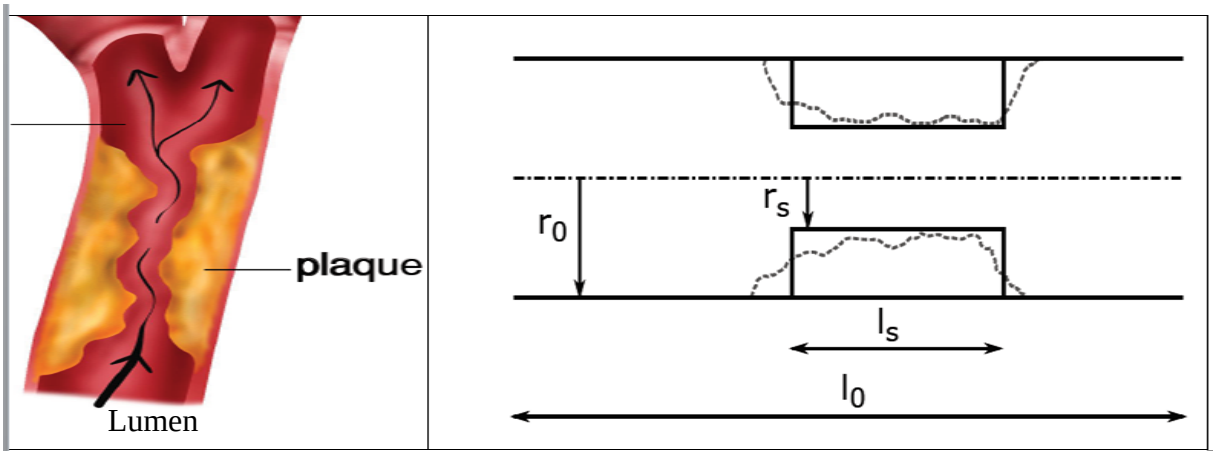


Figure (3): Sketch of arterial vessel segment with stenosis and 0 D-model approaches (Jonas Schollenberger, 2015).

The effective resistance of the vessel stenosis can be calculated as follows:

$$R_p=\frac{8\mu}{\pi r_0^4}(l_0 - l_s)+\frac{8\mu}{\pi r_s^4} l_s \qquad (3)$$

r_0 , l_0 represent the resistance and length of the healthy part of the vessel.
r_s, l_s the resistance and length of the stenosis part of the vessel.
Since our aim is to study the influence of hematocrit on the blood flow of the cerebral circulation, it will be advantageous to confine ourselves on the Fahraors-Lindqvist effect, which will allow us to express the resistance above according to the discharge hematocrit H_D and the vessel diameter D, defining the μ viscosity in vivo as (Prieset et al 1996).

$$\mu = \mu_p[1+ (\mu_1^{0.45} - 1) \frac{(1-H_D^C)-1}{(1-0.45)^C-1}*(\frac{D}{D-1.1})^2]*(\frac{D}{D-1.1})^2 \qquad (4)$$

μ_pIs the viscosity of the initial plasma 9* 10^{-6}mmHgS (Yand and Wang 2013).
$\mu_1^{0.45}$ Is the viscosity of the blood for a hematocrit flow of 0.45
With

$$\mu_1^{0.45}=6 \exp(-0.085D) +3.2-2.44\exp(-0.006D^{0.0645}) \qquad (5)$$

$$C= (0.8+\exp (-0.075D))*(-1+\frac{1}{1+10^{-11}D^{12}}) + \frac{1}{1+10^{-11}D^{12}} \qquad (6)$$

II-3) ARTERY BRAIN POSTERIOR

The vessel compliance (C_p)

The PCA is modeled by a line model. Full PCA compliance (C_p) is considered using a capacitor defined as

$$C_p = \frac{3\pi r_p^2(\frac{r_p}{h_p}+1)^2}{E_p(\frac{r_p}{h_p}+1)} l_p \qquad (7)$$

Where r_p,E_p,h_p and l_p are in respect artery inner radius, Elasticity module, thickness of artery and PAC length

For simulation purposes, it should be noted that the passage of blood through vessels, should expanded or contracted them, so they can keep the blood or release it and this is exactly the behavior of a capacitor.

The blood inertia (L_p)

The blood inertia (L_p) is simulated by inductors: $L_p=\frac{\rho_b}{\pi r_p^2} l_P$ (8)

Where ρ_b,l_P and r_p are in respect blood density, PAC length and artery inner radius.

Reason of this consideration is variability of flow acceleration in pulsatile blood flow, so an inductor can model inertia of blood flow very clearly.

II-4) ARTERIOLAR CIRCULATIONS

The arteriolar resistance and capacity are described by Laplace's law as in (Ursino and Lodi1998). The total tension (T_a) of the arteriolar wall vessel is the sum of the elastic T_{ae}, muscular T_{am} and viscous T_{av} tension given by the following expression:

$T_a=T_{ae}+T_{am}+T_{av}$ (9)

Where $T_{ae}=S_{ae0}(\exp(K_{as}\frac{r_a-r_{a0}}{r_{a0}}-1)-S_{ac}).h_a$ (10)

And muscle tension is defined by:

$T_{am}=T_{amax}\exp(-(\frac{r_a-r_{am}}{r_{at}-r_{am}})^{nam})$ (11)

the viscous tension is giving by:

$T_{va}=\frac{\eta_a dr_a}{r_{a0}\,dt} h_a$ (12)

Where h_a is the thickness of the arteriolar wall, it expression is given according to CBF, it regulated by adjusting T_{amax} in the equation (11) according to this expression

$T_{amax}=T_{amax0}(1+M_s)$ (13)

Where M_s is a measure of smooth muscle activation, which ranges [-1; 1] representing respectively minimum and maximum vasodilatation of constriction.

II-5) STATE EQUATIONS OF LUMPED PARAMETER MODEL

The states equations of Lumped parameter and it parameters are given in this section

II-5-1) ARTERY BRAIN POSTERIOR

The PCA model involves three equations of states:

$\frac{dP_{ai}}{dt}=\frac{2(Q_{ai}-Q_{lp})}{C_p}$ (14)

$\frac{dQ_{lp}}{dt}=\frac{P_{ai}-P_p-Q_{lp}R_p}{L_p}$ (15)

$\frac{dP_p}{dt}=\frac{2(Q_{lp}R_a-P_p+2P_a)}{R_aC_p}$ (16)

Where P_{ai} ,Q_{ai},Q_{lp} C_p,P_p,R_p,R_a and P_a are respectively arterial input, PCA inflow , inertia Flow , compliance, PCA output pressure, PCA resistance, arteriolar resistance and central arteriolar pressure. Although arteriolar complacency is described as a normal capacity in the figure (2), it is written indirectly by Laplace's law, consequently

$\frac{dr_a}{dt}$ Instead $\frac{dP_a}{dt}$ is used in the state equation

$\frac{dr_a}{dt}=\frac{r_{a0}(P_p(R_a+R_v)r_a+P_vR_ar_a-(T_{ae}+T_{am}+P_{ic}(r_a+h_a))(2R_a+R_v)}{R_{a0}r_a^2k_{ar}R_a(R_a+R_v)+\eta_a h_a(2R_a+R_v)}$ (17)

Where R_v, P_v,k_{ar},$\eta_a$and h_a stand respectively for venous resistance, venous pressure, Constant parameter for elastic stress model, wall viscosity and PCA thickness.

II-5-2) VENOUS CIRCULATIONS

Venous circulation is represented by a single equation of state

$$\frac{dP_v}{dt}=\frac{2}{C_v}\left[\frac{P_a-P_v}{R_a-R_v}-\frac{P_v-P_{v0}}{R_v}\right] \quad (18)$$

Where C_v,P_{v0} are venous compliance and Venous output pressure.

II-5-3) DETERMINATIONS OF PARAMETERS

Table1. ***Constant lumped parameter model*** **Symbol, Value, Unit, Description and Reference**

Symbol	value	Unit	Description	Reference
ρ_b	1.05	g/ml	Blood mass density	**Alastruey et al** 2007
E_p	1.6	MPa	Young's modulus of PCA wall	**Mulder** et al 2011
P_{v0}	14	mmHg	venous output pressure	**Ursino and M, Lodi CA 1998**
P_{ic}	10	mmHg	Intracranial pressure	**Mulder** et al 2011
Q_{bl}	53	ml/min	Baseline flow	**Moore 2007**
η_a	47.8	mmHg·	Wall viscosity	**Ursino and M, Lodi CA 1998**
R_p	0.105	cm	PCA inner radius	**Alastruey et al** 2007, **Mulder** et al 2011
L_p	8.6	cm	PCA length	**Mulder** et al 2011
h_p	0.026	cm	PCA thickness	**Alastruey et al** 2007
r_{a0}	0.0075	cm	Vessel inner radius in the condition of unstressed wall	**Ursino and M, Lodi CA 1998**
h_{a0}	0.0025	cm	Wall thickness in the unstressed condition *r*a0	**Ursino and M, Lodi CA 1998**
S_{ae0}	11.19	mmHg	Constant parameter for elastic stress model	**Ursino and M, Lodi CA 1998**
S_{ae0}	11.19	mmHg	Constant parameter for elastic stress model	**Ursino and M, Lodi CA 1998**
K_a	4.5	—	Constant parameter for elastic stress model	(**Ursino and M, Lodi CA 1998**)
S_{ac}	41.32	mmHg	Constant parameter for elastic stress model	**Ursino and M, Lodi CA 1998**
T_{max0}	1.50	mmHg·cm	Optimal smooth muscle tension in basal condition	**Ursino and M, Lodi CA 1998**
r_{am}	0.0128	cm	Radius at which smooth muscle exerts maximal force	(**Ursino and M, Lodi CA 1998**)

r_{at}	0.0174	cm	Constant parameter for smooth muscle tension model	(**Ursino and M, Lodi CA 1998**)
*n*am	1.75	—	Constant parameter for smooth muscle tension model	**Ursino and M, Lodi CA 1998**

III) RESULTS AND DISCUSSIONS

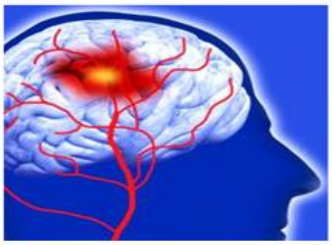
Equations (14, 15, 16, 17 and 18) were integrated numerically using the classical fourth order Runge-kutta algorithm. The integration was performed using the above variables and constants. This was to evaluated the effect of the variation of Hematocrits and stenosis in blood pressure ($\boldsymbol{P_{ai}}$), PCA blood flow ($\boldsymbol{Q_{lp}}$)and the inertial blood flow $\boldsymbol{P_p}$ in order to understand cerebral blood flow and to improve clinical diagnostic.

III-1-1) Variation of PCA blood pressure ($\boldsymbol{P_{ai}}$) as a function of time with variation of hematocrit

Figure (III-1) describes the evolution of Blood pressureP_{ai} as a function of time. The strong line curve (without-stenosis) outlined the sinusoidal evolution of blood pressure (P_{ai}), with different variation of hematocrit. This oscillation is cause by PCA compliance (C_p) in equation (14). It should be noted that the circulation of blood blood through the vessels causes them to expand or shrink, depending on the blood pressure. As such they can keep blood or release it and this is exactly like what a capacitor does (Mirzaee et al 2008). A decrease in the amplitude of oscillation as a function of time is observed. The progressive increase of the friction causes a decrease of the amplitude with each go and return, the oscillations are damped, and the movement is not periodical strictly speaking. This can be reflected by the wall strength due to the presence of hematocrit that increases viscosity of the blood (Shung et al 1992). We also notice that when the hematocrit increases the amplitude of the oscillation reduces due to later reason.

The dashed lines (---) in figure (III-1) represent the evolution of P_{ai} in a stenosis medium
We noticed that in the presence of stenosis with the reduction of 40% of the lumen, the pressure increases brutally, this can be explained by the fact that, the change of radius (r) leads to a great modification of the resistance of the wall R_p (Stoicescuer al 2018). In fact when the wall is blocked, the traffic lumen becomes small, this represents the microcirculation effect (Dellimore et al 1983, Fenton et al 1985, Guibert et al 2010 and pries et al 1989) .This phenomenon has physiological consequences because it modifies the transport of oxygen and other essential metabolites and can even trigger pathology disorders.

We also noticed that, the pressure becomes constant after 90 second, that is when the intensity of the friction force exceeds a critical value (or when the resistance is very large), that is why there is no oscillation. This explained the circulation of blood out of the artery (stroke hemorrhage) figure (4) this is caused by the rupture of the artery. We also see the big difference of hematocrit that increases the pressure (Shadiow et al 2019). A high hematocrit may be a sign of hydration, characterized by the loss of water in the blood plasma to the arterial lumen, which leads to a sudden rise in pressure (Muravyov et al 2018). Here we see that when the hematocrit increases the contribution of hydrodynamic interaction between the red blood cells becomes stronger; it causes an P_{ai} elevated over time as in Pries et al 2010.



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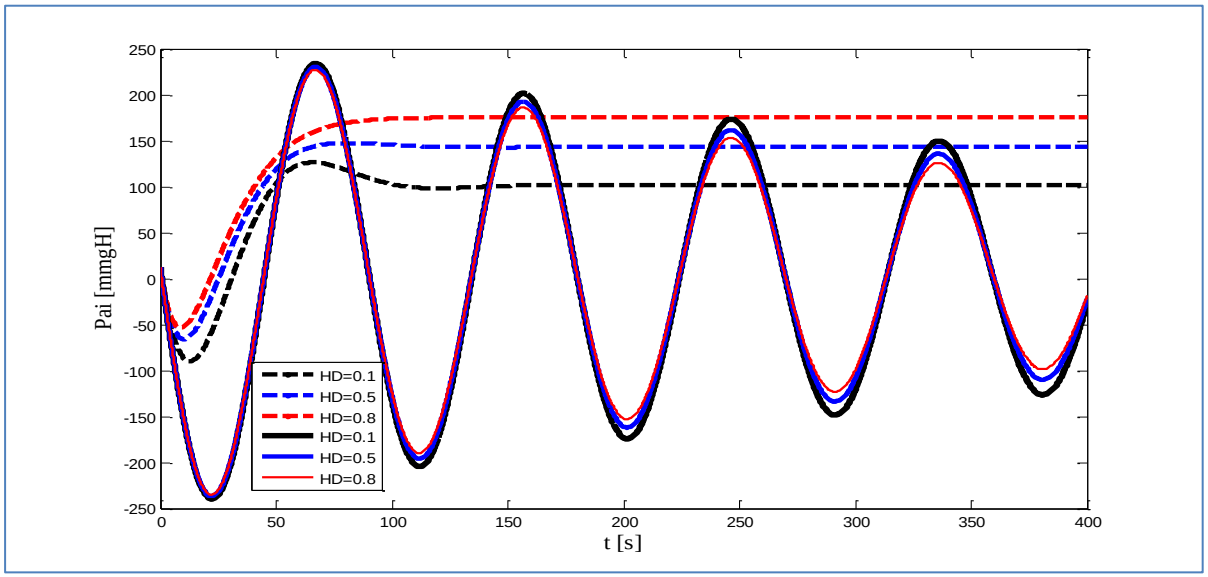

Figure (III-1): Variation of P_{ai} as a function of time with variation of hematocrit. The dashed lines (---) represent the variation of hematocrit with fixed stenosis of 40% of the lumen. The pressure flow increase brutally with the effect of hematocrit, and with 0% of stenosis (strong line) the effect of hematocrit is less.

III-1-2) Variation of PCA blood flow (Q_{lp}) and the inertial blood flow P_p as a function of time with variation of hematocrit

Figure (III-2) and Figure (III-3) show the variation of Q_{lp} and P_p respectively as a function of time. The curves in strong lines (-) show the variation in a clean pipe (without stenosis). One observes a decrease of the amplitude of oscillation which can be caused by the resistance of the wall. Similarly the dotted curves (---) represent Q_{lp} and P_p as a function of time in stenosis conduction. These figures illustrate a sudden change in the mechanical behavior of the circulation, which favors a change in the rheological properties and can often lead to diseases such as Hematologic disorders (Fedoso et al 2011). This understanding of changes in blood flow can improve and lead to new applications of biomedical technology. The difference between the two phenomena is characterized by the presence of stenosis which contributes in the reduction of the diameter of the wall, in fact a small change of the radius r leads to large change of the resistant R_p.

We also see that the increasing of hematocrit reduce the PCA blood flow (Q_{lp}) and the inertial blood flow P_p

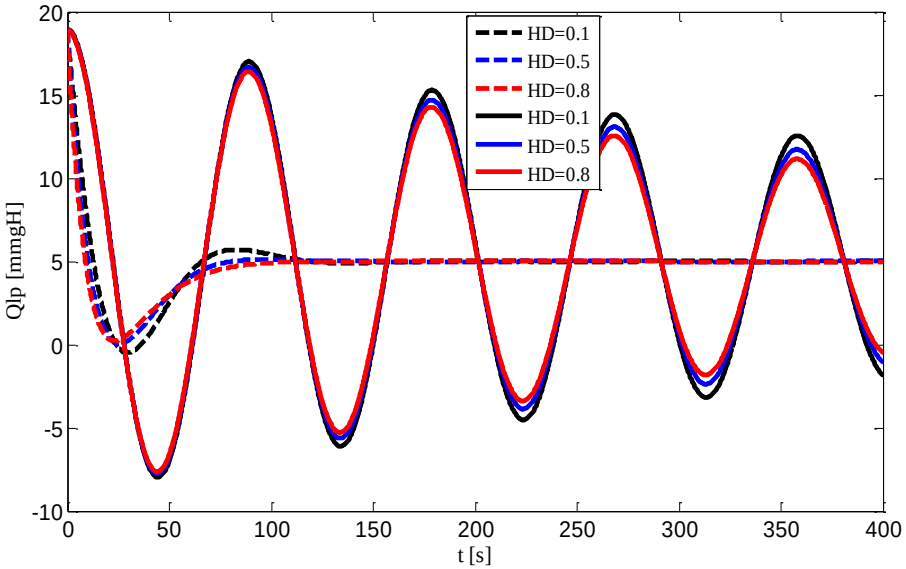

Figure (III-2): Influence of Hematocrit in stenosis and without stenosis. The dashed lines (---) represent the variation of hematocrit with fixed stenosis of 40% of the lumen. PCA blood flow (Q_{lp}) change brutally with the effect of stenosis

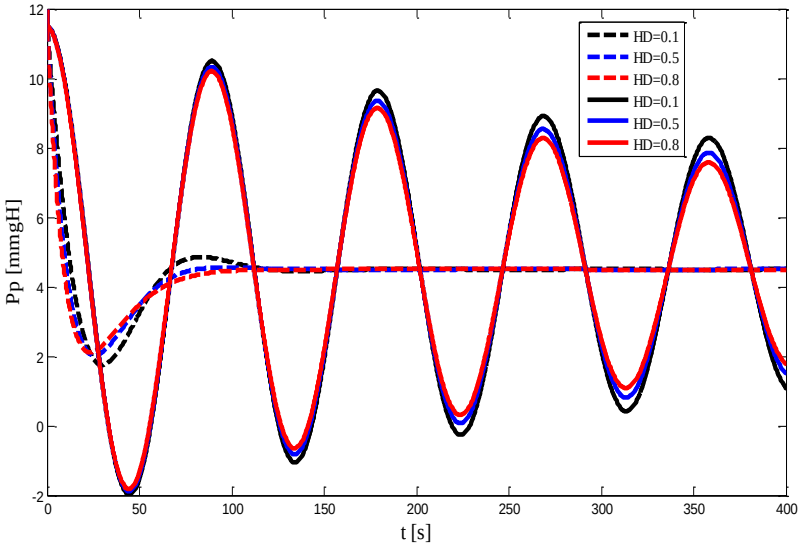

Figure (III-3): Influence of Hematocrit in Stenosis and without Stenosis. The dashed lines (---) represent the variation of hematocrit with fixed stenosis of 40% of the lumen. Inertial blood flow P_p change brutally with the effect of stenosis.

III-2) INFLUENCE OF YOUNG'S MODULUS

Figures (III-4) show the influence of Young's modulus E_P for a time less than 100s. One saw that when increasing E_p, a sudden change in pressure is observed. This is due to the increase E_p which increases the rigidity of the arterial wall .Thus this rigidity causes a rise in PCA blood pressureP_{ai} (Dnna et al 2000, Kashif et al 2012). Over 100s the variation E_p did not influence, this is due to the lesion of the wall which causes the circulation of blood outside the artery as shown in figure (4) (cerebral hemorrhage).

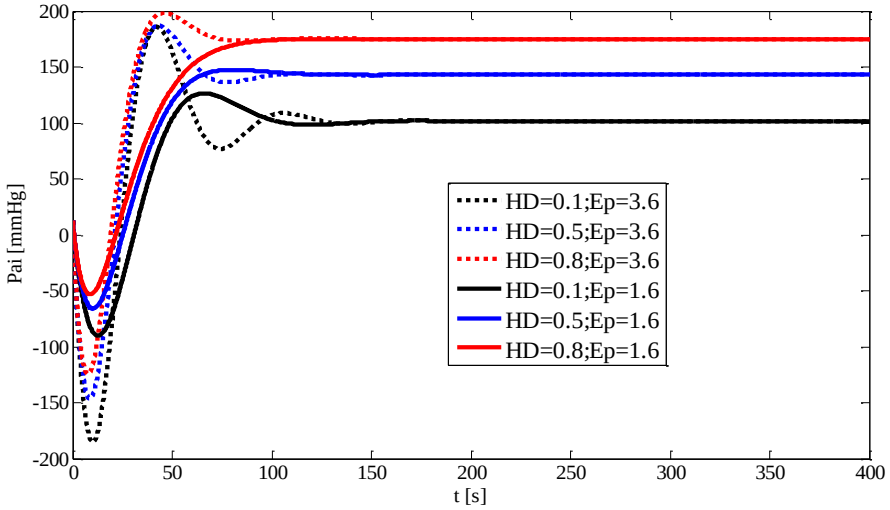

Figure (III-4): Influence of Young's modulus in PCA blood pressure P_{ai}as a function of time

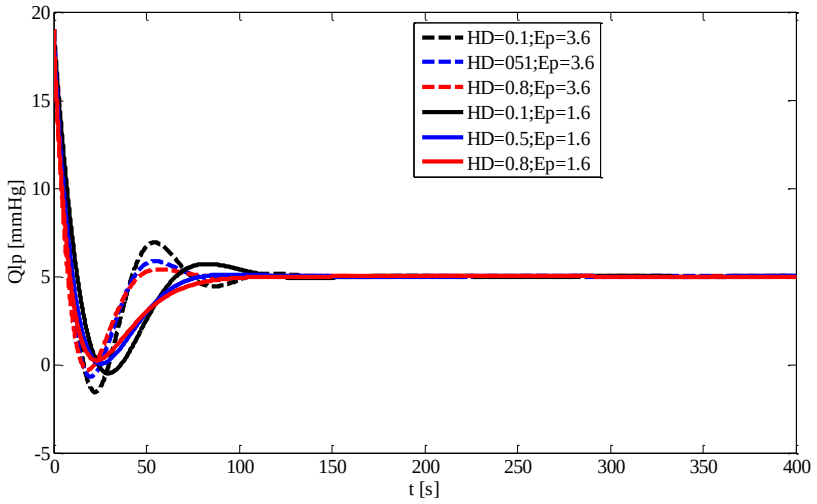

Figure (III-5): Influence of Young's modulus in PCA blood flow (Q_{lp}) as a function of time

III-3)Study of the influence of the longitudinal spread of stenosis on the wall

In figure (III-6), we have fixed the radius $r_s$of the obstacle at 16% of initial radius r_0 and we have varied the length of the obstacle $l_s$to 5%, 60% and 90% of the initial l_0. It is observed that when the length of the stenosis increases longitudinally, the amplitude of the oscillation gradually decreases (Westerhof et al 2019). This shows that the oscillation of pressure decreases proportionally with the resistance offered by the stenosis, which is consistent with the literature (Prakah et al 2015).

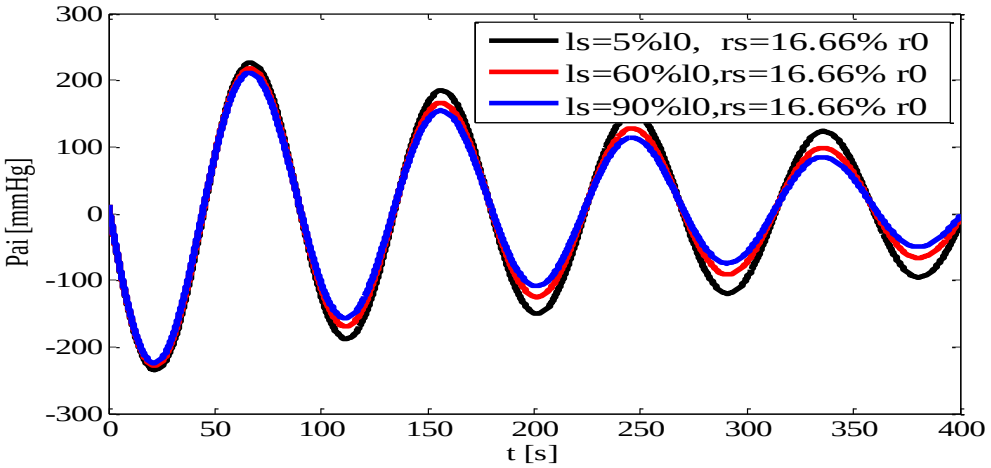

Figure (III-6): Influence of the variation of longitudinal stenosis with fixed radial 16% of the reduction of light P_{ai}as a function of time

III-3)Influence of Atherosclerosis

Atherosclerosis is the pathology of the large and medium-sized arteries, caused by accumulation of stenosis and accompanied by changes in the media (Surabhiet al 2018). The lesions develop in the arterial vessels, preferentially at the site of the stenosis where the laminar flow of the blood is disturbed (decrease of the forces of shear and increase of turbulence) see Figure (III-7), Figure (III-8), Figure (III-9) and Figure (III-10) where the curves are abruptly constant, meaning that there is a rupture leading to hemorrhage, which is consistent with the literature of (Prakah et al 2015). The evolution of this pathology is a slow process, often related to the mode of feeding, and its frequency increases with age or time. Figure (III-7) and Figure (III-10) show that, in the presence of a high stenosis, the pressure gradient-flow relationship becomes much steeper, with a higher pressure gradient for any given flow (Surabhi et al 2018).

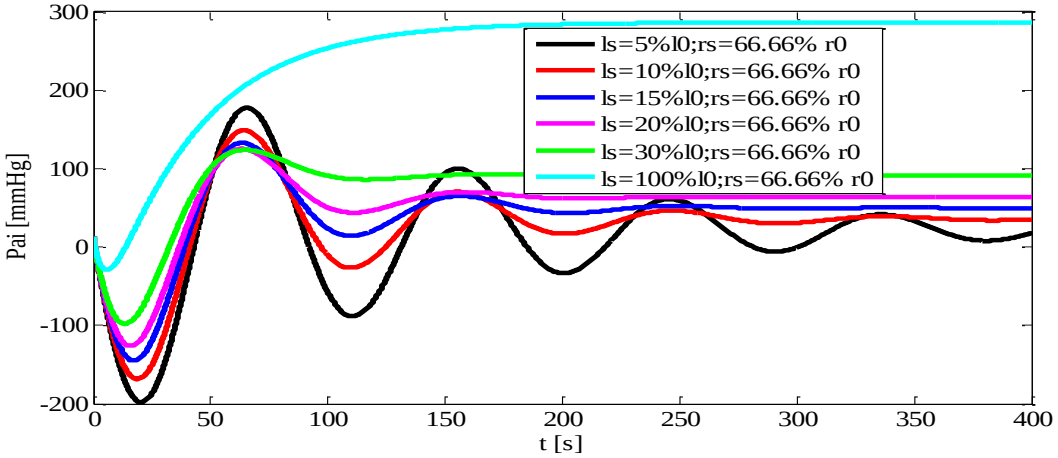

Figure (III-7): Longitudinal fat distribution with 66.66% arterial light reduction:

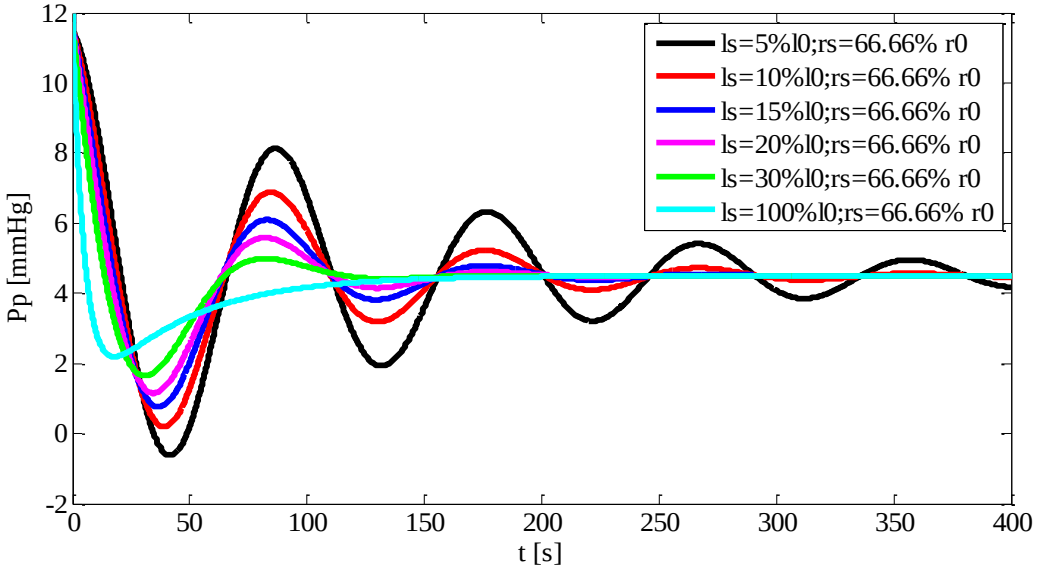

Figure (III-8): Longitudinal Fat Distribution with 66.66% Arterial lumen Reduction

III-4) Study of the influence of the radial spreading of stenosis on the artery wall

In figure (III-9) and figure (III-10), we have fixed the length $l_s$of the obstacle at 20% of initial radius l_0 and we have varied the size of the obstacle $r_s$at 16.66%, 33.34%, 66.67 and 83.34% of the initial length l_0 ,it is observed that when the length of the stenoses increases radicaly, the amplitude of the oscillation gradually decreases, it shows that, the inertial blood flow decreases rapidly due to the blockage of the lumen (Liu and Tang 2019).

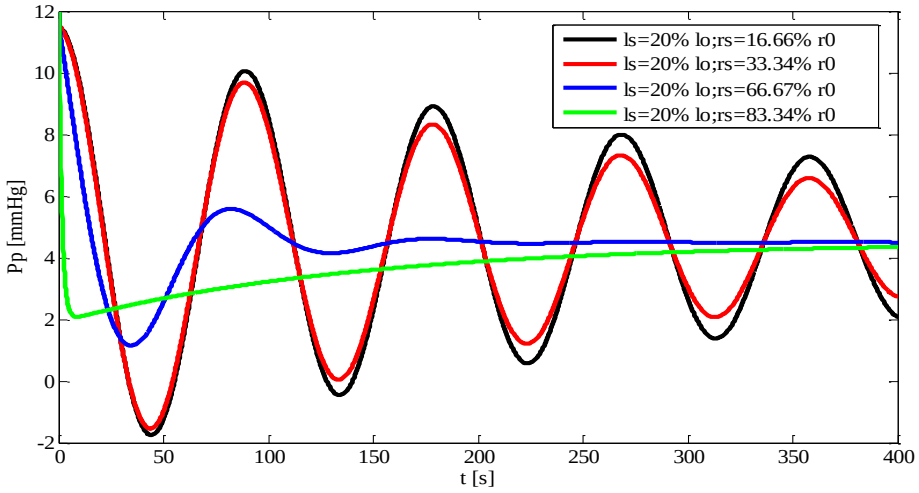

Figure (III-9) Radial distribution of grease with longitudinal spread at 20%:

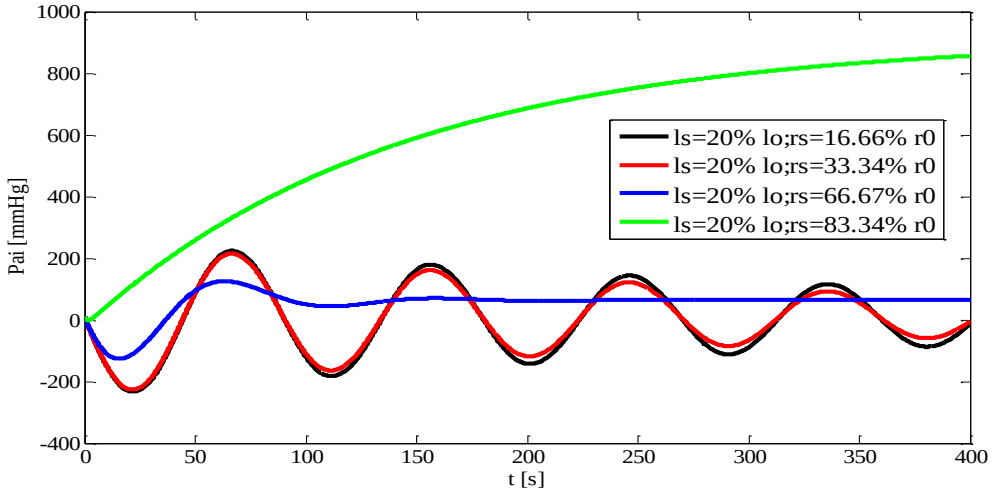

Figure (III-10): Radial distribution of fat with a longitudinal spread at 20%

CONCLUSION

A lumped parameter model associated to the stenosis conduction and hematocrit volume was proposed for the cerebral flow analysis. This considered model is useful to understand radial and longitudinal load distribution of the stenosis on the wall and its influence on the cerebral circulation. Knowledge of radial growth predicts arterial wall rupture and other ricks in cardiovascular systems. The results of this work could help in the early diagnosis of stroke by numerical simulation of noninvasive experimental data.

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