RESEARCH ON TEACHING STRATEGY OF "LOGARITHMIC CONCEPT" FROM HPM PERSPECTIVE

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ABSTRACT

The history of mathematics is an important component of mathematical culture, and its value is of great significance for cultivating students' core mathematical literacy. Therefore, it is necessary to integrate the history of mathematics into mathematics teaching. However, many teachers have limited understanding of how to reasonably and appropriately introduce the history of mathematics into mathematics teaching. In order to address this situation, this article takes the concept of "logarithms" as an example from the perspective of the History of Mathematics and Mathematics Education (HPM), integrates its development history into teaching, and selects appropriate ways to combine the two to carry out teaching design, providing reference suggestions for some teachers who have this problem.

Keywords: HPM; Logarithmic; Mathematics history; teaching design.

INTRODUCTION

(1) Analysis Of Curriculum Standards

The General Senior High School Curriculum Standards (2017 Edition, revised in 2020) point out that in the process of cultivating students' core mathematical literacy, it is necessary to pay attention to the infiltration of mathematical culture, guide students to actively understand the development history of mathematics, stimulate students' interest in learning mathematics, and thereby contribute to the cultivation of core mathematical literacy. The new curriculum standards also require teachers to lead students to understand the essence of mathematics through teaching and establish the connection between new knowledge and old knowledge. Mathematics teachers should respect students' cognitive laws and the disciplinary characteristics of mathematics, slow down the pace, increase experiences, and let mathematics learning occur naturally.(1)In addition, regarding the study of logarithms, the curriculum standards have added that students should search for relevant materials by themselves to understand the significant role that the concept of logarithms has played in human production and life. This helps students deeply understand the concept of logarithms, stimulates their interest and awareness of exploration, and enhances their core mathematical literacy.

(2) Introduction To HPM

At the Second International Congress on Mathematics Education held in Exeter, UK in 1972, Jones from the United States and Rogers from the United Kingdom organized the

International Research Group on the Relationship between the History of Mathematics and Mathematics Teaching (abbreviated as HPM). Since the launch of this congress, it has marked the vigorous development of the academic field of the history of mathematics and mathematics education in various regions.(2)Nowadays, HPM refers to the perspective of studying the relationship between the history of mathematics and mathematics education or integrating the history of mathematics into mathematics teaching, no longer referring to the research groups at that time. The current theoretical exploration of HPM aims to analyze the "why" and "how" issues in the current field, in order to promote the continuous enrichment and improvement of the theoretical and conceptual framework in this field. The research on the HPM topic at the 15th International Congress on Mathematics Education (abbreviated as ICME-15) mainly focuses on the following aspects: theoretical exploration, teaching practice, professional development of teachers, research on the history of mathematics oriented towards education, research on the history of mathematics education, and the history of HPM.(3)Professor Wang Xiaoqin, in light of the specific circumstances both at home and abroad as well as China's national conditions, pointed out four ways to integrate the history of mathematics into mathematics teaching: the addition approach, the replication approach, the adaptation approach, and the reconstruction approach. Specifically, the add-on approach is to attach the history of mathematics as an independent module outside the regular teaching content. Replication means directly restoring historical mathematical problems or methods, allowing students to "retrace the path of the ancients". Adaptation is to moderately adapt historical materials to make them conform to modern teaching logic and students' cognitive levels. The reconstructive approach is to reconstruct the teaching content based on the context of the history of mathematics and integrate the occurrence and development process of knowledge into the teaching design. At present, most mathematics teachers' application of HPM is too superficial. Most of them only introduce it in the form of stories. Although it has stimulated students' interest in learning to a certain extent, the value of the history of mathematics has not been greatly developed. Therefore, from the perspective of HPM, this paper reproduces the calendar invented by logarithms.

DISCUSSION

The following is the teaching strategies design of the concept of logarithms.

(1) Teaching Content Analysis

This section covers the concept of logarithms and the inverse relationship between logarithms and exponents. Before this, the concept and basic properties of exponential functions were learned, which laid the groundwork for the transition to this lesson. This section studies the relationship between the independent variable and the dependent variable of exponential and logarithmic functions without learning the inverse function. Meanwhile, logarithm, as a commonly used mathematical model, has wide applications in solving social life examples. In addition, by understanding the exploration process of mathematicians, one can feel the mathematical ideas from the special to the general, and develop the core literacy of mathematical abstraction, logical reasoning and mathematical operations.

(2) Analysis Of Students' Learning Situation

Before this class, students have already learned the concept and basic properties of exponential functions. Through the previous learning, students have learned from logarithms to logarithmic functions, mastered the research methods of basic elementary functions, experienced the transition from the specific to the general, and from the concrete to the abstract, and have gained a preliminary understanding of functions. However, for students, logarithms, as a new symbol, have certain difficulties in understanding the concept. They may have a retreating mentality when learning new concepts. Therefore, teachers should appropriately control the teaching pace and set up a reasonable teaching process.

(3) Teaching Objectives Design

Understand the concept of logarithms, be aware of the intrinsic connection between exponents and logarithms, be capable of converting exponential and logarithmic expressions, and develop mathematical abstraction literacy. Go through the invention process of logarithms, transform integer operations into logarithmic operations, and develop the literacy of mathematical operations. Feel the important role of logarithmic simplification operations and the necessity of logarithms, and develop logical reasoning literacy.

(4) Key And Difficult Points Of Teaching

Key points of teaching: The concept of logarithms, the mutual transformation between exponential expressions and logarithmic expressions, and the development history of logarithms.

Teaching difficulty: The concept and idea of logarithms.

(5) Teaching Process Design Of "The Concept Of Logarithms"

Part One: Create a Situation and Introduce new knowledge.

Question 1: Please calculate the following equations: (No calculator is used)

- 256×1024
- $8192 \div 128$

Teacher-student activity: Students are doing calculations. The data is too large, and direct calculation is too cumbersome. Design intention: Based on the theory of historical similarity, it is predicted in advance that students will be unable to calculate due to excessive data, allowing them to experience the wisdom of their predecessors firsthand. This, in turn, guides students to independently explore simple calculation methods and cultivate their innovative thinking. [Context of the History of Mathematics] China's space industry is booming. On April 24, 2025, the Shenzhou-20 manned spacecraft was successfully launched from the Jiuquan Satellite Launch Center. However, for a long time, humans were unable to detect the distances between planets in the universe. The teacher presented a fragment of the manuscript of the 16th-century astronomer Tycho Brahe in the PPT, pointing out that the astronomical observation data at that time was huge and involved a large number of complex multiplication and division operations (such as the calculation of the orbital periods of planets). Given that the speed of light in a vacuum is 299,792,468 m/s and the total number of seconds in a year is 31,536,000 seconds, please try to calculate the value of a light-year, students. Teacher-student activity: The teacher leads the students to recall the journey of their predecessors and introduces the topic."The Concept of logarithms". Design intention: By applying the replication method from the perspective of HPM, directly adopting the problems encountered by historical astronomers, delving into the background of astronomy and applying the theory of re-creation, students can understand the origin of logarithms, stimulate their interest in exploration, and lead to the subsequent learning.

Section Two: Think and explore to form concepts.

Question 2: Can you calculate the results of 256×1024 and 8192÷128 based on the information in the following table? What patterns can be discovered through the process of calculation?

x	1	2	3	4	5	6	7	8	9	10
y	2	4	8	16	32	64	128	256	512	1024

x	11	12	13	14	15	16	17	18	...
y	2048	4096	8192	16384	32768	65536	131072	262144	...

Student's assumption: Through the table, it is found that 256 is the 8th power of 2 and 1024 is the 10th power of 2. Adding their exponents equals 18, and it is calculated that 256×1024 equals 262144. According to the table, 262144 is precisely the 18th power of 2. Similarly, 8192 is the 13th power of 2 and 128 is the 7th power of 2. Subtracting their exponents equals 6. The calculation shows that 8192÷128 equals 64. According to the table, 64 is precisely the 6th power of 2.
Question 3: Can this conclusion be drawn based on the discovery just made?
Student's assumption: To calculate the product of two complex numbers, one can find the exponent of 2 corresponding to these two numbers in the table, calculate the exponent according to the corresponding operation rules to obtain the result, and then find the power value corresponding to this result in the table. The power value is the required value.
Follow-up question: This method can facilitate our calculation. Could you summarize how we simplified it?
Students' preset: Transform the multiplication operation of complex numbers into the addition operation of exponential expressions, and the division operation of complex numbers into the subtraction operation of exponential expressions, so that the operations are downgraded.
Design intention: By applying the adaptive expression from the HPM perspective, using Steffel's story of the double sequence as a carrier, mathematical problems are created and students are asked questions. Through problem-driven approach, students are encouraged to continuously explore and think, transforming large number operations into exponential operations, experiencing the essence of simplification, and reproducing the process of knowledge generation in the context of creating the history of mathematics. Develop students' mathematical operation and logical reasoning qualities.
Question 4: Can the above method be used to calculate 299792468×31536000?
The student assumes: Neither of these two numbers is a positive integer power of 2, so it is impossible to calculate based on the above table.
Follow-up question: Then how to calculate? Since the numbers are infinite, we cannot list all the numbers in the form of a table.
Design intention: To restore the problem of Stifel's table, trigger students' exploration, reveal the reason for the creation of logarithms, and lay the groundwork for subsequent learning. Inspired by Stifel, the mathematician Napier created a similar table, published the "Wonderful Logarithmic Table Description", and for the first time proposed the logarithmic principle, namely "logarithm", which was later translated as "logarithm" by the Ming Dynasty mathematician Xue Fengzuo. Napier chose a relatively complex number and then carried out

the above operation.

If the base of a power is not 2 but N, that is,N= a^x ,then how is the exponent expressed? How did we solve similar problems we encountered before?

Student's assumption: By introducing a new symbol "√"

[Teacher] After Napier invented logarithms, predecessors introduced the symbol "log" by analogy with the square root symbol, thereby giving rise to the concept of logarithms.

Generally, if the power of x of a is equal to N (a is greater than 0 and a is not equal to 0), then the number x is called the logarithm of N with a as the base, denoted as, where a is the base of the logarithm and N is the true number.

Design intention: By applying the reconstructive formula of HPM theory, the process of Pinar creating logarithms is reconstructed. Through problem-driven approaches, students' doubts and thoughts are triggered, and the concept of logarithms is derived. This enables students to better understand the origin of logarithms, integrates mathematical culture into it, and stimulates students' interest in learning.

Section 3: Understand the Essence and Apply What You have Learned.

Question 6: Do true numbers have a range of values?

Students assume that the domain of an exponential function is from 0 to positive infinity. According to the transformation relationship between exponents and logarithms, it can be concluded that the value range of a true number N is N greater than 0, while negative numbers and 0 have no logarithms.

The design intention is to cultivate students' habit of classified discussion by discussing the value range of true numbers, laying a foundation for the subsequent learning of the mutual transformation between exponents and logarithms.

After Napier's logarithmic table emerged, his friend Briggs was deeply inspired and further modified the logarithm, replacing it with a logarithmic table based on 10. Since we are accustomed to using the decimal system, this kind of logarithmic table thus has certain advantages. Play "A Brief History of Logarithms", and take the logarithm with base 10 as the common logarithm, denoted as $\lg N$, and the logarithm with base e as the natural logarithm, denoted as $\ln N$.

Example: Calculate the following logarithms

$\log_2 8 \quad \log_{10} 1 \quad \log_a 1 \quad \log_a a$ (a≠0,also,a≠1)

The student assumes: After calculation, it is found that the logarithm of 1 with base a being equal to 0 and the logarithm of a with base a being equal to 1 are identities. Design intention: By applying the addition formula from the HPM perspective, students can understand the common forms of logarithms. Combined with the history of mathematics to impart knowledge, students can well understand and deeply appreciate the interchange relationship between exponents and logarithms. Develop students' mathematical abstraction, logical reasoning and mathematical operation literacy.

Section Four: Reflection and Recall to Enhance Understanding \ nQuestion 7: Please recall what you mainly learned in this class, students? Student's assumption: Simplify complex multiplication and division operations into addition and subtraction operations. The design intention is to guide students to recall what they have learned in this class and deepen their understanding of the content of this class.

Section Five: Assign homework, consolidate and improve the exercises completed after class, and work in groups to search for the specific development process of logarithms. The design intention is to enhance students' teamwork and division of labor abilities in groups, which is conducive to knowledge integration, consolidation of learned content and deepening of understanding.

(6) Reflection on Instructional Design

From the perspective of HPM, instructional design significantly enhances students' understanding of the essence of mathematics through historical reconstruction, the infiltration of mathematical ideas and the integration of cultural values. Through the predicaments of astronomical calculations and the stories of mathematicians, one can intuitively feel the necessity of logarithmic development and stimulate the desire to explore. According to the construction process of students' knowledge, appropriate ways should be selected to integrate mathematical historical materials, adopting the methods of addition, replication, adaptation and reconstruction. Firstly, by adopting a replicative teaching approach, the problems that occurred in history were restored, guiding students to conduct exploration. Then, through an adaptive approach, mathematical problems were compiled in combination with historical materials. Driven by problems, the invention process of logarithms was advanced. Then, through the reconstructive method, the discovery process of Napier and Briggs' knowledge is reconstructed, allowing students to experience the invention process of logarithms firsthand. Finally, the introduction of common logarithms and natural logarithms is an addition. The relevant historical data on logarithms are derived from the search of related materials, which is in line with the scientific nature of the teaching design. Throughout the teaching process, choosing appropriate places to incorporate appropriate mathematical historical materials is conducive to enhancing the interest and cultural nature of teaching. Teaching design based on the HPM perspective is an important manifestation of the new curriculum reform. It enables students to deeply understand and appreciate the process of knowledge occurrence, understand why they are learning, find interest in it, and develop students' core literacy in mathematics.

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